Comparison of Local External Force Functions for Non-rigid Registration of 3D Medical Images

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Abstract. Non-rigid volumetric registration has many applications, including inter-patient image fusion, motion quantification, and automatic atlas-based segmentation. Computation time is often a limiting factor in using current methods in clinical environments. Minimizing computation time requires both the internal and the external force updates to be as efficient as possible. In this article, we concentrate on the choice of the external force function. We compare different methods based on optical flow and propose a new correlation-based external force function. In addition, we propose an acceleration technique and study its effect on image quality and the speed of convergence. The results indicate that the acceleration technique improves both the speed and quality, and increases the stability of all the external force methods considered.

1 Introduction

Registration is the process of aligning two or more medical images so that respective anatomical features coincide as well as possible. This has a number of uses, for example malignant growths can be more discernible in PET images, which however lack the anatomical details visible in MR images. With image fusion, multiple images can be combined to a single one that shares the good characteristics of different modalities. Registration is also used to study changes with respect to time, either short term such as respiratory or cardiac motion, or long term such as registration of pre and post-operation images [1]. Yet another application area for non-rigid registration is correction of MR images that are geometrically distorted, mainly due to inhomogeneous magnetic field [2].

Intra-patient registration is more challenging, since there is a greater mismatch and even disparity between the anatomical features. Ultimately the method relies on the assumption that people have similar anatomical structures on a macroscopic scale. This method has a number of uses, including the measurement and comparison of physical quantities and shapes related to disorders, and building anatomical atlases for automatic classification of tissue types. Due to the challenges of intra-patient registration it is usually desirable to use the same imaging modality with the same imaging parameters.

The model for spatial displacement depends on the application, but in all but the simplest cases a non-rigid model provides more accurate registration. The

choice of the deformation model depends on the assumed differences between images. They range from rigid or affine models through simple polynomial approximations to more generic spline, grid, or local displacement based models. Usually even the most generic models are invertible, continuous, and smooth, but there are exceptions e.g. when matching pre and post-operative images, the removed tissue may necessitate discontinuities in the model.

Usually, non-rigid registration algorithms consist of separate models for internal and external forces. The external force morphs the images to co-align similar features, whereas the internal force maintains a smooth and bounded transform. The internal force is an a priori model of the deformation. Many models for internal forces have been presented, including ones that model elastic materials, fluid dynamics, springs, snakes, or thin plate splines. For an overview of deformable models, see e.g. [3].

The choice of the external force function depends on the features being matched. The simplest ones aim to balance image intensity levels, but more elaborate features can be used, such as user-given landmarks, feature points or lines, crest lines or surfaces, mutual information, or combinations thereof [4] [5] [6].

Another way to classify external forces is by the size of the neighborhood. Some cross-correlation methods search an optimal position in an area or volume that consists of a large number of pixels, whereas single-pixel forces operate on individual pixels and rely on the internal force for local coherence. Single-pixel forces can be further classified by the range of the force. Attractive forces search within a range for suitable matches, whereas local forces operate only within the immediate neighborhood of the pixels. In the nomenclature of Thirion, the former are *attractor-based* and latter *demon-based* [7]. The demon's algorithm by Thirion has been further analyzed, e.g., in [8].

In this paper, we study the effect of different local external force functions on the overall performance of a non-rigid registration algorithm. A pair of CT-based volumetric test images is constructed for quantitative measurements. A simple yet computationally effective structure is chosen as the deformation model. We present an acceleration technique, which improves both the accuracy and the speed of the algorithm, and a novel external force function, which improves speed performance.

2 Methods

Following the approach by Thirion, we use a vector field defining a displacement vector for each voxel of the moving image. In the starting orientation, the image centers are aligned and all displacements are set to zero. Between applications of the external force, the vectors are smoothed with a 3D Gaussian kernel.

2.1 Force Strength

The most important parameter to guide the algorithm is usually the ratio of the external force strength to the internal force strength. A higher ratio means either

faster convergence or more extreme deformations. There are reasons to limit this ratio. On one hand, too high an external force may allow the model to deform too much. This is undesirable in regions where there is an inherent mismatch between the features of the two images. Even more extreme deformations cause the transform to be non-bijective, in other words, the moving image folds on top of itself.

Another problem of a large relative external force is the tendency for oscillations. Interestingly, these oscillations often stem from regions where the match is already almost complete. Consider e.g. a location where the images are only slightly misaligned in one direction. The discrete iteration step may cause the image to jump to the other side of the optimum location. This process is then repeated, causing oscillations. The way to reduce oscillations would be to make sure that the external force is smaller when the distance to optimum location is small. This may be difficult to attain in practice, and in fact many external force expressions are stronger near the optimum location. The tendency of an algorithm for oscillations is often the limiting factor, which in effect prevents setting the convergence ratio as high as would otherwise be desired.

