Cost-Minimizing Algorithm for Replica Allocation and Topology Assignment Problem in WAN

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Abstract. The paper deals with the problem of simultaneously assignment of network topology and server's replica placement in the wide area network in order to minimize the criterion composed of the leasing capacity cost and the building cost of the network. An exact algorithm, based on the branch and bound method, is proposed to solve the problem. Algorithm takes into account the problem of ensuring the reliability of the network. Some interesting features, observed during the computational experiments are reported.

Keywords: resource replication, topology assignment, WAN.

1 Introduction

Designing or modernizing of the Wide Area Network (WAN) consists in the assignment of resource allocation (i.e. servers, replicas of servers), topology and flow routes. The optimal arrangement of resources and optimal allocation of network channels let us obtain the most efficient and economical solution. Designing of the wide area computer networks is always a compromise between the quality of service in the network and reliability of the network from the one hand and the costs needed to build and to support the network from the other. Quality of services and the network costs are criteria often used during the designing process. In [1] we have considered problem based on designing the WAN assuming that the maximal support cost of the network is bounded. In [2] we have proposed algorithm for CFA problem with cost criterion. In those papers the reliability requirements were not considered. In many cases it is useful to formulate the optimizing problem in such a way: to minimize the cost of the network, when the acceptable quality level is known and the reliability level is given. Then in the paper the problem of server's replication and topology assignment with the cost criterion delay constraint and reliability constraint is considered. In our opinion it is well-founded to consider two kind of cost: the building cost of the network, borne once and the supporting cost (i.e. connected with the capacity leasing), borne regularly. Then the criterion function is composed of two ingredients: the regular channel capacity leasing cost and the disposable server cost. We assume that the maximal acceptable total average delay in the network is given as the constraint.

Designing of the wide area network topology consists in assignment of channels location and choosing of channels' capacities. Properly designed network topology should ensure communication between all pairs of nodes in WAN. There must be at least one path between each pair of nodes in the network. In case of the channel failure or node failure, all paths leading through this channel or node become unserviceable. In case when only one path exists between pair of nodes, failure of any of the path elements makes communication between nodes impossible. To ensure the reliability and survivability of the network, there should exist few different paths between each pair of nodes. Paths are different when they do not have any common channel or node, except the source node and destination node. Usually minimal condition imposing to the network is to ensure two paths between each node pair. Some problems and solutions related to the network reliability are presented in [3, 4].

We assumed that the minimal number of paths between each pair of nodes is given as the constraint. It allows to design network topology with denoted reliability level. The problem considered here may be formulated as follows:

- given:* user allocation at nodes, the set of the nodes to which replicas may be connected for each server, maximal value of the total average delay in the network, traffic requirements user-user and user-server, the set of potential channels, capacities and their costs (i.e. cost-capacity function) for each potential channel,
- minimize:* linear combination of the capacity leasing cost and server cost,
- over:* servers allocation, channel's capacity and multicommodity flow,
- subject to:* multicommodity flow constraints, channel capacities constraints, server allocation constraints, total average delay constraint, network reliability constraint (minimal number of different paths between nodes).

We consider the discrete cost-capacity function because it is the most important from practical point of view for the reason that the channels capacities can be chosen from the sequence defined by international ITU-T recommendations. Such formulated problem is NP-complete as more general than the capacity and flow assignment problem (CFA) with discrete cost-capacity function which is NP-complete [5].

The literature focusing on the simultaneous server's replication and topology assignment problem is very limited. Some algorithms for this problem with different delay criterion may be found in [1, 6]. The formulated here problem is more general. It uses cost criterion and take into account the maximal acceptable average delay in the WAN as the constraint. Moreover it takes into account some aspects of reliability of the designed network. In the literature such formulated problem has not been considered yet.

2 Problem Formulation

Let n be the number of nodes of the wide area network and b be the number of potential channels which may be used to build the network. For each potential channel i there is the set $\bar{C}^i = \{c_1^i, \dots, c_{s(i)-1}^i\}$ of alternative values of capacities from which exactly one must be chosen if the i -th channel was chosen to build the WAN. Let d_j^i

be the cost of leasing capacity c_j^i [€/month]. Let $c_{s(i)}^i = 0$ for $i = 1, \dots, b$. Then $C^i = \bar{C}^i \cup \{c_{s(i)}^i\}$ be the set of alternative capacities from among which exactly one must be used to channel i . If the capacity $c_{s(i)}^i$ is chosen then the i -th channel is not used to build the wide area network. Let x_j^i be the decision variable, which is equal to one if the capacity c_j^i is assigned to channel i and x_j^i is equal to zero otherwise. Since exactly one capacity from the set C^i must be chosen for channel i , then the following condition must be satisfied:

