Fast Non-linear Elastic Registration in 2D Medical Image

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Abstract. Non-linear image registration can match two images' local areas exactly. Linear elastic model that obeys the Navier-stokes equilibrium equations is especially fit for image registration with small deformation. Bro-Nielsen derived a linear elastic convolution filter from the eigenfunctions of the Navier-stokes differential operator. Based on the elastic filter, the elastic partial differential equation (PDE) is easy to be solved although the filter is mainly used in viscous PDE of fluid model at first. Gaussian filter used in the 'demon'-based registration method of Thrion could be regarded as an approximation to the elastic filter. Because of the complexity of the elastic filter and the poor performance of the gaussian filter, we propose a new simple filter, two-sided exponential filter, to approach the elastic filter. The results of experiments also show the new filter improves the algorithm's convergence speed and the precision greatly and its performance is superior to the other two filters.

1 Introduction

Image registration, which aims to match two images by searching a best transformation between them, has been widely used in many fields. Especially in medical image field, changes in different images of the same object over periods of time need to be observed, or the images between patient and normal personal need to be compared. So it is very important for the involved images to be matched in structure. Usually there exist two kinds of matching algorithm in medical image registration. They are linear and non-linear registration. Linear registration, such as rigid or affine transformation, is often applied to matching two images in global shape, while non-linear registration can match two images locally and finely. In most cases, non-linear matching algorithm also need initial linear alignment.

Non-linear registration usually transforms images in higher-dimensional anatomical mappings in order to match local variability across different anatomies. Physical continuum models can be used in non-linear registration because they allow extremely flexible deformations and the degrees of freedom can be as many as the number of the voxels in the images. Bajcsy et al. [2] were the first to construct the deforming image as 3D elastic solid and derived the body force from the gradient of the intensity cor

relation. Because of the limitation of small deformation assumptions for linear elasticity model, Christensen [5] proposed a viscous fluid model to estimate large image deformations for keeping the non-linear topological behavior. In this model, internal restoring forces that exist in the linear elastic model disappear and large magnitude displacement can't be penalized any more by regularization methods based on linear elasticity. So the precise deformation can be achieved. However, because the viscous fluid PDE is solved on a discrete lattice, the speed of the algorithm is too slow to be implemented on PC.

Based on the work of Christensen, Bro-Nielsen [3] derived a convolution filter for linear elasticity from the eigenfunctions of the Navier-stokes differential operator $L = \mu \nabla^2 + (\lambda + \mu) \nabla(\nabla \cdot)$. And the algorithm speed got an increase of several orders of magnitude. Although the elastic filter improved the algorithm, the time cost of producing the filter still increased exponentially with the size of the image. And it will be much more difficult to get the filter in the 3D images. Therefore, the more easily the filter can be got, the more efficient the algorithm will be. In fact Bro-Nielsen [3] also demonstrated that the 'demon'-based registration method of Thirion [11] could be seen as using a gaussian filter to approach the elastic filter. Because big differences exist between the two filters, gaussian filter still has many disadvantages.

And because Navier-stokes differential operator can be extracted from elastic as well as fluid model, the linear elastic filter can be applied to the two models easily. By viewing the global shape of the 2D elastic filter, we find it is more like the shape of two-sided exponential, especially in the middle areas acted by driving force. And because two-side exponential filter is also lowpass just like the elastic filter, we propose the new simple filter as an approximation of the elastic filter. In our work, MR images from two different subjects are matched in global shape initially then we mainly apply elastic registration by using the new two-side exponential filter. Comparing the three filters, the performance of the new filter is obviously better than the gaussian filter, its final precision can almost reach the same level as the elastic filter and its algorithm convergence speed is much faster than the elastic filter.

2 Theory

2.1 Image Registration Based on Elastic Models

We assume two images, an object image (O) and a template image (T), are to be registered. And a best smooth deformation field $\mathbf{u}(\mathbf{x})$ needs to be found, where $\mathbf{x}$ is the particles of O. When applying the transform $\mathbf{u}(\mathbf{x})$ to the coordinates of the particles in O, the transformed image should match the template image very well.

Considering the deformed image to be embed in elastic media, the displacement field $\mathbf{u}(\mathbf{x})$ resulting from internal deformation forces $\mathbf{F}(\mathbf{x})$, which is called body force, obeys the Navier-stokes equilibrium equations for linear elasticity [12].

$$\mu \nabla^2 \mathbf{u}(\mathbf{x}) + (\lambda + \mu) \nabla(\nabla^T \cdot \mathbf{u}(\mathbf{x})) + \mathbf{F}(\mathbf{x}) = 0 \quad \forall \mathbf{x} \in R \quad (1)$$

Here $\nabla^T \cdot \mathbf{u}(\mathbf{x}) = \sum \partial u_j / \partial x_j$, ∇^2 is the Laplacian operation. Lamé's coefficients λ and μ controls the elastic properties of the medium. Body force can be derived from the gradient of a cost function, such as intensity correlation.

