Synthesis of the Supervising Agent in MAS

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Abstract. The systematic approach to the synthesis of the agent-supervisor supervising a group of agents is presented. It is based on results of the DES (discrete event systems) control theory. The agents as well as the agent-supervisor are understood to be DES and they are modelled by Petri nets (PN). The method based on PN place invariants is used in order to synthesise the supervisor. The illustrative example is introduced to explain the elementary steps of the proposed technique. A counter-example illustrating the limits of the approach is introduced too.

Keywords: Agent, invariants, MAS, modelling, Petri nets, supervisor.

1 Introduction

Agents are usually understood to be persistent (not only software, but also material, personal, etc.) entities that can perceive, reason, and act in their environment, and communicate with other agents. Hence, multiagent systems (MAS) can be apprehended as a composition of collaborative agents working in shared environment. The agents together perform a more complex functionality. Communication enables the agents in MAS to exchange information. Thus, the agents can coordinate their actions and cooperate with each other. However, an important question arises here: What communication mechanisms enhance the cooperation between communicating agents?

In this paper the synthesis of the supervising agent is presented. It utilizes some results of the DES (discrete event systems) control theory, especially those concerning the theory of place invariants [8,6] and the supervisor [9] and/or controller synthesis based on this theory. Mathematical and/or graphical description of the agents as well as the global MAS will be dealt on the background of place/transition Petri nets (P/T PN) described e.g. in [8].

Consider two simple agents A_1 , A_2 representing, respectively, two processes P_1 , P_2 given in Fig. 1a). While the resources used by the agents are sufficient, the agents can work autonomously. However, when the resources are limited a coordinated action is necessary. In such a case a supervisor or a controller is necessary. The procedure of the mutex (mutual exclusion) frequently occurs in

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different systems when it is necessary to solve conflicts among parallelly operating subsystems. It is very frequent also in DES like flexible manufacturing systems (e.g. a robot serves two transport belts), communication systems (two users want to use the same channel), transport systems (two trains want to enter the same segment of the railway), etc. The PN model of the simplest form of the mutex - the mutex of two processes - is given in Fig. 1b). It is not very complicated to synthesize such a supervisor in this simple case. Contrariwise, in case of more agents and more complicated conditions for co-

Fig. 1. The PN-based model of the a) two simple processes and b) their mutex

operation among them, founding the supervisor is by far more intricate. Consider the group of 5 autonomous agents $Gr_A = \{A_1, A_2, A_3, A_4, A_5\}$ given in Fig.2. Consider e.g the situation when it is necessary to ensure that only one agent from the subgroup $Sgr_1 = \{A_1, A_4, A_5\}$ and only one agent from the subgroup $Sgr_2 = \{A_2, A_4, A_5\}$ and only one agent from the subgroup $Sgr_3 = \{A_3, A_4, A_5\}$ can simultaneously cooperate with other agents from Gr_A . In other words, the agents inside the designated subgroups must not work simultaneously. Even, the agents A_4 and A_5 can work only individually (any cooperation with other agents is excluded). However, the agents A_1, A_2, A_3 can work simultaneously. In such a situation the synthesis of the supervisor realizing such a mutex (i.e. deciding which agents can simultaneously work and when) is not so simple. Consequently, a systematic (orderly, not stray) approach has to

Fig. 2. The PN-based model of the group of 5 simple agents

be found. There exists the DES control synthesis method based on the PN place invariants (P-invariants) in DES control theory [11,7,2,4]. Its principle can be utilised for our purposes.

2 The Supervisor Synthesis

Before starting the supervisor synthesis it is necessary to introduce the mathematical model of P/T PN. Namely, the ability to express the states of agents, the ability to test properties by means of the PN invariants and the reachability graph (RG) as well as the ability to synthesize the supervisory controllers by means of the invariants and RG [3,5] utilizing linear algebra predestinate PN to be frequently used. The model has the form as follows

$$\mathbf{x}_{k+1} = \mathbf{x}_k + \mathbf{B} \cdot \mathbf{u}_k \quad , \quad k = 0, N \quad (1)$$