$$\sum_{j=1}^{s(i)} x_j^i = 1 \quad \text{for } i = 1, \dots, b. \quad (1)$$

Let $W^i = \{x_1^i, \dots, x_{s(i)}^i\}$ be the set of variables x_j^i , which correspond to the i -th channel. Let X_r' be the permutation of values of all variables x_j^i for which the condition (1) is satisfied, and let X_r be the set of variables, which are equal to one in X_r' .

Designing the wide area network topology (channels allocation) the reliability of the network should be ensured. Particularly, in case of a failure of a link (channel) or a node, some routes must be redirected to another path. Let PN be the least number of different paths between each pair of nodes in the network. Paths between two nodes are different only when they do not have any common nodes or channels. Let MPN be the minimal number of paths, which have to exist between all pairs of nodes of the WAN. Ensuring MPN number of paths is very important for reliability of the network.

Let K denotes the total number of servers, which must be allocated in WAN and let LK_k denotes the number of replicas of k -th server. Let M_k be the set of nodes to which k -th server (or replica of k -th server) may be connected, and let $e(k)$ be the number of all possible allocation for k -th server. Since only one replica of server may be allocated in one node then the following condition must be satisfied

$$LK_k \leq e(k) \quad \text{for } k = 1, \dots, K. \quad (2)$$

Let y_{kh} be the decision binary variable for k -th server allocation; y_{kh} is equal to one if the replica of k -th server is connected to node h , and equal to zero otherwise. Since LK_k replicas of k -th server must be allocated in the network then the following condition must be satisfied

$$\sum_{h \in M_k} y_{kh} = LK_k \quad \text{for } k = 1, \dots, K. \quad (3)$$

Let Y_r be the set of all variables y_{kh} , which are equal to one. The pair of sets (X_r, Y_r) is called a selection. Let $\mathfrak{R}$ be the family of all selections. X_r determines

the network topology and capacities of channels and Y_r determines the replicas allocation at nodes of WAN.

Let $T(X_r, Y_r)$ be the minimal average delay per packet in WAN in which values of channel capacities are given by X_r and traffic requirements are given by Y_r (depending on server replica allocation). $T(X_r, Y_r)$ can be obtained by solving a multi-commodity flow problem in the network [7]. Let $U(Y_r)$ be the server cost and let $d(X_r)$ be the capacity cost.

Let $Q(X_r, Y_r)$ be linear combination of the capacity cost and the server cost:

$$Q(X_r, Y_r) = \alpha D(X_r) + \beta U(Y_r) \quad (4)$$

where α and β are the positive coefficients; $\alpha, \beta \in [0, 1]$, $\alpha + \beta = 1$.

Let $T^{\max}$ be the maximal acceptable average delay per packet in WAN. Then, the considered servers allocation, capacity and flow assignment problem in WAN with total average delay constraint is formulated as follows:

$$\min_{(X_r, Y_r)} Q(X_r, Y_r) \quad (5)$$

subject to

$$PN_{X_r} \geq MPN \quad (6)$$

$$(X_r, Y_r) \in \mathfrak{R} \quad (7)$$

$$T(X_r, Y_r) \leq T^{\max} \quad (8)$$

Where PN_{X_r} is the least number of different paths between each pair of nodes in the network in which values of channel capacities are given by X_r .

3 Calculation Scheme of the Branch and Bound Algorithm

Assuming that $LK_k = 1$ for $k=1, \dots, K$ and $C^i = \overline{C}^i$ for $i=1, \dots, b$ and omitting the constraint (6), the problem (5-7) is resolved itself into the “host allocation, capacity and flow assignment problem”. Since the host allocation, capacity and flow assignment problem is NP-complete [6, 7] then the problem (5-8) is also NP-complete as more general. Then, the branch and bound method can be used to construct the exact algorithm. Starting with the selection $(X_1, Y_1) \in \mathfrak{R}$ we generate a sequence of selections (X_s, Y_s) . Each selection (X_s, Y_s) is obtained from a certain selections (X_r, Y_r) of the sequence by complementing one variable x_j^i (or y_{kh}) by another variable from W^i (or $\{y_{km} : m \in M_k \text{ and } m \neq h\}$).

