A Modified Total Variation Denoising Method in the Context of 3D Ultrasound Images

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Abstract. Ultrasound volumes are corrupted by a multiplicative noise, the speckle, which makes high level analysis difficult. Within each resolution cell a number of elementary scatterers reflects the incident wave toward the sensor. This paper proposes a method of restoration based on variational principles adapted to ultrasound images statistics. The contribution of this paper is two-fold: we first derive a modified TV scheme to integrate the multiplicative nature of the ultrasound noise and we propose to tune the parameter λ automatically accordingly to the local noise distribution thanks to the kurtosis information. We present qualitative and quantitative results on various ultrasound volumes.

1 Introduction

Almost invasive nature, low cost and real time image formation, all of these features make ultrasound images an essential tool for medical diagnosis. This article aims at improving the use of ultrasound images so as to improve diagnosis and therapeutic treatment. However, the presence of multiplicative noise [1], called speckle, makes the interpretation of these images difficult.

This phenomenon is a common feature to all the images produced by coherent systems (SAR, laser, ultrasound, ...). For each resolution cell, it renders the spatial arrangement of the observed environment in terms of different scatterings which are not discriminated by ultrasound waves. The backscattered coherent waves with different phases undergo a constructive or a destructive interference in a random manner. Speckle statistical properties are linked to those of elementary scattering which make up the scanned area. We take into account this modeling to adjust our model based on total variation method, proposed by Rudin, Osher and Fatemi [2].

After a brief presentation of ultrasound images statistics, we will discuss briefly variational methods theory before describing our methods and comparing it to existing ones.

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Table 1. Applications of the statistical models in ultrasonic imagery

Domain	The most adapted model(s)
cardiology	K-distribution
obstetrics and gynecology	K-distribution
abdominal	Rice, Homodyned K-distribution
Doppler	Rayleigh

2 Modeling Ultrasound Images

B-scan ultrasound images represent the back-scattering of an ultrasound beam from structures inside the body. Four models of noise are generally proposed to characterize an ultrasound image. Each noise distribution depends on two criteria, the density and the type of scatterers. Under some conditions, the classical distribution used for a random walk phenomenon is a Rayleigh distribution. It is valid only for a fully developed speckle, i.e. with a large number of incoherent scatterers. According to the number of scatterers and their space arrangement, several statistical models have been introduced. The type and distribution of scatterers is intrinsically related to the type of tissue that the ultrasound beam is passing through. Therefore this modeling is particularly adapted for restoration and segmentation purposes.

The K distribution is a generalization of the rayleigh distribution and models the presence of few diffuse scatterers. The Rician distribution is another generalization of the Rayleigh distribution and describes the presence of many coherent scatterers. The last one, introduced by Jakeman [3], the homodyned K distribution, describes the presence of few coherent scatterers.

Pragger [4] proposes to summarize these different cases (see Fig.2, left). We indicate in Tab.1 the comon application domains of these noise distribution.

Fig. 1. Left: Distributions of noise in ultrasound images. Knowing the type and density of scatterers, appropriate distributions model the noise of ultrasound images. This illustrates the relationship between the type of noise and the type of tissue that the ultrasound beam is imaging. Middle and right: Ultrasound Image and corresponding kurtosis. The kurtosis is a relevant information to characterize edges and regions in the context of ultrasound images.

3 The Total Variation (TV) Restoration Concept

The TV anisotropic diffusion model was introduced by Rudin, Osher and Fatemi [2], and is now a popular tool for image restoration. The TV method tends to approximate the original image $u(x, y)$ by minimizing an energy.

To estimate a true signal in noise, the most frequently used methods are based on the least squares criteria which L_2 norm depends on. Rudin and Osher resort to the L_1 norm, which provides some discontinuities. Nevertheless, in practice, numerical approximation offers good results without the residual oscillations, equivalently to the Wiener filter.