$$\mathbf{B} = \mathbf{G}^T - \mathbf{F} \quad (2)$$

$$\mathbf{F} \cdot \mathbf{u}_k \leq \mathbf{x}_k \quad (3)$$

where k is the discrete step of the dynamics development; $\mathbf{x}_k = (\sigma_{p_1}^k, \dots, \sigma_{p_n}^k)^T$ is the n -dimensional state vector of DEDS in the step k ; $\sigma_{p_i}^k \in \{0, c_{p_i}\}$, $i = 1, \dots, n$ express the states of the DEDS elementary subprocesses or operations - 0 (passivity) or $0 < \sigma_{p_i} \leq c_{p_i}$ (activity); c_{p_i} is the capacity of the DEDS subprocess p_i as to its activities; $\mathbf{u}_k = (\gamma_{t_1}^k, \dots, \gamma_{t_m}^k)^T$ is the m -dimensional control vector of the system in the step k ; its components $\gamma_{t_j}^k \in \{0, 1\}$, $j = 1, \dots, m$ represent occurring of the DEDS elementary discrete events (e.g. starting or ending the elementary subprocesses or their activities, failures, etc. - 1 (presence) or 0 (absence) of the corresponding discrete event; $\mathbf{B}$, $\mathbf{F}$, $\mathbf{G}$ are structural matrices of constant elements; $\mathbf{F} = \{f_{ij}\}_{n \times m}$, $f_{ij} \in \{0, M_{f_{ij}}\}$, $i = 1, \dots, n$, $j = 1, \dots, m$ expresses the causal relations among the states of the DEDS (in the role of causes) and the discrete events occurring during the DEDS operation (in the role of consequences) - 0 (nonexistence), $M_{f_{ij}} > 0$ (existence and multiplicity) of the corresponding causal relations; $\mathbf{G} = \{g_{ij}\}_{m \times n}$, $g_{ij} \in \{0, M_{g_{ij}}\}$, $i = 1, \dots, m$, $j = 1, \dots, n$ expresses very analogically the causal relations among the discrete events (causes) and the DEDS states (consequences); because $\mathbf{F}$ and $\mathbf{G}$ are the arcs incidence matrices the matrix $\mathbf{B}$ is given by means of them according to (2); $(\cdot)^T$ symbolizes the matrix or vector transposition.

The main idea of the approach to the supervisor synthesis consists in imbedding of additional PN places (slacks) and finding the structural interconnections between them and the original PN places. Consequently, a new PN subnet representing the supervisor and its interface with the original system are found and added to the PN model. Thus, the desired behaviour of agents is forced. The definition of the P-invariant $\mathbf{v}$ of PN in general is the following

$$\mathbf{B}^T \cdot \mathbf{v} = \mathbf{0} \quad (4)$$

with $\mathbf{v}$ being n -dimensional vector and $\mathbf{0}$ is m -dimensional zero vector. However, usually there are several invariants in PN models. Hence, the set of the P-invariants of PN is created by the columns of the $n \times n_x$ -dimensional (n_x expresses the number of invariants) matrix $\mathbf{V}$ being the solution of the equation

$$\mathbf{V}^T \cdot \mathbf{B} = \mathbf{0} \quad (5)$$

In PN with the slacks mentioned above we have to use the following structure with augmented matrices. Consequently, the equation (5) has to be written in the form

$$[\mathbf{L}, \mathbf{I}_s] \cdot \begin{bmatrix} \mathbf{B} \\ \mathbf{B}_s \end{bmatrix} = \emptyset$$

where $\mathbf{I}_s$ is $(n_x \times n_x)$ -dimensional identity matrix and $\mathbf{L}$ describes conditions (as to marking of some places in the PN models of agents) of the desired MAS behaviour. While $\mathbf{B}$ is the known structural matrix of the agents (without the supervisor), $\mathbf{B}_s$ is the supervisor structure which is searched by the synthesis process. Thus,

$$\mathbf{L} \cdot \mathbf{B} + \mathbf{B}_s = \emptyset; \quad \mathbf{B}_s = -\mathbf{L} \cdot \mathbf{B}; \quad \mathbf{B}_s = \mathbf{G}_s^T - \mathbf{F}_s \quad (6)$$

where the actual structure of the matrix $\mathbf{L}$ has to be respected. The augmented state vector and the augmented structural matrices (of the original system together with the supervisor) are the following

$$\mathbf{x}_a = \begin{bmatrix} \mathbf{x} \\ \mathbf{x}_s \end{bmatrix}; \quad \mathbf{F}_a = \begin{pmatrix} \mathbf{F} \\ \mathbf{F}_s \end{pmatrix}; \quad \mathbf{G}_a^T = \begin{pmatrix} \mathbf{G}^T \\ \mathbf{G}_s^T \end{pmatrix}$$

where the submatrices $\mathbf{F}_s$ and $\mathbf{G}_s^T$ correspond to the interconnections of the incorporated slacks with the actual (original) PN structure.