For each selection (X_r, Y_r) we constantly fix a subset $F_r \in (X_r, Y_r)$ and momentarily fix a set F_r^t . The variables in F_r are constantly fixed and represent the path

from the initial selection (X_1, Y_1) to the selection (X_r, Y_r) . Each momentarily fixed variable in F_r^t is the variable abandoned during the backtracking process. Variables, which do not belong to F_r or F_r^t are called free in (X_r, Y_r) . There are two important elements in branch and bound method: testing operation (lower bound of the criterion function) and branching rules. Then, in the next section of the paper, the testing operation and choice operation are proposed.

The lower bound LB_r and branching rules are calculated for each selection (X_r, Y_r) . The lower bound is calculated to check if the “better” solution (selection (X_s, Y_s)) may be found. If the testing is negative, we abandon the considered selection (X_r, Y_r) and backtrack to the selection (X_p, Y_p) from which selection (X_r, Y_r) was generated. The basic task of the branching rules is to find the variables for complementing to generate a new selection with the least possible value of the criterion function. The detailed description of the calculation scheme of branch and bound method may be found in the paper [8].

4 The Lower Bound

Since the traffic requirements in the network depend on the server allocation, then obtaining the lower bound for the problem (5-8) is difficult. We propose, the lower bound may be obtained by relaxing some constraints and by approximating the discrete cost-capacity curves with the lower linear envelope [5, 7].

To find the lower bound LB_r of the criterion function (4) we reformulate the problem (5-8) in the following way:

- we assume that the variables $x_j^i \in X_r - F_r$, such that $X^i \cap F_r = \emptyset$ are continuous variables. Then we can approximate the discrete cost-capacity curves (given by the set C^i) with the lower linear envelope. Let Z' be the set of such channels i , for which variables x_j^i are continuous. The criterion function (4) turns itself into:

$$Q(X_r, Y_r) = \alpha \left(\sum_{i \in Z'} c^i d^i + \sum_{i: x_j^i \in F_r} d^i \right) + \beta \sum_{y_{kh} \in Y_r} y_{kh} u_{kh},$$

where $d^i = \min_{x_j^i \in X^i} (d_j^i / c_j^i)$ and c^i is capacity of channel i (continuous variable).

- we assume that the variables $y_{kh} \in Y^k - F_r$, for $k = 1, \dots, K$, such that $Y^k \cap F_r = \emptyset$ are continuous variables. We create the model of the WAN in the following way. We add to the considered network $2K$ new artificial nodes, numbered from $n+1$ to $n+2K$. The artificial nodes $n+k$ and $n+K+k$ correspond to the k -th server. Moreover we add to the network directed artificial channels $\langle n+k, m \rangle$, $\langle m, n+K+k \rangle$, $\langle n+K+k, n+k \rangle$, for all $m \in M_k$ and $k = 1, \dots, K$, such that

$Y^k \cap F_r = \emptyset$. The capacities of the new artificial channels are following:
 $c(n+k, m) = \infty$, $c(m, n+K+k) = \infty$, $c(n+K+k, n+k) = \sum_{h=1..n} u_{kh}$. The leasing costs of all artificial channels are equal to zero.

Then, the lower bound LB_r of minimal value of the criterion function $Q(X_s, Y_s)$ for every possible successor (X_s, Y_s) generated from (X_r, Y_r) may be obtained by solving the following optimization problem:

$$LB_r = \min_{\underline{f}} \left(\alpha \left(\sum_{i \in Z'} d^i f_i + \frac{\left(\sum_{i \in Z'} f_i \sqrt{\frac{d^i}{c^i}} \right)^2}{\mathcal{T}^{\max} - \sum_{x_j^i \in F_r} \frac{f_i}{x_j^i c_j^i - f_i}} + \sum_{i: x_j^i \in F_r} d^i \right) + \beta \sum_{y_{kh} \in Y_r} y_{kh} u_{kh} \right) \quad (9)$$

subject to

$$f_i \leq c^i \quad \text{for } i \in Z' \quad (10)$$

$$f_i \leq x_j^i c_j^i \quad \text{for } x_j^i \in F_r \quad (11)$$

$$f_i \leq c_{\max}^{ir} \quad \text{for each } i \in Z' \quad (12)$$

where $c_{\max}^{ir}$ is maximal capacity connected with variables $x_j^i \in X^i - F_r^t$, and F_r^t is the subset of momentarily fixed variables.

