Experimental Comparison of *Call String* and *Functional* Approaches to Interprocedural Analysis

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Abstract. The techniques used to implement (non-trivial) interprocedural data flow analyzers can be generally divided into two subsets: the *call string* approach and the *functional* approach as presented in [Sharir and Pnueli, 1981]. Both differ in time and space complexity as well as in precision due to properties of the abstract domains and transfer functions. We have developed a data flow analyzer generator PAG which is able to produce interprocedural analyzers for both techniques. We specified two variants of constant propagation working in an ANSI-C compiler; a *copy constant* propagation that uses *distributive* transfer function and can be solved precisely, even interprocedurally [Sagiv et al., 1995], and a *full constant* propagator which includes an interpreter for expressions of the language. We present the practical relevant results applying both analyzers to a fair set of real-world programs and compare the space/time consumption of the analyzers versus their precision.

1 Introduction

The need for interprocedural analyses results from a conflict between the goals of software engineering and performance [Metzger and Stroud, 1993]. The usage of procedural and functional abstraction generalizes the code and makes it more extensible and maintainable. Optimizing compilers try to compute information about program points that can be used for code improvements. Intraprocedural analysis assumes the worst case (loss of all information) at procedure boundaries, whereas interprocedural analysis propagates information across procedure/function boundaries or eliminates the calls by inline expansions for non-recursive calls. There exist two different approaches to interprocedural analysis which have been discussed in [Sharir and Pnueli, 1981]: the *call string* and the *functional* approach. The *call string* approach separates call chains (and the related data flow information) to a procedure differing in a suffix of a fixed length. Thus precision may increase with longer call strings. The maximally useful length for non-recursive programs is the height of the call DAG. For recursive programs the number of possible call strings is infinite. If there are no recursive procedures

the number of call strings can be exponential in the number of procedures. Thus the call string length has to be limited in any practical implementation to some constant K .

The functional approach, on the other side, computes a function for each procedure describing the “abstract” effect of the procedure. These functions are then used in a standard (intraprocedural-like) algorithm. However, the computation of the abstract functions may not terminate for lattices of infinite cardinality, even if they are of bounded height.

We designed and implemented an interprocedural data flow analyzer generator PAG [Martin, 1998], that produces analyzers using either the call string approach with a length parameter or the functional approach. Using this analyzer generator we generated two analyzers, which have been included in a ANSI-C compiler, and measured the efficiency and precision for practical relevant applications.

By creating PAG we hope to close the gap between the theory of abstract interpretation and practical useful analyzers. PAG is available under a free academic license on request from the author or in form of a simplified WWW interface (www.cs.uni-sb.de/~martin/pag).

The paper is organized as follows: a brief introduction to the framework of dataflow analysis is followed by the discussion of the two interprocedural approaches. Then these are compared based on experimental data for two data flow problems.

2 Techniques

The data flow analysis practiced nowadays is based on the work described in [Kam and Ullman, 1977]. It uses a *control flow graph* $CFG=(V,E)$ that contains a node for every statement or basic block in a procedure and an edge for a possible flow of control. The set of vertices V is a disjoint union of the set of nodes V_i for each subroutine p_i in the program¹. Furthermore unique entry nodes s_i and exit nodes e_i for each procedure p_i are added. A *data flow analysis problem* (DFP) is a combination of such a graph with a complete lattice of values, called the *underlying lattice* ($\mathcal{L}$), and a family of functions (one for each node). These functions express the local semantics and are called *transfer functions* ($\llbracket \bullet \rrbracket : \mathcal{L} \rightarrow \mathcal{L}$). If every transfer function is monotone (distributive) the problem is called a *monotone (distributive) problem*. To describe the solution of a data flow problem the semantics of a path $\pi = n_1, \dots, n_k$ in the CFG is defined as:

$$\llbracket \pi \rrbracket = \begin{cases} id & \text{if } \pi = \epsilon^2 \\ \llbracket (n_2, \dots, n_k) \rrbracket \circ \llbracket n_1 \rrbracket & \text{otherwise} \end{cases}$$

¹ The existence of a main routine p_0 is assumed.

