

# A Multivariate Hit-or-Miss Transform for Conjoint Spatial and Spectral Template Matching

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**Abstract.** The Hit-or-Miss transform is a well-known morphological operator for template matching in binary and grey-level images. However it cannot be used straightforward in multivalued images (such as colour or multispectral images) since Mathematical Morphology needs an ordering relation which is not trivial on multivalued spaces. Moreover, existing definitions of the Hit-Or-Miss Transform in grey-level use only spatial templates (or structuring elements) which could be insufficient for some feature extraction problems. In this paper, we propose a multivariate Hit-or-Miss Transform operator which combines spatial and spectral patterns to perform template matching. We illustrate its relevance with an application in the remote sensing field, the extraction of coastline from very high (spatial) resolution images.

**Keywords:** Mathematical Morphology, Hit-or-Miss Transform, multivalued image, template matching, remote sensing.

## 1 Introduction

Nowadays, image sensors produce multivalued images in a wide set of applications. So the need for reliable operators adapted to multivalued data become more and more critical, in order to deal with various problems such as object detection, segmentation, classification, filtering, etc. Among these problems, object detection and recognition are of primary importance in many domains such as remote sensing, colour video processing, astronomy, and can be solved using template matching. An efficient template matching operator will help to automatically process huge collections of images. Mathematical Morphology (MM) offers several tools for image processing, including a template matching operator called the *Hit-or-Miss Transform* (HMT). This operator is effective on binary and grey-level images but has not been defined on multivalued images (such as colour or multispectral images) yet. So the contribution of this paper is to propose an adequate solution for morphological template matching in multivalued images, by defining an original multivariate hit-or-miss operator.

In this paper, we first recall main existing definitions of the Hit-Or-Miss Transform for monovalued images and discuss their possible extension to multivalued images. We believe that none of these existing approaches seems to be reliable to be extended to multivalued images in case of problems where both spatial and spectral knowledge have to be taken into account. So we introduce a multivariate Hit-or-Miss Transform relying on a set of structuring elements combining spatial and spectral descriptions. We finally illustrate the relevance of the proposed operator with the problem of coastline extraction in remote sensing.

Let us introduce notations used in this article.  $D \subset \mathbb{N}^2$  is the spatial domain of images, a grey-level image is represented by a function  $f : D \rightarrow \mathbb{N}$ , while a multivalued image is noted  $F : D \rightarrow \mathbb{N}^d$ ,  $F_b$  being a multivalued image band (i.e. a grey-level image  $f$ ),  $f^-$  and  $f^+$  are respectively the lowest and highest bounds of the pixel value range in  $f$ . Thus  $f(p)$  represents a scalar value in case of grey-level image while  $F(p)$  denotes a vectorial value in case of multivalued image. Reflection is denoted  $\check{f}$  and defined by  $\check{f}(p) = f(-p)$ . Duality is denoted  $f^*$  and defined by  $f^* = -\check{f}$ . Support of a function is defined by  $\text{supp}(f) = \{p \in D | f(p) > -\infty\}$ .

## 2 Monovalued Hit-or-Miss Transform

In this section, we briefly recall basic foundations of MM before reviewing existing definitions of the Hit-or-Miss Transform. Finally we explain how these definitions can be applied to multivalued images.

### 2.1 Foundations of Mathematical Morphology

Mathematical morphology is a theoretical framework introduced 40 years ago by G. Matheron and J. Serra [1] to compute quantitative description of geometrical structures through a spatial-based analysis. It is based on two operators called *erosion* ( $\varepsilon$ ) and *dilation* ( $\delta$ ) respectively defined as :

$$\varepsilon_g(f)(p) = \inf_{y \in \text{supp}(g)} \{f(p + y) - g(y)\} \tag{1}$$

$$\delta_g(f)(p) = \sup_{y \in \text{supp}(g)} \{f(p - y) + g(y)\} \tag{2}$$

A pattern  $g$  called Structuring Element (SE) is involved in most of the morphological operators. We consider here the general case of functional SE, where  $g : D \rightarrow \mathbb{N}$ . Let us state that flat structuring elements which are used most often in MM can be expressed as particular cases of functional SEs where values of all SE's points are set to the additive identity of the value space (most often zero).

These two operators can be combined to build most of other MM operators, such as *opening* and *closing*. MM also offers a template matching operator called *Hit-or-Miss Transform* which has been defined for binary[1] and grey-level images[2].