The solution of problem (9–12) gives the lower bound LB_r . To solve the problem (9–12) we can use an efficient Flow Deviation method [5, 7].

5 Branching Rules

The purposes of the branching rules is to find the normal variable from (X_r, Y_r) for complementing and generating a successor of the selection (X_r, Y_r) with the least possible value of criterion function (4). The choice criterions should be constructed in such a way that complementing reduces the value of (4) and the increase of the total average delay in the network is as minimal as possible.

Complementing variables x_j^i causes the capacity change in channel i . Then, the values of average delay in the network and the capacity cost changes, and the value of server cost does not change. Moreover, complementing variable x_j^i , $j < s(i)$ by $x_{s(i)}^i = 0$ causes that network topology changes - constraint (6) can not be violated. We propose the following choice criterion for complementing variable $x_j^i \in X_r$ by the variable $x_j^i \in X_s$:

$$\Delta_{jl}^i = \begin{cases} \frac{\frac{f_i}{c_l^i - f_i} - \frac{f_i}{c_j^i - f_i}}{\alpha(d_j^i - d_l^i)} & \text{if } f_i < c_l^i \text{ and } PN_{X_s} \geq MPN \\ \infty & \text{otherwise} \end{cases}$$

where f_i is the flow in the i -th channel obtained by solving the multicommodity flow problem for network topology and channel capacities given by the selection X_r .

The choice criterion for complementing variable $y_{kh} \in Y_r$ by the variable $y_{km} \in Y_s$ may be formulated as follows:

$$\delta_{hm}^k = \begin{cases} \frac{\frac{1}{\gamma} \sum_{x_j^i \in X_r} \frac{\tilde{f}_i}{x_j^i c_j^i - \tilde{f}_i} - T(X_r, Y_r)}{\beta(u_{kh} - u_{km})} & \text{if } \tilde{f}_i < x_j^i c_j^i \text{ for } x_j^i \in X_r \\ \infty & \text{otherwise} \end{cases}$$

where the flow $\tilde{\mathbf{f}} = [\tilde{f}_1, \dots, \tilde{f}_b]$ was constructed as follows: flow from all users to k -th server was moved from the routes leading from users to node h to the routes leading from users to node p . The calculation scheme for obtaining flow $\tilde{\mathbf{f}}$ may be found in [1].

Let $E_r = (X_r \cup Y_r) - F_r$, and let G_r be the set of all reverse variables of normal variables, which belong to the set E_r . We want to choose a normal variable the complementing of which generates a successor with the possible least value of criterion (4). We should choose such pairs $\{(x_j^i, x_l^i) : x_j^i \in E_r, x_l^i \in G_r\}$ or $\{(y_{kh}, y_{kp}) : y_{kh} \in E_r, y_{km} \in G_r\}$, for which the values of the criterion Δ_{jl}^i or δ_{hm}^k is minimal.

6 Computational Results

The presented algorithm was implemented in C++ code. Extensive numerical experiments have been performed with this algorithm for many different networks. The experiments were conducted with two main purpose in mind: first, to examine the impact of various problem parameters on the solution (i.e. on the value of the criterion Q) and second, to test the computational efficiency of the algorithm.

The typical dependence of the optimal value of Q on maximal acceptable total average delay per packet $T^{\max}$ for different values of parameters α and β is presented in the Fig. 1. It follows from computational experiments that the dependence of Q on $T^{\max}$ is decreasing function. The following conclusions follows from the computer experiments (Fig.1).

Conclusion 1. There exist such acceptable total average delay per packet $T_*^{\max}$, that the problem (5-8) has the same solution for each $T^{\max}$ greater or equal to $T_*^{\max}$.

The typical dependence of the optimal value of D on the value of parameter α ($\beta = 1 - \alpha$) is presented in the Fig. 2. It follows from computational experiments and from Fig. 2 that the dependence of D on α is increasing function.

We have examined the impact of reliability parameter MPN on the solution. Typical dependences of the optimal value of the criterion Q and total average delay in the network on the minimal number of different paths are presented in the Fig. 3 and Fig. 4. Experiments were conducted for $MPN=1, 2$ and 3. Obtaining of more than three different path between each pair of nodes is difficult or even impossible for small and medium wide area networks. To ensure $MPN=4$ there must exists at least four channels adjacent to each node of the network. It makes the network very expensive, because the leasing costs of channels increase fast and the cost of nodes (WAN switches) increases as well.