The desired solution of the *DFP* is the union of the semantics of all paths applied to bottom ($\perp$), called the *merge over all paths solution*:

$$\mathbf{MOP}(n) = \bigsqcup \{ \llbracket \pi \rrbracket(\perp) \mid \pi \text{ is a path from } s_0 \text{ to } n \}$$

for every $n \in V$. As the set of all paths from s to n is usually infinite, this solution is in general not computable. Therefore, the *minimal fixed point solution* was introduced:

$$\mathbf{MFP}(n) = \begin{cases} \llbracket n \rrbracket(\perp) & \text{if } n = s_0 \\ \llbracket n \rrbracket(\bigsqcup \{ \mathbf{MFP}(m) \mid m \text{ predecessor } n \}) & \text{otherwise} \end{cases}$$

If $\mathbf{MFP}(n) = \mathbf{MOP}(n)$ for all n , then the **MFP** solution is called *precise*. Kam has proved in [Kam and Ullman, 1977] that for every monotone data flow problem the **MFP** is greater (with respect to the ordering of the lattice) than the **MOP** solution, and is therefore a safe approximation. Moreover, if the *DFP* is distributive the **MFP** solution is precise. The interprocedural version of this theorem is presented in [Knoop and Steffen, 1992].

2.1 Call String Approach

This approach considers procedure calls and returns as ordinary transfers of control, but avoids propagation along interprocedurally invalid paths. This can be implemented by tagging the propagated data with a call string, which is a sequence of call nodes, whose calls are still unfinished. As the number of call strings may be infinite their length is bounded by a constant K . This results in an approximate solution by merging the information belonging to call strings with the same K -suffix. For a more practical implementation a vector of data flow elements is used. If a call site c has to be appended to a call string $\gamma = c_1 c_2 \dots c_K$ which already has length K the first element c_1 of γ is removed, which results in $c_2 \dots c_K c$. This method is turned into a vector based implementation by encoding the call strings as numbers which will correspond to positions in the data flow array: assign to every call site c a unique number between one and $M - 1$, where M is the number of call sites in the given program plus one. Then a call string $\gamma = c_{i_1} \dots c_{i_n}$ corresponds to a K digit number m to base M ($m = i_1 \dots i_n$). Converting this number to the decimal system is done in usual way: multiply the n -th digit by M^n and add this for all digits ($m' = \sum_{1 \leq j \leq n} i_j * M^j$). This results in a number between zero and M^K where zero denotes the empty call string ϵ . Fortunately not all of these call strings are valid for each procedure. So all invalid numbers can be omitted for a fixed procedure and the array can be compressed, so that every position is used.

As an example consider the flow graph from the left side of Fig. 1 with $K = 1$. Valid call strings for **proc** are $c_1, c_1 c_2, c_1 c_2 c_2, \dots$. If they are restricted to length one c_1 and c_2 remain. For **main** the empty call string is the only valid call string. So in the procedure **main** position zero of the data flow array of length three is

² ϵ denotes the empty path.

Fig. 1. Example of the vector based call string approach with $K=1$

used and in procedure `proc` the positions one and two are used. This results in the graph on the right side of Fig. 1.

If the underlying lattice $\mathcal{L}$ is finite a vector implementation can be constructed that results in the precise solution of the *DFP*. Call chains of length up to $|\mathcal{L}|^2 * M$ must be separated.

The vector based approach can also be used to implement more sophisticated methods which can also be used for the analysis of loops [Martin et al., 1998].

2.2 Functional Approach

The functional approach computes for every procedure in the program a function which maps the data flow value at the entry of the procedure to the corresponding data flow value at the exit of the procedure. Because a procedure can be simultaneously recursive it is necessary to do this computation for all procedures in an interleaved fashion. The function is computed by tabulating input/output pairs. This table is guaranteed to be finite if the lattice is of finite cardinality. As an optimization only those input/output pairs are computed for each procedure where the input value actually occurs.