## 2.2 Grey-Level Hit-or-Miss Transform

The Hit-or-Miss Transform has been first defined for binary images. In such images, this operator[1] is quite trivial and uses two disjoint structuring elements: the first has to match the foreground while the second has to match the background. Both matches are necessary in order the operator to give a positive matching response. Extension of the HMT operator to grey-level images led to several definitions which have been reviewed and unified by Naegel *et al.*[2]. The two main approaches are:

**Soille :**

$$HMT_{g,h}^{Soille}(f)(p) = \max\{\varepsilon_g(f)(p) - \delta_h^*(f)(p), 0\} \quad (3)$$

with  $g, h$  flat structuring elements.

**Ronse :**

$$HMT_{g,h}^{Ronse}(f)(p) = \begin{cases} \varepsilon_g(f)(p) & \text{if } \varepsilon_g(f)(p) \geq \delta_h^*(f)(p) \\ 0 & \text{otherwise} \end{cases} \quad (4)$$

with  $g, h$  functional structuring elements.

Ronse and Soille approaches are both composed of two steps for each pixel: first a *fitting* to check if the given pixel matches the pattern defined by the SE, and second a *valuation* to give to the fitted pixel a value in the resulting image. Fitting of these methods ensures the translation invariance property. Thus they focus on the difference between the results of the erosion and dilation. This is particularly relevant in the general case, but as soon as some template knowledge on spectral minimal or maximal values is available, the interest of these methods decreases. Moreover, matching non-uniform features composed of several parts with different spectral or intensity behaviour (e.g. plane with white fuselage, black wings parked on a grey airway) will be impossible because of the fixed number of SEs (i.e. 2).

## 2.3 Extension to Multivalued Images

In order to extend Soille and Ronse definitions to multivalued images, lattice theory has to be considered to formulate MM operators, thus requiring an ordering relation between pixels values. Contrary to grey-level images where pixel scalar values can be easily ordered using the single natural order, the case of multivalued is complex since various possible vectoring ordering schemes are available, none of them being widely accepted. The reader will find more details on vectorial ordering for multivariate MM in the recent survey from Aptoula and Lefèvre[3]. Two main options can be followed:

**Marginal strategy :** it consists in processing each band separately and independently from the others. Despite its ease of use, its disadvantages are the loss of correlated information between bands (thus the spectral signature of the pixels) and the possible lack of vector preservation (values which were not in the

input image may appear in the resulting image, thus resulting in new spectral signatures).

**Vectorial strategy** : it uses a vectorial ordering, such as one of the following: *conditional orderings* which give priority to some particular bands, *reduced orderings* which reduce vectors to scalar values and then order these values or *partial orderings* which cluster vectors into groups of equivalence with an ordering relation between groups. This strategy preserves vectors and use the correlated information between bands, but its main default is the need for the chosen vectorial ordering to be meaningful for the problem under consideration.

These two strategies can be used to extend grey-level HMT definitions to multivalued images. However, in the frequent case where template matching should be performed using spectral properties (thus needing to keep the correlated information) but where no meaningful vectorial ordering scheme seems adapted to the problem to be solved, both strategies may be irrelevant. Another way to perform morphological template matching in multivalued images is thus necessary.

### 3 Multivariate Hit-or-Miss Transform

We introduce here a new definition of the Hit-or-Miss Transform for multivalued images, thus resulting in a *Multivariate Hit-or-Miss Transform* (MHMT). We also see how this operator can be used to deal with grey-level images and point out its advantages over other existing HMTs.

#### 3.1 Definition

The proposed MHMT is not based on marginal or vectorial strategies explained in section 2.3. Indeed a new strategy is necessary to avoid the limits of these two strategies and to correctly take into account spectral properties without selecting a (possibly inadequate) vectorial ordering. Each band is processed independently like in the marginal strategy, but with a particular SE (while the same SE is used for all bands in marginal strategy). Here each SE is dedicated to a given band (multiple SEs can be related to the same band), thus the behaviour of the HMT is specific to each band. Moreover, the HMT will return a positive result if all the SE have been correctly matched. The spectral properties are considered through the merging of the results obtained for each SE. This strategy does also not need a vectorial ordering which is particularly relevant when no vectorial ordering seems to be meaningful for the problem to be solved. Compared to the two other strategies, here the result image is not multivalued but will contain scalar values, which can be easily processed by the end-user (e.g. display, thresholding, etc).