2.2 Optical Flow

Thirion gives us two expressions for the local external force $\mathbf{p}$, which are based on optical flow between the images f and g [7]. The simpler form requires the calculation of gradient from only the fixed image f . The term $(g - f)^2$ in the denominator is intended to keep the force bounded in cases where the gradient ∇f is close to zero.

$$\mathbf{p} = \frac{(g - f)\nabla f}{\|\nabla f\|^2 + (g - f)^2} \quad (1)$$

It may be useful to introduce a factor to scale the gradient ∇f , as there is not necessarily any natural scaling between a function and its derivative.

The expression (1) may create artificial dilation or compression forces in cases where there is a lower or higher intensity feature in one image against the flat background of another. In general, these anomalies are hard to avoid. The suggested alternative is

$$\mathbf{p} = \frac{4(g - f)\nabla f \|\nabla f\| \|\nabla g\|}{(\|\nabla f\|^2 + \|\nabla g\|^2)(\|\nabla f\|^2 + \|\nabla g\|^2 + 2(g - f)^2)} \quad (2)$$

2.3 Correlation Force

Here we describe a novel computationally effective external force, which is based on correlation. Let $g(\boldsymbol{\xi})$ be a three-dimensional image indexed by an integer vector $\boldsymbol{\xi}$, and $\mathbf{t}(\boldsymbol{\xi})$ is the respective displacement. Also, let $f(\mathbf{p})$ be a tri-linear interpolation of the other image. Now given the translation $\mathbf{t}$, perturb the translation at point $\boldsymbol{\xi}$ by a small displacement vector $\boldsymbol{\delta}_x = s_x \mathbf{i}$. The considered locations

are thus $\mathbf{t}(\boldsymbol{\xi}) - \boldsymbol{\delta}_x$ and $\mathbf{t}(\boldsymbol{\xi}) + \boldsymbol{\delta}_x$, and we calculate the difference of the squared error of aligning $g(\boldsymbol{\xi})$ with either of these points. The result is

$$p_x = 2g(\boldsymbol{\xi})(f(\mathbf{t}(\boldsymbol{\xi}) - \boldsymbol{\delta}_x) - f(\mathbf{t}(\boldsymbol{\xi}) + \boldsymbol{\delta}_x)) + f(\mathbf{t}(\boldsymbol{\xi}) + \boldsymbol{\delta}_x)^2 - f(\mathbf{t}(\boldsymbol{\xi}) - \boldsymbol{\delta}_x)^2 \quad (3)$$

This term can now be directly used as a force component in the x direction. Other components are obtained in an analogous manner.

2.4 External Force Acceleration

Usually there is similarity in the external force steps in consecutive iterations. A naïve approach to take advantage of this by increasing the strength of the external force will only usually lead to oscillations, as explained in section 2.1.

This observation leads us to a new technique called external force acceleration, where consecutive applications of similar external force are accumulated. Whereas the normal update for displacement $\mathbf{t}_i(\boldsymbol{\xi})$ at iteration i , given force $\mathbf{p}_i(\boldsymbol{\xi})$ and external force strength α , is given by

$$\mathbf{t}_i = \mathbf{t}_{i-1} + \alpha \mathbf{p}_i, \quad (4)$$

the accelerated force adds an intermediate accumulation term $\mathbf{a}_i(\boldsymbol{\xi})$ such that

$$\begin{aligned} \mathbf{a}_i &= \beta \mathbf{a}_{i-1} + \alpha \mathbf{p}_i \\ \mathbf{t}_i &= \mathbf{t}_{i-1} + \mathbf{a}_i, \end{aligned} \quad (5)$$

where β is an exponential decay term. A search of values of β with steps of 0.05 shows that $\beta = 0.9$ results in lowest error in this experiment.

The acceleration serves two purposes. First, it accumulates consecutive external force fields pointing to the same direction, thus effectively increasing the force strength. But it also dampens oscillations, since if the external force in subsequent iterations points to the opposite direction, the magnitude of $\mathbf{a}_i$ is reduced.

2.5 Construction of Test Image Pair

Our objective is to study the effect of different external force functions. For this purpose, a pair of test images is needed. In the literature, different test settings have been used for this purpose. One possibility is to use images from two different patients and to manually delineate the structures, and then calculate which fraction of pixels would be correctly delineated by applying the result of the deformation to the pixels. But it is difficult to separate the effect of differences in manual delineation from errors in registration. Another possibility would be to segment one of the images and to do the mapping twice by exchanging the roles of moving and target images [7]. But this measures more the symmetry of the non-rigid registration method than the actual registration quality.

In this study, we constructed a pair of test images by taking a 3D CT image set and deforming it artificially with a sinusoidal deformation field that is applied

to each pixel with trilinear interpolation. The deformation field was created by applying a displacement of amplitude $a_z \sin(\omega_x x) \sin(\omega_y y)$ to all pixels in the z column. Similar displacement was applied to x and y columns. The amplitudes were 10 mm peak to peak. For the test, the images were subsampled to $96 \times 126 \times 74$ size, with a pixel spacing of 1.7, 1.7 and 2.0 mm. The ω parameters were chosen such that the wavelength was exactly 25% of the respective image axis. An example slice from both 3D image sets are shown in Fig. 1. The original image and the artificially deformed image are then used to test the algorithms. The test image pair is constructed so that the deformations are moderately but not prohibitively large, similar to those found in inter-patient registration.