2.1 The Illustrative Example

It can be seen that in the above mentioned case of the five agents the form of $\mathbf{L}$ follows from the imposed conditions prescribing the limited cooperation of agents expressed mathematically as

$$\sigma_{p_2} + \sigma_{p_8} + \sigma_{p_{10}} \leq 1 \quad (7)$$

$$\sigma_{p_4} + \sigma_{p_8} + \sigma_{p_{10}} \leq 1 \quad (8)$$

$$\sigma_{p_6} + \sigma_{p_8} + \sigma_{p_{10}} \leq 1 \quad (9)$$

The form of the augmented matrix $[\mathbf{L}, \mathbf{I}_s]$ follows from the modified conditions (introducing the slacks) which are given mathematical terms as

$$\sigma_{p_2} + \sigma_{p_8} + \sigma_{p_{10}} + \sigma_{p_{11}} = 1 \quad (10)$$

$$\sigma_{p_4} + \sigma_{p_8} + \sigma_{p_{10}} + \sigma_{p_{12}} = 1 \quad (11)$$

$$\sigma_{p_6} + \sigma_{p_8} + \sigma_{p_{10}} + \sigma_{p_{13}} = 1 \quad (12)$$

where p_{11} , p_{12} , p_{13} are the additional places - i.e. the slacks s_1 , s_2 , s_3 (in general, the number i of such slacks s_i can also be different from $i = 3$, of course) - that ensure the desired eliminations of the agents cooperation activities. The number of the slacks depends on the number of conditions and on their form. As it is clear from the equations (10)-(12), only one place from the set of four places

p_2, p_8, p_{10}, p_{11} can be active while the rest ones have to be passive. Analogically, the same is valid for the sets p_4, p_8, p_{10}, p_{12} and p_6, p_8, p_{10}, p_{13} .

$$\mathbf{F} = \begin{pmatrix} \mathbf{F}_{A_1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{F}_{A_2} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{F}_{A_3} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{F}_{A_4} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{F}_{A_5} \end{pmatrix}; \quad \mathbf{G} = \begin{pmatrix} \mathbf{G}_{A_1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{G}_{A_2} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{G}_{A_3} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{G}_{A_4} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{G}_{A_5} \end{pmatrix} \quad (13)$$

$$\mathbf{F}_{A_i} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}; \quad \mathbf{G}_{A_i} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \quad \mathbf{B}_{A_i} = \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix}; \quad i = 1, \dots, 5 \quad (14)$$

$$\mathbf{L} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \end{pmatrix}; \quad \mathbf{B} = \begin{pmatrix} -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \end{pmatrix} \quad (15)$$

$$\mathbf{B}_s = -\mathbf{L} \cdot \mathbf{B} = \begin{pmatrix} -1 & 1 & 0 & 0 & 0 & 0 & -1 & 1 & -1 & 1 \\ 0 & 0 & -1 & 1 & 0 & 0 & -1 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 & -1 & 1 & -1 & 1 & -1 & 1 \end{pmatrix} \quad (16)$$

Hence, by virtue of (6), we have

$$\mathbf{F}_s = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \end{pmatrix}; \quad \mathbf{G}_s^T = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \end{pmatrix} \quad (17)$$