Now a more formal description of this method is given: For each procedure p_i and each node $n \in V_i$ we define an element $\Phi(s_i, n) : \mathcal{L} \rightarrow \mathcal{L}$ which describes the effects to elements of $\mathcal{L}$ when they are propagated from the start s_i of procedure p_i to n . These functions are defined by:

$$\begin{aligned}\Phi(s_i, s_i) &= \llbracket s_i \rrbracket \\ \Phi(s_i, n) &= \sqcup_{(m,n) \in E} \mathcal{E}(n) \circ \Phi(s_i, m)\end{aligned}$$

Fig. 2. Functional Approach

and $\mathcal{E}$ is defined as either the normal transfer function for intraprocedural edges or Φ for interprocedural ones:

$$\mathcal{E}(n) = \begin{cases} \llbracket n \rrbracket & \text{if } \forall j : n \neq s_j \\ \Phi(s_j, e_j) & \text{if } \exists j : n = s_j \end{cases}$$

The results of the above equation system are used to compute the solution of the data flow problem:

$$\begin{aligned} \mathcal{F}(s_{main}) &= \llbracket s_{main} \rrbracket(\perp) \\ \mathcal{F}(s_i) &= \sqcup_{(c, s_i) \in E} \mathcal{F}(c) \\ \mathcal{F}(n) &= \Phi(s_j, n)(\mathcal{F}(s_j)) \text{ for } n \neq s_j, n \in V_j \end{aligned}$$

The iteration algorithm in PAG is implemented with some optimizations that are specially developed to handle large equation systems with a dynamic number of variables (the elements in the abstract function tables correspond to the variables, so their number is dynamic as the size of the tables is determined in a demand driven way). These optimizations turned out to be very good for this class of equation systems (cf. [Fecht, 1997, Nielson et al., 1998]).

2.3 Comparison

We briefly summarize the properties of both approaches (assuming a language with recursive procedures and table based implementation of the functional approach) for a given lattice $\mathcal{L}$. While the call string approach computes only values of $\mathcal{L}$, the functional approach computes values from $\mathcal{L} \rightarrow \mathcal{L}$. In order to guarantee termination it is required that this function space is finite. Thus the call string approach is always guaranteed to terminate, but may deliver worse results. Additionally there is the problem of finding an appropriate length K .

The functional approach is guaranteed to terminate for finite lattices, but may be exponential in the size of the lattice. It delivers best results.

An other point for the reliable use in a compiler is the the speed of the analysis. The speed for interprocedural analyses depends mainly on how often the procedures of the programs are visited during the analysis. In the K call string approach this is the number of K -paths to the procedure in the call graph. In the functional approach it is the number of different data flow values that occur at the beginning of the procedure. In order to keep this number small one tries usually to benefit from the locality properties of the programming language: if it is known that a procedure p can only read and write a certain part of the data flow information, the rest of the data flow information is passed directly from the call to the return of the procedure. This is modeled in PAG by the `local` edge from Fig. 1 and 2.

But for a constant propagation in C one can not bypass any part of the data flow information since any procedure can modify any other variable through pointer accesses. This is different for other analyses and programming languages. E.g. in Pascal all variables local to the caller can be passed by any call.

3 Practical Evaluation

We generated data flow analyzers for *copy constant propagation* (ccp) which is a *distributive* problem and for *full constant propagation* (fp) which is not. $\mathcal{L}_{ccp}$ is finite and therefore of bounded height and the $\mathcal{L}_{fp}$ is infinite but has bounded height. The ccp only processes assignments of the form $x := y$ or $x := c$ where x, y are variables and c is a constant, whereas fp includes a full interpreter for expressions. Furthermore, we use a simplified form of a technique called *downset completion* [Cousot and Cousot, 1992]; instead of combining the data flow information directly where two (or more) control flow edges come together, we first apply the transfer function and combine the results: $f(a \sqcup b)$ is replaced by $f(a) \sqcup f(b)$. This results in better solutions for non distributive $\sqcup$.

