Influence of Network Structure on Market Share in Complex Market Structures

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Abstract. We study an dynamical model on complex networks. The model is based on a multi agent modeling which is aimed for representing ‘Network Effect’ in a market of personal communication services. By series of numerical simulations using various models of complex networks, we found two classes in resulting dynamics which are dependent on underlying network structures as well as parameter settings. We also apply the model on a real social network data, and discuss the simulation results in relation with the network models.

1 Introduction

Study on complex networks has been attracting wide attention in physics, engineering and other fields as a basic representation for the analysis and understanding of a variety of complex systems. In such systems, complex network structures are considered to be responsible for characteristic dynamical phenomena that take place on the systems. Under this idea a number of examples of complex networks including physical, technological, biological and social systems have been studied [1–3]. Many statistical properties which are commonly seen in many instances in networks are revealed, and theoretical models of complex networks which realize these properties have been proposed [4–7].

In our preceding study, we reported that even a simple spin-like model of dynamical process on complex networks is widely affected by underlying network structure, and inherent characteristics of network structures that are not fully represented by the statistical properties can be classified in relation with the dynamical process [8].

On the other hand, real world examples of dynamical processes are more complicated than such the simple physical modeling. For example, interaction and decision making of human in a real world market are much more complicated with various constraint and uncertainty, which often lead to specific real world phenomena, such as ‘Network Effect’ that is often referred to in business and marketing science literature [9]. In such processes of human interactions, it is not an obvious question that the structural complexity of network structures of interaction patterns will be associated with and accountable for the resulting phenomena, even with relatively complicated models of

dynamical process. In this paper, we investigate a relationship between a more complicated dynamical model and underlying network structures, by extending our previous works [8,9]. The dynamical model is based on a multi agent modeling for an artificial market simulation. Through a number of numerical simulations using complex network models and a structural data of a real social network, we study resulting dynamics on the system and their classifications.

2 The Model: Dynamics of Network Effect

We consider a dynamical model inspired by so-called ‘Network Effect’, one of a market characteristic that often occurs in a market where two or more products or services are competing. Generally, the network effect, or network externalities, is defined as an effect whereby the benefit of a product or a service increase with the number of users who also use the same product or service [10]. This has been considered to lead to emergence of market phenomena such as ‘winner-takes-all’. So far, a number of studies have examined the network effect, mainly for the purpose of understanding market dynamics and constructing an efficient market or effective market strategies [11–16]. Above all, Wendt and Westarp [14], and Weitzel et al [16] reported that network structures, or topologies, of interaction patterns of users have a strong influence on the magnitude of the network effect.

In the present paper, a multi agent simulation model that represents a market of a personal communication service (i.e. mobile phone or short messaging service) is constructed. The basic concept of the modeling is as follows. Individual users are represented by agents. Each user agent has a contact list of friends with whom to communicate. Users pay a charge for the service, which is proportional to the amount of communication they do. In the market, there are assumed to be two competing service providers. Users choose one of the providers so as to minimize their overall costs. Consequently, the communication pattern of users can be represented by a network

Fig. 1. Schematic diagram of the network

structure. Figure 1 shows a schematic representation of this model. Each circle denotes a user. For example, the user of the provider S in the center of the figure has six friends to communicate with, four of whom use the different provider D and the rest are using the same provider S.

2.1 Model of User

We consider that users can select either S or D service provider. Users pay for the service in proportion to the amount of communications. Here, we assume that the charge within the same provider is cheaper than that to the different provider. We also assume that an additional cost (typically an initial setup cost) is required if a user wants to switch to the other provider.

We can formulate this model as follows. The overall cost c that user i pays in a unit period is

$$c(x_i, \alpha^S, \alpha^D, k_i^S, k_i^D, \delta) = \min \left\{ (\alpha^S k_i^S + \alpha^D k_i^D) x_i, (\alpha^S k_i^D + \alpha^D k_i^S) x_i + \delta \right\}, \quad (1)$$

where x_i is the amount of communication per a friend in a unit period of user i , k_i^S and k_i^D are the numbers of friends of user i who are using the same provider and the different provider as user i , respectively, and α^S and α^D are the unit charges of communication to users of the same provider and the different provider, respectively. We assume α^S and α^D as constant values that satisfy $\alpha^S < \alpha^D$. Here, δ is the cost to switch providers, which is also assumed to be a constant.