In fact, Rudin and Osher propose to estimate an image u that minimizes the following energy:

$$E(u) = \int_{\Omega} |\nabla u| + \lambda \int_{\Omega} (u - u_0)^2 \quad (1)$$

where Ω is the image domain and λ is a real positive parameter. Minimizing this energy on a set of functions amounts to finding a function u_0 penalizing the oscillations. The Euler-Lagrange equation which corresponds to this energy is:

$$-\nabla u + \lambda(u - u_0) = 0,$$

For an additive noise $\sim \mathcal{N}(0, \sigma)$, this leads to the following constraints:

$$\int_{\Omega} u dx = \int_{\Omega} u_0 dx \text{ and } \frac{1}{|\Omega|} \int_{\Omega} (u - u_0)^2 dx = \sigma^2. \quad (2)$$

The first constraint is satisfied thanks to TV norm translation invariance ($TV[u+c] = TV[u]$), and the corresponding gradient descent becomes:

$$\frac{\partial u}{\partial t} = \text{div}\left(\frac{\nabla u}{|\nabla u|}\right) + \lambda(u_0 - u), \quad (3)$$

To avoid singularities when $\nabla u = 0$, we modify the denominator of the equation (3) as mentioned in [6], and more precisely

$$-\nabla \cdot \left(\frac{\nabla u}{\sqrt{|\nabla u|^2 + \epsilon}} \right) + \lambda(u - u_0) = 0 \text{ with } \epsilon > 0. \quad (4)$$

The steady state solution of this equation is the denoised image.

4 Application to Ultrasound Images

The purpose of this paper is to propose a modified TV restoration scheme adapted to the specific nature of ultrasound images. The afore mentioned statistics of the ultrasound images make difficult their integration in the numerical scheme directly. The contribution of this paper is two-fold: (a) we first derive a modified TV scheme to integrate the multiplicative nature of the ultrasound noise in subsection 4.1 and (b) we propose to tune the parameter λ automatically accordingly to the local noise distribution thanks to the kurtosis information in section 4.2.

Fig. 2. Comparison of the original TV and our modified TV scheme for three ultrasound volumes. Top row: abdominal phantom, middle row: Carotid sequence and bottom row: Egg-shape phantom. Left column: original image, middle column: original TV scheme and right column: our modified TV scheme (multiplicative constraint and kurtosis-adaptive λ). Our modified TV scheme was able to smooth homogeneous regions while improving the edges of anatomical structures.

4.1 Multiplicative Noise

We take into account the multiplicative nature of the noise in the formulation of the total variation. The image u is multiplied by a noise η

$$u_0 = u\eta$$

Fig. 3. Line profiles of the egg-shape phantom. Top left: image with white line for which the profile is computed. We compare image profiles of the original image, of the original TV scheme and of our modified TV scheme. We observe a reduction of the oscillation on homogeneous area and the enhancement of edges.

with the assumption of η following a normal distribution $\sim \mathcal{N}(0, \sigma)$. The constraints thus become:

$$\int \frac{u_0}{u} = 1, \quad \int \left(\frac{u_0}{u} - 1 \right)^2 = \sigma^2.$$

A total variation denoising is applied on the image following the above constraints.

4.2 Kurtosis as a Tissue and Edge Descriptor

The kurtosis β_2 is a statistical measure corresponding to the fourth central moment. It measures the degree of steepness of the intensities distribution relative to a normal distribution. Kurtosis is defined as

$$\beta_2(x) = \frac{1}{N} \sum_{i \in V(x)} \left(\frac{x_i - \mu}{\sigma} \right)^4, \quad (5)$$

where $V(x)$ is the neighborhood of point x , N is the number of neighbors, μ is the mean gray level, σ is the standard deviation.

In the context of ultrasound images, a recent paper [5] characterizes the type of noise (Rayleigh, K, ...) according to the value of kurtosis. This value gives information about the type and density of the scatterers, consequently about the nature of the tissues. Therefore, it is a relevant information to be used for a restoration method as a region descriptor in the context of ultrasound images. In addition to this, the kurtosis is very sensitive to high transitions as any other high moment. Therefore it is a relevant information to characterize the edges of the image to be restored. Fig. 2 shows a typical kurtosis image.

Fig. 4. Left: Gradient of phantom abdomen, Middle: gradient of denoised image using the original TV scheme. Right: gradient of denoised image using our modified TV scheme.