As it follows from Fig. 3 the value of combined cost criterion Q is quite similar for $MPN=1$ and $MPN=2$ and rapidly increases for $MPN > 2$. Similar dependences were discovered for all examined networks.

Typical dependence of the optimal value of the total average delay in the network, obtained by solving problem (5-8), on the minimal number of different paths is decreasing function (Fig. 4). In most cases dependency of T on MPN can be approximated with the linear function.

Basing on the results, partially presented in the Fig. 3 and Fig. 4, we can formulate the following conclusion.

Conclusion 2. For small and medium wide area networks the optimal value of minimal number of paths between each pair of nodes is equal to two.

Computational properties of the presented algorithm were tested during experiments. Let $NT = ((T^{\max} - T_{\min}) / (T_*^{\max} - T_{\min})) \cdot 100\%$ be the normalized maximal acceptable total average delay per packet in the network - problem (5-8) has no

Fig. 1. Typical dependence of the optimal value of criterion Q on maximal acceptable total average delay per packet

Fig. 2. Typical dependence of the optimal value of D on the coefficient α

solution for $T^{\max} < T_{\min}$. This normalized value let us compare the results obtained for different wide area network topologies and for different numbers and locations of servers. Let $P(u, v)$, in percentage, be the arithmetic mean of the relative number of iterations for $NT \in [u, v]$ calculated for all considered network topologies and for different servers locations. Fig. 5 shows the dependency of P on divisions $[0\%, 10\%)$, $[10\%, 20\%)$, ..., $[90\%, 100\%]$ of NT . It follows from Fig.3 that the exact algorithm is especially effective from computational point of view for $NT \in [60\%, 100\%]$.

Fig. 3. Typical dependence of the optimal value of criterion Q on the minimal number of different paths

Fig. 4. Typical dependence of the optimal value of the total average delay in the network on the minimal number of different paths

7 Conclusion

In the paper the exact algorithm for solving the servers replication and topology assignment problem with cost criterion, delay constraint and reliability constraint is proposed. Such formulated problem has not been considered in the literature yet. It follows from computational experiments that the presented algorithm is effective from computational point of view for greater values of acceptable average delay in

Fig. 5. The dependence of P on normalized maximal average delay per packet NT

the WAN (Fig.5). We are of the opinion that the WAN property formulated as Conclusion 2 is very important from practical point of view. It shows the optimal value of minimal path's number between each node pairs in the small or medium networks. Moreover, properties presented in the section 4 and 5 may be very useful to construct effective approximate algorithm for solving the problem (5-8).

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References

1. Markowski, M., Kasprzak, A.: The Three-Criteria Servers Replication and Topology Assignment Problem in Wide Area Networks. In: Gavrilova, M.L., Gervasi, O., Kumar, V., Tan, C.J.K., Taniar, D., Laganá, A., Mun, Y., Choo, H. (eds.) ICCSA 2006. LNCS, vol. 3982, pp. 1119–1128. Springer, Heidelberg (2006)
2. Markowski, M., Kasprzak, A.: An Exact Algorithm for the Servers Allocation, Capacity and Flow Assignment Problem with Cost Criterion and Delay Constraint in Wide Area Networks. In: Shi, Y., van Albada, G.D., Dongarra, J., Sloot, P.M.A. (eds.) ICCS 2007. LNCS, vol. 4487, pp. 442–445. Springer, Heidelberg (2007)
3. Koidea, T., Shinmorib, S., Ishiic, H.: Topological Optimization with a Network Reliability Constraint. *Discrete Applied Mathematics* 115, 135–149 (2001)
4. Yi-Kuei, L.: Reliability of a Flow Network Subject to Budget Constraints. *IEEE Transactions on Reliability* 56(1), 10–16 (2007)
5. Pioro, M., Medhi, D.: Routing, Flow, and Capacity Design in Communication and Computer Networks. Elsevier, Morgan Kaufmann Publishers, San Francisco (2004)
6. Chari, K.: Resource Allocation and Capacity Assignment in Distributed Systems. *Computers Ops Res.* 23(11), 1025–1041 (1996)
7. Fratta, L., Gerla, M., Kleinrock, L.: The Flow Deviation Method: an Approach to Store-and-Forward Communication Network Design. *Networks* 3, 97–133 (1973)
8. Wolsey, L.A.: Integer Programming. Wiley-Interscience, New York (1998)