In this modeling, we assume the total amount of communication X_{total} that users make in a unit period follows a normal distribution $N(\mu, \sigma^2)$ for each user. Therefore, x_i in Eqn. (1) can be rewritten as $x_i = X_{total}/k_i$, where $k_i = k_i^S + k_i^D$ is the number of friends of user i . Note that because of this assumption, the amount of communication is asymmetric; the amount of communication from user i to user j and from j to i is different if the number of friends are different. Consequently, Eqn. (1) can be rewritten as

$$c(k_i^S, k_i^D, X_{total}) = \min \left\{ (\alpha^S k_i^S + \alpha^D k_i^D) X_{total} / (k_i^S + k_i^D), (\alpha^S k_i^D + \alpha^D k_i^S) X_{total} / (k_i^S + k_i^D) + \delta \right\}. \quad (2)$$

At each step t , a user is randomly selected and a choice is made as to whether the user should switch providers in order to minimize the overall cost according as Eqn. 2. In other words, user i will switch the provider if

$$(\alpha^S k_i^S + \alpha^D k_i^D) X_{total} / (k_i^S + k_i^D) \geq (\alpha^S k_i^D + \alpha^D k_i^S) X_{total} / (k_i^S + k_i^D) + \delta,$$

and pay the switch cost.

Table 1. Structural properties of networks

	$P(k)$	L	C	r
lattice	uniform	large	large	no
ER random graph	Poisson	small	small	no
WS model	Poisson-like	small	large	no
BA model	power-law	small	small	no
KE model	power-law	small	large	negative
CNN model	power-law	small	large	positive

2.2 Model of Interaction Pattern by Complex Network Models

In the present modeling, communication pattern among users in the market is modeled on a kind of social networks. It has been revealed by recent studies that social networks in the real world have a unique characteristics in their structures [2,3]. Examples include power-law degree distribution, short average path length, high clustering coefficients and positive degree correlations. These characteristics are surely considered to give an effect on dynamical processes on such social networks, which we assume are not be reproduced using simple structural models, such as lattices or random graphs.

We assume to be able to realize the characteristics of real social networks on the market dynamics by considering network models as an interaction patterns of the multi agent modeling. In order to investigate into the effect, the main purpose of this paper is to analyze the dynamics driven by the above mentioned dynamical model under various structures of complex networks, and to find out suitable structural models for an analysis on real world problems.

In the present paper, we use four complex network models: the WS model [4], the BA model [5], the KE model [6] and the CNN model [7]. The WS model is a so-called ‘small-world’ model that is based on a random rewiring procedure of the edges from a one-dimensional lattice. The remaining three models realize a power-law degree distribution. They are models of a mechanism of network growth, in which different types of growth algorithms produce different characteristics in other structural properties. The characteristics of the models are briefly summarized in Table 1. See related references for the details of the models. For comparison, we also test the relatively simple structures of the one-dimensional lattice and the Erdős-Renyí (ER) random graph.

3 Numerical Studies

3.1 Network Models

First, we perform numerical simulations using the network models described in the previous section. The total amount of communication X_{total} is defined as a random value that follows normal distribution $N(\mu, \sigma^2)$, where $\mu = \overline{X}\langle k \rangle$ and $\overline{X}\langle k \rangle/3$, $\langle k \rangle$ is

Fig. 2. Temporal evolutions of a typical realization. $\langle k \rangle = 10$, $\overline{X} = 100$, $\alpha^S = 20$, $\alpha^D = 25$ and $\delta = 100$. (A) regular lattice, (B) ER random graph, (C) WS network, (D) BA network, (E) KE network, (F) CNN network.

average degree, and $\overline{X}$ is a constant value. $X_{total} = 0$ if a probabilistic random variable $N(\mu, \sigma^2)$ is less than zero. The number of agents (users) is set 3,600. Initial conditions are randomly given. At the initial state, the two competing providers have randomly selected 50% share of the users. Here, all of the parameter of the two providers are the same, which means neither providers is advantageous. However, we found that the system reaches an equilibrium state as time progresses, that one provider becomes advantageous (becomes the ‘winner’) at around a particular share. Examples of temporal evolutions of shares in typical realizations are shown in Fig. 2.

We focus on the winner’s share after the equilibrium is reached. Figs. 3 and 4 represent average shares of the winner as functions of switch cost δ , comparing on different average degrees and different amount of communications. The values are averaged over last 50 steps after $t = 450$, and averaged over 50 simulations with different initial conditions.