Figure 3 shows the structure and size of the used test programs in ANSI C, where most of them are well known everyday life programs (a ‘*’ indicates programs with procedural variables). The columns give from left to right: the name of the program, a short description, the number of lines without comments, the number of procedures, the number of control flow nodes, and the number of variables.

For each of the programs the *functional* analyzer was applied. It finds the precise solution for any program in case of the *copy constant propagation*. This is also true for the *full constant propagation*, but the termination of the functional analyzer is not guaranteed. Also the *call string* analyzers have been applied with call strings up to length two or less if they find the same number of constants as the functional approach.

For each program four mayor columns are printed in Fig. 4 for the copy constant propagation and in Fig. 5 for the full constant propagation: three for call string approach of length zero, one, and two ($\mathcal{C}_0$, $\mathcal{C}_1$, $\mathcal{C}_2$), and one for the

program	description	lines	flow nodes	procedures	variables
bison	parser generator	6438	11722	155	3575
cdecl	C++ compiler part	2831	2401	33	841
cloop	benchmark	1488	2176	26	622
dhry	dhystone	446	319	14	240
ed	editor	1506	2745	47	796
flex	scanner generator	5985	8688	129	2314
flops	benchmark	723	353	3	139
gzip*	compress	4056	6043	47	796
linpack	benchmark	821	796	13	278
twig	code generator	2092	3198	81	1085
xmodem	communication	2060	2965	31	1159

Fig. 3. The set of test programs

Prog	$\mathcal{C}_0$			$\mathcal{C}_1$			$\mathcal{C}_2$			$\mathcal{F}$		
	Prec	Fold	Time	Prec	Fold	Time	Prec	Fold	Time	Prec	Fold	Time
cdecl	615	10	15,66	627	10	66,41				627	10	113,93
twig	756	6	6,83	865	6	45,13	891	6	308,22	993	6	1746,14
dhry	81	2	1,82	109	2	1,54	109	2	1,45	109	2	2,06
ed	1098	42	7,81	1175	44	26,04	1198	44	37,05	1198	44	27,25
cloop	9839	145	20,95	9865	145	200,30				19649	145	6309,13
flex	3362	22	16,40	3496	22	88,76	9714	22	378,66	9731	22	193,54
flops	6897	107	0,52	7973	119	0,50				7973	119	0,70
bison	2699	10	24,45	2974	10	157,12	2978	10	372,43	2978	10	303,74
linpack	355	78	5,17	355	78	17,60	355	78	24,36	355	78	5,72
gzip	2744	16	9,52	2822	16	91,68	2822	16	1122,28			∞
xmodem	2716	6	88,77	3269	6	256,61	3269	6	326,55	4806	6	802,53

Fig. 4. Copy Constant Propagation

functional approach ($\mathcal{F}$). For each analysis method tree numbers are given: as a measurement of (theoretical) precision the number of available constants in the program is used. This is the number of constant variables at all control flow nodes, i.e.

$$\sum_{n \in V} |\{var \mid (\sqcup_{i=1, \dots, Arity(n)} flow(n)[i])(var) \neq \perp, \top\}|$$

As a measurement of the usability of the calculated information the number of source code transformations is given that can be done with the information obtained by the constant propagation. This is the number of replacements of variables by their values. The last number is the runtime in seconds on a SUN SPARCsystem-600 (Ross-RT625 cpu, 128Mb memory), running SunOS 4.1.4.