Unlike the grey-level HMT operators presented in section 2.2, the MHMT can involve more than two SEs. It allows complex templates with very different parts (e.g. an object with red, blue, green and yellow parts) to be matched. To facilitate the understanding of the proposed HMT, we consider here flat SEs associated with threshold values instead of functional SEs. However, our transform can be directly expressed with functional SEs. Since we involve threshold values,

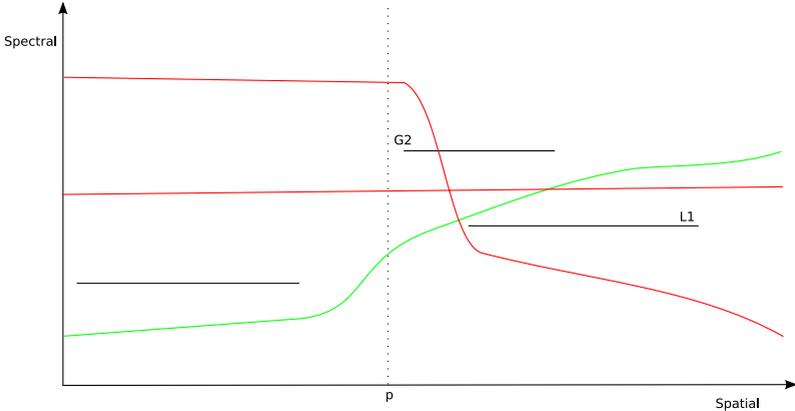
not necessarily a couple of SEs (foreground/background), and multiple spectral bands, we define extended SEs with:

- *shape* (sh) : the specific shape of the SE defined on the spatial domain,
- *band* (b) : the spectral band of the image assigned to the SE,
- *threshold* (th) : a threshold defined in  $\mathbb{N}$  (which may be avoided by using functional SEs),
- *type* (ty) : the type of the SE, either a greater bound (G) to be processed with a dilation or a lower bound (L) to be processed with an erosion.

The proposed model of extended SE includes spatial information like classical SE but also spectral information. SEs are defined to match a specific pattern, so parameters can be set from user problem knowledge or automatically. SEs are then used as follows for extraction of predefined templates.

First, we perform a *fitting* which will determine for each pixel if it matches the pattern defined by the set of SEs. Then, we perform a *valuation* which will assign a resulting value to each fitted pixel.

Fitting consists in checking for each pixel and for each SE if all pixel neighbours match the SE (not only spatially). If SE type is *greater* (respectively *lower*), SE threshold must be greater (resp. lower) than all neighbouring pixel (the neighbourhood being defined by the SE shape) value for the considered spectral band, as illustrated in Fig. 1.



**Fig. 1.** Fitting example for a pixel  $p$  with a set of 3 linear SEs related to the same spectral band but with various shapes. We can observe that only the green curve is fitted and the red curves are not (G1 and G2 are greater SEs, L1 is a lower SE).

The fitting is formally defined for a given SE  $s$  as:

$$\text{Fitting}_s(F)(p) = \begin{cases} \varepsilon_{s_{sh}}(F_{s_b})(p) \geq s_{th} & \text{if } s_{ty} = L \\ \delta_{s_{sh}}(F_{s_b})(p) \leq s_{th} & \text{if } s_{ty} = G \end{cases} \quad (5)$$

To be valued, a pixel must match all the SEs, i.e. global fitting is defined as follows:

$$\text{Fitting}_S(F)(p) = \bigcap_{s_i \in S} \text{Fitting}_{s_i}(F)(p) \tag{6}$$

where  $S$  is the set of cardinality  $|S|$  of SEs defined by  $S = \{s_i\}$ .

Valuation consists in assigning a scalar value to all pixels. For non-fitted pixels, valuation process assigns the value 0 (similar to other HMTs, see section 2.2). For fitted pixels, the valuation is more complex and defined for each SE  $s$  by the following formula:

$$\text{Valuation}_s(F)(p) = \begin{cases} \frac{\varepsilon_{s_{sh}}(F_{s_b})(p) - s_{th}}{F_{s_b}^+ - s_{th}} & \text{if } s_{ty} = L \\ \frac{\delta_{s_{sh}}(F_{s_b})(p) - s_{th}}{F_{s_b}^- - s_{th}} & \text{if } s_{ty} = G \end{cases} \tag{7}$$

Global valuation is then defined as follows :

$$\text{Valuation}_S(F)(p) = \begin{cases} \frac{1}{|S|} \sum_{s_i \in S} \text{Valuation}_{s_i}(F)(p) & \text{if } \text{Fitting}_S(F)(p) \\ 0 & \text{otherwise} \end{cases} \tag{8}$$

We can notice that MHMT results are normalized into  $[0, 1]$  and do not depend anymore on the value range of the different spectral bands of the processed image. This specificity can be used or not, depending on the application (and its need for fuzzy or soft results).