Here we can observe a number of characteristics. From Fig. 3, the resulting phenomena can be divided into two classes. One is ‘ordered’ dynamics, which is observed in the ER random graph, the BA network and the WS network. In this class, the shares of the winner well converge to particular values with little deviation. The convergence values are increasing function of the switch cost δ . This class is also observed in the CNN network with large $\langle k \rangle$, however, the convergence values are rather small than the rest three networks. The other is ‘fluctuated’ class, in which the shares of the winner fluctuate with a large deviation. The average values of the share is constant, or gradually decreasing, function of δ . In regular lattice, the dynamics belongs to this class and the values are around 50%, which was the initial share. This is considered to be the inherent characteristics of the model of interaction itself, without the effects of complex network structure. In the KE network, the deviation is larger than the regular lattice so that the average values are rather large.

Fig. 3. Average share of the winner as a function of switch cost δ , comparison on average degree. $\bar{X} = 100$, $\alpha^S = 20$ and $\alpha^D = 25$. (A) $\langle k \rangle = 8$, (B) $\langle k \rangle = 10$, (C) $\langle k \rangle = 14$.

Fig. 4. Average share of the winner as a function of switch cost δ , comparison on amounts of communication. $\langle k \rangle = 10$, $\alpha^S = 20$ and $\alpha^D = 25$. (A) $\overline{X} = 50$, (B) $\overline{X} = 150$.

From Fig. 4, the system tends to be ‘fluctuated’ if the amount of communication $\overline{X}$ is small and δ is large. However, it strongly depends on network structures which class will occur at particular $\overline{X}$ and δ .

3.2 Real Social Network

We then perform simulations using a real structure of an e-mail correspondence network at a Spanish university [17]¹, which we assume reflects the characteristics of an human communication pattern in the real world. The data contains 1,133 users and 5,451 friendships (edges) $\langle k \rangle = 9.62$. The clustering coefficient is $C = 0.254$, which is 10 times larger than a random graph of the same number of agents, and the average

¹ The dataset can be obtained from

<http://deim.urv.cat/%7Eaarenas/data/welcome.htm>.

Fig. 5. Temporal evolutions of a typical realization on a real e-mail correspondence network. (A) $\delta = 0$, (B) $\delta = 100$, (C) $\delta = 2900$.

Fig. 6. Average share of the winner as a function of switch cost δ on a real e-mail correspondence network, comparison on amounts of communication

path length is $L = 3.606$. The degrees of the network follows an exponential degree distribution [17].

Fig. 5 shows example of temporal evolution of shares in typical realizations with different δ . We can confirm that the system reaches an equilibrium state with the real data as well as in the networks models. Fig. 6 shows convergence values of the average share of the winner with different values of $\overline{X}$. From Fig. 6, we can still observe the ‘ordered’ and the ‘fluctuated’ classes as in the network models. With large $\overline{X}$, the system is ‘ordered’; the convergence shares of the winner are increasing function of δ , with little deviation. With smaller $\overline{X}$, we can still observe the ‘ordered’ dynamics as long as δ is small. With such values of $\overline{X}$, the convergence shares of the winner are ordered and increases as δ increases, then reach the maximum at particular value of δ^* . As δ further increases $\delta > \delta^*$, the system does a transition to the ‘fluctuated’ class; the convergence

share decreases, with large deviations. With $\overline{X} \ll \delta$, the convergence shares reaches to 50%, which was the initial share.

4 Discussion and Conclusion

In the series of the numerical studies, we found two classes, ‘ordered’ and ‘fluctuated’, on the dynamics driven by the proposed model. From the simulations, we can say that the ‘ordered’ dynamics is likely to emerge in small δ , large $\langle k \rangle$ and large $\overline{X}$. The classes have a dependence on network structures as well. The ER, BA and WS network tend to exhibit ‘ordered’ dynamics in most cases, while in the KE network and lattice there is always ‘fluctuated’. The CNN network is intermediate of the two, which is also seen in the real data of e-mail correspondence network. The values of convergence shares of the ‘winner’ are also dependent on network structures. In practice, the winner’s share can be interpreted as a magnitude of ‘Network Effect’; if stronger network effect works on the market, then the winner can take a larger share consequently due to the network effect. Therefore, we can analyze which network structure can enhance or decrease the network effect, and which model of network is suitable for studying the real social network, from the simulation results.

In summary, we investigated an influence of network structures on a process of market dynamics. The dynamical model is based on a multi agent modeling for artificial market simulation, which intends of representing a dynamics of ‘Network Effect’. Series of numerical studies using the simulation model showed two classes of dynamics, which are strongly dependent on network structures. The simulation with an empirical data shows the pattern like that of the CNN model, which implies the CNN model reflects the characteristic of structures of the real interaction patterns of users. In conclusion, it is confirmed that the structure of interaction patterns surely gives strong effect on the resulting dynamics on it. By using appropriate models of the real interaction patterns of users, it becomes possible to analyze dynamics of market more precisely.

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