To compare the different analysis approaches for each analysis three bar charts are used: Fig. 6, 7, 8 for the copy constant propagation and Fig. 9, 10, 11

Prog	$\mathcal{C}_0$			$\mathcal{C}_1$			$\mathcal{C}_2$			$\mathcal{F}$		
	Prec	Fold	Time	Prec	Fold	Time	Prec	Fold	Time	Prec	Fold	Time
cdecl	885	10	15,93	903	10	65,08	911	10	135,02	911	10	101,05
twig	995	6	8,31	1130	6	47,05	1156	6	230,13	1258	6	1813,02
dhry	87	3	1,26	141	15	1,49	141	15	1,43	141	15	1,49
ed	2503	50	6,99	2714	52	24,56	2763	52	30,63	2763	52	21,58
cloop	10367	149	21,90	10711	149	260,06	10803	149	728,21	21952	151	2402,53
flex	4329	22	16,82	4597	22	94,54	10890	24	397,14	10911	24	209,26
flops	7080	188	0,74	7306	217	0,58				7306	219	0,67
bison	5591	10	24,03	5960	10	152,13	55973	10	1674,14			∞
linpack	385	80	4,57	385	80	10,15	385	80	16,64	385	80	5,31
gzip	4117	24	8,53	4916	24	83,00	5321	24	1604,51			∞
xmodem	3870	12	94,23	4756	12	226,18	4756	12	323,80	7521	12	593,11

Fig. 5. Full Constant Propagation

Fig. 6. Copy Constant Propagation: Percentage of available constants compared to the functional approach

Fig. 7. Copy Constant Propagation: Percentage of constant foldings compared to the functional approach

Fig. 8. Copy Constant Propagation: Runtimes

Fig. 9. Full Constant Propagation: Percentage of available constants compared to the functional approach

Fig. 10. Full Constant Propagation: Percentage of constant foldings compared to the functional approach

Fig. 11. Full Constant Propagation: Runtimes

for the full constant propagation. The first bar charts (Fig. 6, 9) show the relative number of available constants of the call string approaches compared to the functional approach, the second bar charts (Fig. 7, 10) show the same comparison for the number of foldings, and the third bar charts (Fig. 8, 11) show the runtime of the different approaches.

3.1 Results

As can be seen in (Fig. 4, 5) the functional analyzers find always the largest number of constants, if they terminate. As termination is not guaranteed for the full constant propagation and the functional approach, one would not use this in a production compiler. One can also see that runtimes of the functional analyzer are unsatisfying for programs like `twig`, `cloop`, `bison`, and `gzip`.

This can be understood by the discussion from Sect. 2.3, where it is stated, that the runtime of the functional analyzer depends on the locality properties of the analysis and the programs, as well as the programming language. And these are very bad for full ANSI C and the constant propagation.

So one can deduce from the theoretical discussion that the functional analyzer may have bad runtimes for some programs, and in fact this claim is supported by the practical measurements.

On the other hand one does not expect runtime explosions for the call string analyzer for K equals zero and one, since then the number of runs through a

procedure is determined by one or the number of call sites to the procedure, which is small, at least for hand written programs.

The shortcoming of the call string approach is that it may deliver potentially worse analysis results than the functional approach. Also this claim can be verified by the measurements for the theoretical precision of the analyses (Fig. 6, 9). But the practical use that can be gained from this higher precision is very low. Only for `cloop` and `flex` in the full constant propagation the number of foldings is higher for the functional approach than for the call string 1 approach (Fig. 10).

One can also see from the experiments that the full constant propagation delivers more foldings than a copy constant propagation, and has only a minor impact on the run times (Fig. 4, 5).

4 Conclusion and Further Work

Although no new ideas are presented in this paper, it presents a fair experimental evaluation of the two techniques, *call string*- and *functional approach* for interprocedural data flow analysis applied to two data flow problems.

We have shown that a full constant propagation delivers better results than a simple one (Fig. 4 and 5). Furthermore one can see that the set of call strings which should be kept separately (1) is small for real programs, but leads to similar results as the functional approach.

So we can draw the conclusion that an expensive functional approach can be replaced by a cheaper call string approach, at least for constant propagation in languages similar to C.

In the future we will test if these results also hold for other analyses.

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