2.2 Convolution Filters in the Solution of the Navier-Stokes Equation

From equation 1, we can see PDE must be solved in order to get deformation field. In order to simplify the solving of PDE, Bro-Nielsen developed the convolution filter to solve the linear PDE [3]. A Navier-stokes differential operator L can be extracted from equation 1.

$$L = \mu \nabla^2 + (\lambda + \mu) \nabla(\nabla \cdot) \quad (2)$$

When applying the linear operator L to the corresponding variable $\mathbf{u}(\mathbf{x})$, elastic models can be simplified like equation 3.

$$L\mathbf{u}(\mathbf{x}) + \mathbf{F}(\mathbf{x}) = 0 \quad (3)$$

In fact, equation 3 describes a linear system with $\mathbf{F}(\mathbf{x})$ as the input and $\mathbf{u}(\mathbf{x})$ as the output. Based on the eigen-functions of the linear operator L , Bro-Nielsen derived the filter $\mathbf{H}$ as an approximation to the impulse response of an applied unit force in the middle of an image. A linear filter of size $D \times D$ for the x direction has the form of Equation 4.

$$H_1(\mathbf{x}) = \frac{4}{\pi^2 \mu (2\mu + \lambda)} \sum_{i,j=0}^{D-1} \frac{p(c)}{(i^2 + j^2)^2 \Gamma} ij \begin{bmatrix} -(i^2 \mu + (2\mu + \lambda)j^2)p(\mathbf{x}) \\ (\mu + \lambda)ijq(\mathbf{x}) \end{bmatrix} \quad (4)$$

And detailed derivation can refer to Bro-Nielsen in [3]. However, time cost to create the filter increases rapidly with the size of image.

2.3 Properties of Linear Elastic Filter

As $\mathbf{H}_1(\mathbf{x})$ is a vector field with a unit force acting in x direction, each point in the field is a vector that consists of two components in x and y dimensions separately. So $\mathbf{H}_1(\mathbf{x})$ can be decomposed into two scalar fields, $H_x(\mathbf{x})$ along x direction and $H_y(\mathbf{x})$ along y direction. Fig. 1 shows displacement field of $\mathbf{H}_1(\mathbf{x})$ with size 15×15 . Force from x direction makes the $H_x(\mathbf{x})$ must be much larger than $H_y(\mathbf{x})$. Then $H_y(\mathbf{x})$ can also be ignored for simplicity.

Because a unit force acts on the center of the field, the center displacement around the force direction is much larger than the other areas. From fig. 1, we can see the scalar field $H_x(\mathbf{x})$ is almost symmetrical about x and y directions and the displacements of the five middle rows of x and y can nearly dominate the whole field. In order to study the property of the filter further, the middle five rows and columns of $H_x(\mathbf{x})$, corresponding to x and y direction separately, are extracted. And only three of them are drawn because of the field's symmetry.

Fig. 1. Displacement vector field $H_1(x)$ (left) and the scalar field $H_x(x)$ of $H_1(x)$ shown in 3D space (right).

Fig. 2 shows those curves' shape along x and y directions are very similar. The main difference among them is that the curves' width along x direction is larger than that along y direction. That demonstrates the filter must be anisotropic.

In order to contrast gaussian filter with elastic filter, three middle rows in x and y direction of gaussian filter are also drawn in fig. 3. Comparing fig. 3 with fig. 2, we can see there are big differences between them.

2.4 2D Two-Sided Exponential Filter

Some problems may appear in the stability and precision of the deformation fields when applying the gaussian filter although the filter is very easy to be created. So the main work becomes whether we can find a new simple lowpass filter that is closer to the elastic filter than gaussian. When observing fig. 2 and 3 carefully, we can see that the curve's shape in fig. 2 is sharper and isn't like gaussian function at all, but more like two-sided exponential function in equation 5.

1D two-sided exponential filter:

$$H(x) = e^{\left(\frac{-|x|}{\alpha}\right)} \quad (5)$$

2D two-sided exponential filter:

$$H_1(x, y) = e^{\left(\frac{-|x|}{\alpha_1} \frac{|y|}{\alpha_2}\right)} \quad H_2(x, y) = e^{\left(\frac{-|x|}{\alpha_2} \frac{|y|}{\alpha_1}\right)} \quad (6)$$

Here the parameters α_1 and α_2 control the variation of data just like σ in gaussian function. If the two parameters are not identical, this filter is anisotropic too.

The subscript 1 and 2 of $H(x, y)$ denotes that the center unit force comes from x and y direction respectively.