The quality of the resulting match could be estimated directly by comparing the resulting deformation field to the original one. This measure could however be inaccurate in large flat regions. So instead we calculate an error measure directly as the sum of squared differences of the intensity values.

Fig. 1. The slice on the left is a transversal slice of the test 3D image. The slice on the right is from the artificially deformed 3D image, which is obtained by applying a sinusoid-based deformation field to the image on the left. The algorithm is tested using this 3D image pair, each image containing 74 slices.

3 Results

We compare the result of non-rigid registration with three different expressions for external force: the demons force in equation (1), the enhanced demons force in equation (2), and the correlation force in equation (3). In addition, we studied the effect of applying the acceleration enhancement proposed in (5). The internal force strength, characterized by the radius σ of the Gaussian kernel, was set to $\sigma = 5.0$ mm in all experiments.

3.1 Sensitivity to External Force Strength

Each of the algorithms has an external force parameter, and the scaling depends on the algorithm. Usually this parameter has to be determined experimentally. We studied the sensitivity of the algorithm to variations in this parameter.

The least-squares error was determined after 40 iterations. The error measure was relatively stable after 10 iterations, but 30 more iterations were run to make sure that the final result is reached. The error measure is only intended for relative comparison of the algorithms. Also the relative force strength varies between algorithms, so the location of the minimum may change, but the overall shape of the curve reveals the sensitivity of the chosen external force model. A low smooth plateau is a desirable shape.

As shown in Fig. 2, the behavior was as expected: increasing the external force first leads to better results, but eventually leads to oscillations, and the error starts to rise sharply. Of the three force expressions considered, the best results were obtained by the standard demons force. On the other hand, this force has the sharpest rise after the minimum, demonstrating its sensitivity to the choice of the external force strength.

The most noteworthy observation from the experiment is however that all three force expressions significantly benefit from the acceleration method, with an order of magnitude improvement on the error metric.

3.2 Convergence

The convergence of the algorithms was studied by calculating the error metric at each iteration. The strength of the external force was set to the optimum value for each algorithm, obtained at earlier experiment.

In all cases, convergence was rapid, as shown in Fig. 3. The relative order of the algorithms was established very early, and after 10 iterations there was very little change. It should especially be noted that all the accelerated forces considered here behave in a very similar way.

3.3 Performance

The algorithm has been implemented in optimized C++ using Microsoft Visual Studio 6.0. The experiment was performed on a $2 \times 1.7\text{GHz}$ Intel Pentium IV. The implementation is multi-threaded for maximum speed. The run time of the algorithms did not vary significantly and was within the range of 0.93 s to 1.07 s for each full iteration of the 3D image, with the non-accelerated correlation force being the fastest. About 75% was spent on application of the external force and the rest to the Gaussian shaped internal force.

4 Discussion and Conclusions

In our experiment, it appears that the differences between external force functions are rather small. However, the acceleration of the external force resulted

Fig. 2. The relative error of the different external force functions, as measured by sum of squares difference. The normal and enhanced demons forces are compared with correlation-based force. In addition, the effect of acceleration on each force expression is calculated.

Fig. 3. The error function recorded at each iteration of the algorithm for the first 14 iterations. The error does not significantly improve after 10 iterations. The external force strength was at optimum setting, and as a result, the non-accelerated forces already show signs of oscillations.

an order of magnitude improvement in the minimum of the registration error. It should also be noted that the acceleration method dramatically reduces the quality differences between external force functions, allowing the use of computationally simpler forms.

The acceleration method also appears to decrease the sensitivity to the choice of the external force strength. Since this parameter usually needs to be determined experimentally, a good initial guess can be of great benefit.

Using per-voxel displacement vectors may seem too fine-grained and inefficient. The aim is to have a computationally simple structure, and per-voxel displacements are very efficient to evaluate for both the internal and external forces. But it is true that the simplicity and efficiency are obtained partly at the expense of increased memory consumption. Per-voxel displacement will necessitate allocation of three floating-point values for each image voxel, typically 6 times that of a 16-bit volume image. A similar trade-off applies to the accelerated external force technique. It requires the storage of another vector field, further doubling the memory consumption.

The test image pair used in the experiment was intended to have similar deformations than those observed in inter-patient registration, and yet allow for quantitative analysis. We have also run similar tests with other types of test

images and also more qualitative tests with real patient images, and the results agree with those presented here.

Given the results obtained with the external force acceleration method, the question arises whether acceleration could be applied to the internal force as well. In our initial experiments it appears that there is no similar benefit. The internal force is a reactive force with the purpose of counter-acting the external force, indicating that resistance to change is a useful property.

In this paper, we have presented two novel methods for non-rigid matching, namely the correlation-based external force and the acceleration technique. We have shown that the correlation-based force provides minor improvements in calculation speed, and the external force acceleration is an excellent method that can be used to improve both the speed and the quality of an external force function and to increase the stability.

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