Moreover, the proposed MHMT can also be applied on grey-level images. Indeed, a grey-level image is a multivalued image with a number of bands equal to one. The previous definitions can then be used relying on a single spectral band  $s_b$ .

### 3.2 Comparison with Related Work

MHMT offers several advantages compared to other HMT definitions. First, whereas Ronse’s and Soille’s HMTs use only spatial information through the shape of SEs (and their intensity profile with Ronse’s HMT), MHMT also relies on spectral information with bands and thresholds. The combined use of spatial and spectral features allows extraction of well-defined templates in terms of spectral signature within a spatial configuration. Second, it can use more than two SEs which allows complex shapes to be matched (moreover SEs do not need to be disjoint anymore). Third, MHMT results are normalized in  $[0, 1]$  which have two main interests: in one hand it can lead to fuzzy or soft interpretation of the results, in the other hand it allows comparison between a set of single MHMT results even if the spectral bands do not have the same value range. Finally, contrary to the HMT from Ronse or Soille which requires the two SEs to be fully applied on the image, our definition of MHMT allows a fast implementation. Indeed, the MHMT fitting needs each SE to be fitted: so, as soon as one SE is not fitted, processing the other SEs is useless. Furthermore, MHMT fitting compares erosion and dilation

results to a threshold, pixel values could directly be compared to threshold because of the same principle: as soon as one pixel value is lower than the threshold while processing an erosion with a lower SE, this SE is not fitted (making the processing of the rest of the SE useless). By simply extending Ronse's and Soille's definitions to multivalued space (cf. 2.3) using multivariate morphology [3], these interesting properties cannot be reached.

We will now give a practical illustration of the proposed MHMT for template matching in multivalued images, and focus on coastline extraction in multispectral images.

## 4 Application to Template Matching

We do not provide here a generic method for template matching by using MHMT, but we give some clues for elaborating methods adapted to the problem to solve and illustrate it by an application to coastline extraction.

### 4.1 MHMT-Based Template Matching

MHMT is a powerful tool for template matching, it can be used to build feature extraction methods. Due to its specificities, the set-up of MHMT-based template matching methods is quite simple and consists mainly in two steps (cf. figure 2).

The first step for the elaboration of feature extraction method is to formalize the problem and to wonder how the feature can be defined from both spatial and spectral points of view. Spatial part consists in definition of the shape and the position of each SE (related to its center), while spectral part is the introduction of expert knowledge about spectral response of feature to be extracted.

The second step concerns specificities of the problem to be solved, introduction of multiple SE's orientations if the sought template can have different orientations, or processing of MHMT at various scales if the template can have different sizes. These specification examples are obviously not exhaustive.

### 4.2 Example: Coastline Extraction

Coastline is typically defined as the border between sea and land. Its extraction is a well-known problem in remote sensing imagery[4]. Even if there exist several methods which give accurate results on low or medium resolution, few are relevant on very high spatial resolution (VHR) satellite imagery where a pixel represents an area lower than 5x5 meters square. Here, we define a method for coastline extraction using the MHMT.

The proposed MHMT has been used to build a coastline detection method for VHR images. Spatial definitions of SEs needed for coastline extraction are deduced from coastline definition. Coastline is defined as the border between sea and land, we defined two linear SEs which are both aligned and separated by only one pixel ( $S^1$  and  $S^2$  in figure 3). Using only these two SEs leads to extract all borders between water and land (e.g. lake border, river bank, etc...),