Thus, we obtain the structure of MAS given in Fig. 3 where the supervisor is represented by the structure consisting of the places p_{11}, p_{12}, p_{13} together with the interconnections joining them with the elementary agents. Such MAS exactly fulfils the prescribed conditions. The reachability graph (RG) [8] of the system is given in Fig. 4. Its nodes are the states reachable from the initial state $\mathbf{x}_0$ and its edges are weighted by corresponding PN transitions enabling the state-to-state transit. The control synthesis based on RG is described in the author's works [3,5]. Consequently, this problem is not discussed here. In the light of the above introduced facts we can illustrate also the systematic synthesis of the simplest mutex presented in Fig. 1. Here we have

$$\sigma_{p_2} + \sigma_{p_4} \leq 1 \quad (18)$$

$$\sigma_{p_2} + \sigma_{p_4} + \sigma_{p_5} = 1 \quad (19)$$

$$\mathbf{L} = (0, 1, 0, 1); \quad \mathbf{B} = \begin{pmatrix} -1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & -1 \end{pmatrix}; \quad \mathbf{L} \cdot \mathbf{B} = (1, -1, 1, -1) \quad (20)$$

$$\mathbf{B}_s = -\mathbf{L} \cdot \mathbf{B} = (-1, 1, -1, 1); \quad \mathbf{F}_s = (1, 0, 1, 0); \quad \mathbf{G}_s^T = (0, 1, 0, 1) \quad (21)$$

Of course, the result is the same like that given in the Fig. 1.b) - compare (21) with Fig. 1b).

Fig. 3. The PN-based model of the controlled cooperation of the agents

Fig. 4. The RG of the five cooperating agents a) in the form where the edges are oriented bidirectionally, but weights of the elementary edges are not introduced because they are different depending on the orientation; b) in the form where both the orientation of edges and their weights are introduced implicitly

3 Applicability of the Approach

The method described above is suitable for the so called pure PN. P/T PN is pure when there is no 'loop' between a place and a transition - i.e. it is the net without the loops of the form given in Fig. 5a). Namely, in such a case

the matrices $\mathbf{F}$, $\mathbf{G}^T$ have nonzero element equal to 1 in the same place - in general, when there exists the 'loop' between p_i and t_j the element $f(i, j)$ of the matrix $\mathbf{F}$ is equal to the element $gT(i, j)$ of the matrix $\mathbf{G}^T$. The effect is, that the difference $\mathbf{G}^T - \mathbf{F} = \mathbf{B}$ gives us a deformed information about the system structure, because the elements $f(i, j)$, $gT(i, j)$ eliminate each other (namely, $gT(i, j) - f(i, j) = 0$). Fortunately, in PN theory a simple solution of such a problem is possible. It is sufficient to replace the transition in question by the fictive structure introduced in Fig. 5b). The auxiliary place p' together with its input and output transitions t' and t'' replace the original transition t . It means that the impureness in the form of a 'loop' can be replaced (without changing any dynamic property of the net) very simply by the replacement of the transition in question by two fictive transitions and a fictive place set between them. Such a replacement is invariant. After removing the 'loop' problems the original method of the supervisor synthesis can be utilized also for the systems modelled by the 'impure' P/T PN.

Fig. 5. The replacement of the loop. The original loop is on the left. The structure replacing the loop is given on the right.

3.1 Dealing with Problems of Mutual Interactions Among Agents

Many times, another form of demands on the cooperation among agents occurs. Especially, the situation - see e.g. [1] - when there exist mutual interactions among the agents to be supervised. In such a case a particular marking (i.e. the state of a concrete place) of one agent affects (e.g. stops) firing a transition in another agent. The situation is illustrated in Fig. 6a), where the transition A_1t of the agent A_1 is prevented from firing under marking the place A_2p of the agent A_2 . As we can see in that picture, a figment that the place p' represents the supervising agent A_3 (or, better said, its part) can be accepted and consecutively, the above described approach to the supervisor synthesis can be utilized. Namely, the transition A_1t can be substituted by means of the auxiliary fictive place p'' together with its fictive input and output transitions t' , and t'' analogically to that displayed in Fig. 5b). Hence, the new situation - i.e. new structure - is displayed in Fig. 6b). It can be seen that it is the classical mutex. Such a substitution can be utilized in synthesis of the supervisor in case of agents with more complicated structure. For example, the well known kind of the reader-writer cooperation named as the Owicki/Lamports mutex (defined e.g. in [10]) can be automatically synthetised using this substitution.