Obviously two-sided exponential filter's shape (Fig. 4) is much more similar to linear elastic filter (Fig. 2) than that of Gaussian filter (Fig. 3). Therefore it can be inferred that the new filter must be superior to the Gaussian approximation and can also be created easily.

Fig. 2. Three middle rows of the scalar field $H_x(x)$ with 128×128 size along x (left) and y(right) direction. Row 62, 63 and 64 correspond to dashed, solid and dotted line.

Fig. 3. Three middle rows of gaussian scalar field with 128×128 size along x (left) and y(right) direction. Row 62, 63 and 64 correspond to dashed, solid and dotted line.

Fig. 4. Three middle rows of two-sided exponential scalar field with 128×128 size along x(left) and y(right) direction. Row 62, 63 and 64 correspond to dashed, solid and dotted line.

3 Experiments and Results

Two experiments based on elastic model are presented to demonstrate how the two-sided exponential filter is superior to the other two. And we focus on comparing the registration precision, convergence speed and time cost of the three filters under the

same iterative times after the filters have been created. All programs were run in Matlab 6.1 software.

3.1 2D Square to Rectangular

In the first experiment, the object image was a square and the template was a rectangular (see fig. 5A). The dimension of the images is 128×128 pixels. Three filters, elastic, gaussian and two-sided exponential filters, were used to transform the square into the rectangular. Among three filters, two-sided exponential filter spent the least time, which is 4.7 seconds, with 120 iterative times, while elastic filter took 6.45 seconds and gaussian filter took 9.85 seconds. Fig. 5B shows that two-sided exponential filter got better precision than the other two filters. And from Fig. 5C, we can see that the convergence speed of two-sided exponential filter is also the fastest while the gaussian filter is the slowest among them.

Fig. 5. A, Object (left) image and template (right) image. B, Transformed images after 120 iterative times using the linear elastic (left), Gaussian (middle) and two-sided exponential filter (right) respectively. And their corresponding mean squared errors are 4.20×10^{-4} , 0.0137 and 1.54×10^{-7} . C, Changes of mean squared error with iterative times, dotted line denotes elastic filter and the dashed line denotes gaussian filter.

3.2 2D MR Images of Human Brain

In medical field, medical images are often need to be matched so that some regional areas can be compared. In order to observe how the nonlinear transformation acted on image locally, we selected two 2D MR images from 3D MRI brain data of two subjects. The size of two images is 91×109 pixels. Only two-sided exponential filter and elastic filter are compared from the speed of convergence and the result precision.

Fig. 6. A, Object (left) image and template (right) image. B, Transformed images after 120 iterative times using the linear elastic (left) and two-sided exponential filter (right). And their corresponding mean squared errors are 0.095 and 0.066. C, Changes of mean squared error with iterative times. The solid line denotes two-sided exponential filter and dotted line denotes elastic filter.

Fig. 6 shows that after nonlinear transformation, the original object image's global shape is almost consistent with the template and some local areas are also matched. Two-sided exponential filter get better result than elastic filter. And its convergence speed is also much faster.

4 Conclusion

Fast fluid registration based on the linear elastic filter has been a classical method for large displacement field. And because of the similarity between fluid and elastic model, the elastic filter can be used in the elastic model for small deformation. As the filter is based on the perfect physical model theory, it is also very robust for image registration in any cases. However, the creation speed of the linear elastic filter becomes slower and slower with the increase of the image size, which may increase the computation time cost greatly and reduce the algorithm's efficiency and practicability. When the new two-sided exponential filter is used in elastic model, the algorithm's time efficiency and result's precision are improved greatly because it has the following advantages. Firstly the filter's creation and the extension to 3D cases become much simpler than the elastic filter for images with any sizes. And that can make it very practicable especially for large size images. Secondly, the property of the new filter is much closer to the elastic filter than the Gaussian filter. So the result won't be worse than that of Gaussian filter and sometimes can even reach or overpass the elastic filter's precision. Lastly, its separable property, just like gaussian filter, improves the convolution speed greatly especially for 3D images with large sizes. It is all these attractive advantages that make the novel filter superior to the other filters in some aspects.

There are still some shortages in the two-sided exponential filter. Firstly, it is almost impossible for the two-sided exponential filter to have the same parameters in any cases. Specific images may correspond to specific parameters. And a bad parameter can degenerate deformation field greatly. So the parameters must be selected very carefully. Secondly, if we observe the two filters carefully in 3D space, we will find that the elastic filter's shape is much smoother than the two-sided exponential filter, which also makes its deformation field is smoother than the two-sided exponential filter. So the new filter may not be as robust as the elastic filter. And how to find a filter, which is smoother, more convenient and closer to the elastic filter than the two-sided exponential filter, is also worth studying further.

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