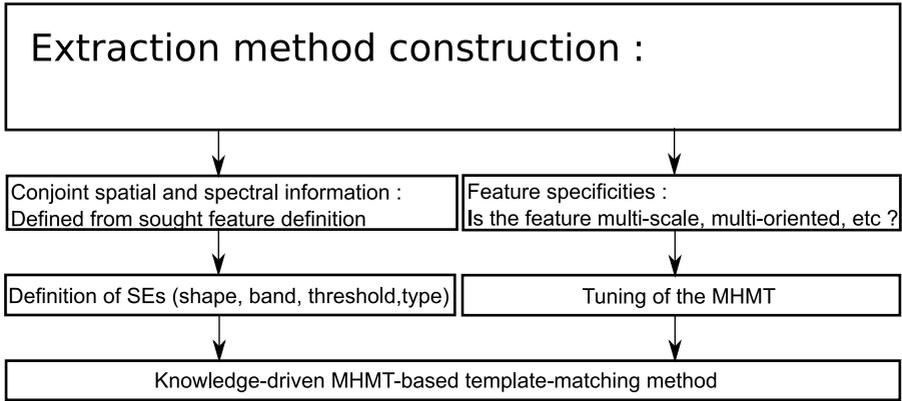


Fig. 2. Flowchart of Mhmt-based method elaboration

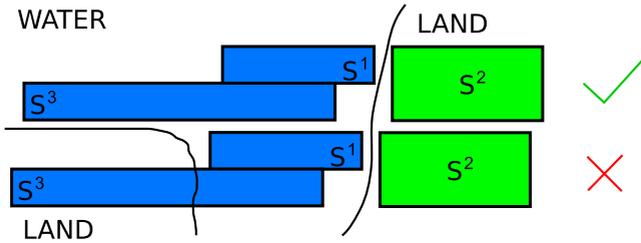


Fig. 3. Spatial definition of SEs used for coastline extraction with matching and unmatching conditions

so we have to add one more SE representing water further from the coastline ( $S^3$  in figure 3). As coastline can have any orientation, Mhmt is then applied with various directions. Now, we have defined the spatial part of our coastline extraction method.

The spectral parameters have been defined by a geographical expert, based on knowledge about spectral response of land (i.e. sand) or sea (i.e. water) in the area observed with a given satellite sensor. They are defined as follow :

$$\Omega = \left\{ \begin{array}{l} \{s_{sh}^1 = 180 + 0, s_b^1 = NDVI, s_{th}^1 = 0.5, s_{ty}^1 = G\} \\ \{s_{sh}^2 = -180 + 0, s_b^2 = NDVI, s_{th}^2 = 0.5, s_{ty}^2 = L\} \\ \{s_{sh}^3 = 360 + 12, s_b^3 = R, s_{th}^3 = 0.05, s_{ty}^3 = L\} \end{array} \right. \quad (9)$$

where  $s_{sh} = i + j$  means a line of length  $i$  meters shifted of  $j$  meters from the origin (minus representing opposite direction). R and NDVI denote normalized images respectively built from the red band and the normalized difference vegetation index.

Using only Mhmt with parameters defined in (9) could lead to extract uncontinuous coastline due to satellite picture defaults, noise, etc. To improve results



**Fig. 4.** Coastline extraction on Normandy coast QuickBird image at resolution of 2.4m per pixel ((c)Digitalglobe)

on this specific application, we added a binary post-processing step to obtain a connected coastline. It consists in enlarging the initial thresholded MHMT result (all pixels with non null values are set to 1) to get a connected region and then compute and filter the morphological skeleton of this region to generate the final coastline result. The input of the skeleton algorithm, i.e. the connected region or mask build from the initial MHMT result, is obtained through a double threshold operator [5]. More precisely, another MHMT is applied with some more tolerant spectral parameters (i.e. thresholds) and is thresholded (with non null values set to 1) and filtered (with a morphological closing) to remove holes. Then we perform a geodesic reconstruction using the first MHMT as a marker and the second MHMT as a mask. On this reconstructed image is then extracted the skeleton considered as the final coastline result.

Fig. 4 illustrates the results obtained on a QuickBird image (2.4m/pixel spatial resolution) and shows the relevance of MHMT for the extraction of coastline in VHR imagery. Compared to existing approaches, the MHMT gives particular promising results for VHR imagery [6], due to the simultaneous usage of both spatial and spectral knowledge. In particular, MHMT-based method gives less false-positive pixels and is more accurate than existing methods [7,8,9]. Moreover, such an extraction could not been achieved with classical HMTs (such as those from Ronse or Soille) due to their inability to consider more than two shapes and to deal with spectral properties. Similarly, spectral approaches could not extract coastline as well. Indeed, spectral approaches can only determine borders between water and land regions, so they will also extract lake border and riverbank as coastline.

## 5 Conclusion

In this paper, a Hit-or-Miss Transform dedicated to multivalued images has been proposed. Since an image with a single band is a particular case of multivalued

images, this method can also be used on grey-level images. The proposed operator takes into account spectral properties without requiring a vectorial ordering. It relies on both spatial and spectral knowledge to define the set (of variable cardinality) of extended structuring elements (involving spectral features) and thus to extract specific templates. Its relevance for multivalued template matching has been underlined by solving the problem of coastline extraction in VHR remote sensing.

Future works will consist in using the MHMT in other application fields. To ensure a higher robustness to noise, a soft or fuzzy version of the MHMT operator will also be elaborated by replacing the global fitting and valuation steps by some more advanced merging techniques.

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