However, it is necessary to say, that there exist cases where the synthesis of the supervisor based on invariants is not so simple (i.e. fully automatized) or even, it is impossible. Here, the experience and invention of the creator of the

Fig. 6. The prevention of the transition A_1t (the part of the Agent A_1) from firing under the marking A_2p (the part of the Agent A_2) by means of p' - the part of the supervisor, i.e. the Agent A_3 . a) The original schema; b) The substitutive schema.

agents supervisor has to be used. In the next illustrative example such a system is presented and described.

3.2 The Counter-Example

In order to illustrate such a counter-example let us consider the so called Peterson's mutex algorithm [10] given in Fig. 7. This system consists of two agents A_1 (writer) and A_2 (reader). The PN places describe the following activities: p_1 - pending 1 of A_1 ; p_2 - pending 2 of A_1 ; p_3 - critical A_1 ; p_4 - finished A_1 ; p_5 - pending 0 of A_1 ; p_6 - quiet A_1 ; p_7 - pending 1 of A_2 ; p_8 - pending 2 of A_2 ; p_9 - critical A_2 ; p_{10} - finished A_2 ; p_{11} - pending 0 of A_2 ; p_{12} - quiet A_2 ; p_{13} - at A_1 (left); p_{14} - at A_2 (right).

The state p_4 signals to A_2 that A_1 is presently not striving to become critical. This allows A_2 to "easily" access its critical region, by the action represented by the transition t_{12} . Likewise, the state p_{10} allows A_1 to access its critical state, by the action represented by the transition t_5 . The shared token alternates between the states p_{13} and p_{14} . The step from p_1 to p_2 results in the token in p_{13} : by action expressed by the transition t_2 in case A_1 obtains the token from p_{13} , or by action expressed by the transition t_3 in case at A_1 held the token anyway. The step from p_7 to p_8 likewise results in the token in p_{14} . Hence, the token is always at the site that executed the step from 'pending 1' to 'pending 2' most recently.

After leaving p_6 along the quiescent action represented by the transition t_7 , the A_1 takes three steps to reach its critical state p_3 . In the first step, the fair action represented by the transition t_6 brings A_1 from p_5 to p_1 and removes the state p_4 . Fairness of the transition t_6 is local, because the transition t_6 is local to A_1 , with p_4 the only forward branching the place in the transition t_6 , which is connected to the A_2 by the loop (p_4, t_{12}) . The second step, from p_1 to p_2 , results in the shared token in the place p_{13} , as described above. The third step brings A_1 to p_3 , with action expressed by the transition t_5 in case A_2 signals with p_{10} ,

Fig. 7. The PN-based model of the two agents communication - the writer (agent A1) and the reader (agent A2) - in the form of the Peterson's mutex

that it is presently not interested in going critical, or with action represented by the transition t_4 in case A_2 more recently executed the step from p_7 to p_8 .

The algorithm's overall structure guarantees that one of p_{10} or p_{14} will eventually carry a token that remains there until eventually either the event represented by t_5 or t_4 occurs. The two agents are structurally symmetrical, but the initial state favors A_2 (if the place p_{14} contains the token) or A_1 (if the place p_{13} contains the token).

In this example the invariants-based approach to the supervisor synthesis fails. Namely, any satisfying matrix $\mathbf{L}$ cannot be found. Searching for reasons it was detected that the problem consists in the specific structure of the P-invariants of the global system presented in Fig. 7. As we can see from the following matrix of the global system P-invariants the invariants of the agents A_1 , A_2 are mutually disjoint

$$\mathbf{V}^T = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

4 Conclusion

The method of the DES control theory based on PN place invariants was utilized for the synthesis of the agent (with prescribed properties) supervising a group of agents in MAS. The illustrative example documenting the soundness of the method was introduced. It was shown that the method is simply applicable for autonomous agents modelled by pure PN. Additionally, it was described how the applicability can be extended for the agents modelled by impure PN. Namely,

a simple substitution of the transition in the loops of the PN model is sufficient to remove the impureness. As a result of the research it can be said that the method is very useful for the synthesis of the agents supervising a group of autonomous agents. In case of a group of cooperating agents some problems can occur. Therefore, the counter-example was presented. It was shown that e.g. for the reader-writer in the form of the so called Peterson's mutex the supervising agent cannot be synthesised by the proposed method. Hence, there exists a challenge for the future investigation on this way, especially, to define general conditions for the validity of the approach.

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