The parameter λ , which is the balance between the data term and the regularization term, can then be automatically tuned accordingly to the kurtosis in the following way:

$$\forall x \in \Omega, \lambda(x) = \frac{\beta_2^3(x)}{\max_{(\Omega)}(\beta_2^3)}$$

For a contour point, λ is maximum and the restoration scheme will preserve the contour. On the contrary, λ is minimal in homogeneous areas leading to a higher regularization.

5 Results

All the volume considered were acquired using a 3 free-hand ultrasound system. We present results on three types of images: a carotid (size $220 \times 240 \times 200$), the acquisition of a 3D multimodal abdominal phantom (size $531 \times 422 \times 127$) and a ultrasound phantom (egg shape, size $143 \times 125 \times 212$). The results presented here are the steady state of the restoration algorithm. Computation time on a standard PC are approximately 60 seconds, depending on the volume size.

5.1 Qualitative Assessment

The result of the denoising (Fig. 4) on the three kinds of volumes, with different number of scatterers, and thus different statistics, are visually satisfying. Indeed, for the three volumes, we obtain an piecewise smooth image with a good preservation of the structures. Our modified TV scheme seems visually better at smoothing homogeneous regions while improving the edges of anatomical structures.

5.2 Quantitative Assessment

Quantitative assessment in the context of restoration is an open problem. We here present some experiments including moments of the restored image on homogeneous areas, restored gradient images and line profiles of the images.

Table 2. Comparison of variance and mean in a 3D context on homogeneous areas.

Homogeneous area	Mean	Variance
Abdomen	50.4	289.6
Classical TV3D	53.5	66.4
TV3D, additive constraint, kurtosis based λ	53.5	66.4
TV3D, multiplicative constraint, kurtosis based λ	58.31	64.19
Carotid	54.1	74.1
Classical TV3D	67.4	36
TV3D, additive constraint, kurtosis based λ	67.4	36
TV3D, multiplicative constraint, kurtosis based λ	68.91	35.2
Egg	87.8	379.12
Classical TV3D with $\lambda = 0.1$	78.7	176.61
Classical TV3D with $\lambda = 0.01$	95.5	35
TV3D, additive constraint, kurtosis based λ	97.03	42
TV3D, multiplicative constraint, kurtosis based λ	96.18	58

Firstly, we have specified homogeneous areas -anatomically speaking - for each image sequence. Within these areas, we compute the mean and variance of image intensities. Results are presented in table 2. We compare three methods here: the original TV scheme, a first modified TV scheme (additive constraint and kurtosis-adaptive λ) and our modified TV scheme (multiplicative constraint and kurtosis-adaptive λ). We also present results with different values of parameter λ for the original TV scheme.

For the abdomen and carotid volumes, we obtain similar or better results than classical TV without tuning any parameter. Better results are obtained taking into account a multiplicative noise. The results concerning the egg volume point out the fact that the coefficient λ is critical in the classical scheme. An optimal tuning of this parameter leads to good results with the original TV scheme. For the abdominal and carotid sequence, our modified TV scheme gave better results than the original TV scheme, even for a wide range of variation for parameter λ . Based on these results, we now only compare the original TV scheme and our modified TV scheme (multiplicative constraint and kurtosis-adaptive λ). For the egg-shaped sequence, we present the profile of the image along a given line in figure 4.1. This result shows the effectiveness of our modified TV scheme since edges are enhanced and homogeneous areas are correctly smoothed.

We have also computed the image gradient. Results are presented in figure 4.2. Results show that in both restoration cases, edges are enhanced as denoted by strong image gradients. In our modified TV scheme, image gradients appear more consistent in terms of “shape”.

6 Conclusion

After describing ultrasound images and fundamental concepts inherent to analysis by total variation, we propose to adapt the TV restoration scheme in two ways: we first include the constraint of a multiplicative noise and introduce a kurtosis-based parameter. In the context of ultrasound images, kurtosis is related to noise distribution and therefore to tissue properties indeed. This leads to an automatic tuning of parameter λ , which we think is useful since we experienced that the tuning of this parameter can be problematic.

Qualitative and quantitative results on various image sequences have shown the effectiveness of our modified TV scheme in smoothing homogeneous regions while enhancing edges.

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