Velocity Based Segmentation in Phase Contrast MRI Images^{*}

Jan Erik Solem, Markus Persson, and Anders Heyden

School of Technology and Society
Malmö University, Sweden

{jes,markus.persson,heyden}@ts.mah.se

Abstract. Phase contrast MRI is a relatively new technique that extends standard MRI by obtaining flow information for tissue in the human body. The output data is a velocity vector field in a three-dimensional (3D) volume. This paper presents a method for 3D segmentation of blood vessels and determining the surface of the inner wall using this vector field. The proposed method uses a variational formulation and the segmentation is performed using the level set method. The purpose of this paper is to show that it is possible to perform segmentation using only velocity data which would indicate that velocity is a strong cue in these types of segmentation problems. This is shown in experiments. A novel vector field discontinuity detector is introduced and used in the variational formulation. The performance of this measure is tested with satisfactory results.

1 Introduction

Magnetic Resonance Imaging (MRI) provides three-dimensional (3D) images that are useful for diagnostic purposes. Phase contrast MRI also provides velocity measurements that can be used for analysis of the blood flow and tissue motion. Segmentation is a well studied problem within medical image analysis and have great impact on diagnostic performance. This paper presents a method for 3D segmentation of blood vessels. After segmentation, when the shape of the vessels has been determined, volumetric flow data can be obtained from the velocities. Blood volume, pressure and velocity of the blood flow and the motion and shape of the vessel walls are examples of useful measurements that can aid in the diagnosis and are also important for research within the medical field.

This paper deals with the problem of segmentation using velocity data without the assumption that the walls are stationary. This is the case for many medical images taken for diagnostic purposes, such as dynamic cardiac images. Many different approaches have been proposed to analyze and extract shape information from cardiac images and 3D model-based methods have shown to improve the diagnostic value [1].

The most basic approach to segmentation is to threshold the images. However, this method only works on very simple images. Later approaches makes

^{*} This work has been supported by the SRC project 621-2002-4822.

use of moving interfaces, e.g. snakes and other active contours, cf. [2]. During the latest years level set approaches have become popular, since they can handle global properties and are based on a solid mathematical framework, cf. [3,4].

The level set method [5] is a popular technique for representing and tracking dynamic interfaces. The surface is represented implicitly as a level set of a function. The sign of the level set function gives a natural partitioning and is frequently used in segmentation. Our segmentation problem is solved using a variational level set framework. The optimal solution, i.e. the optimal position of the surface separating the blood from the vessel walls, corresponds to the minimum of a functional driven by the velocity data.

1.1 Relation to Previous Work

In [6] blood is segmented in cardiac phase contrast MR images using the fact that the heart wall has a periodic motion and resumes its position after a completed heart cycle. The method is based on a particle trace technique for time-resolved 3D velocity vector fields, combined with magnitude image data. In [7] the myocardium is segmented from MRI intensity and phase contrast images. The segmentation is performed in 2D images using level set curve evolution. Three different constraints determine the curve motion, the intensity gradient, the velocity magnitude and the coherence of the velocity direction. In [8] segmentation of curvilinear structures in MR angiography images is performed using evolution techniques for implicit curves. In [9] blood vessels are segmented from MR angiography images using the criterion that the normal of the blood vessel boundary should be orthogonal to the vector flow field and thus minimizes the flux through the surface. The underlying assumption is that the velocity of the vessel wall is zero. Our proposed method solves the segmentation problem with moving vessel walls. An approach like the one in [9] would not work since the velocity field of the walls are approximately orthogonal to the blood flow.

1.2 Contribution of the Paper

This paper presents a method for three-dimensional segmentation of blood vessels and determining the surface of the vessel wall using the vector field obtained from phase contrast MRI measurements. It improves upon the performance of previous methods in that it can handle wall motion, a feature that is lacking in e.g. [9]. A measure for finding discontinuities in this vector field is introduced. The segmentation problem is then formulated as a variational problem with a new functional using this measure. The proposed method uses only the information contained in the velocity vector field and this gives support to the claims that velocity is an important cue for these types of segmentation problems.

2 Background

2.1 Phase Contrast MRI

Phase contrast MRI is based on the property that a uniform motion of tissue in a magnetic field gradient produces a change in the MR signal phase, Φ . This change is proportional to the velocity of the tissue, $\mathbf{v}$. The MR signal from a volume element accumulates the phase [10]

$$\Phi(r, t) = \gamma B_0 T + \gamma \mathbf{v} \cdot \int_0^T \mathbf{G}(r, t) dt \ ,$$

$$\Phi(r, t) = \gamma B_0 T + \gamma \mathbf{v} \cdot \overline{\mathbf{G}} \ , \quad (1)$$

during time T , where B_0 is a static magnetic field, γ the gyro-magnetic ratio and $\mathbf{G}(r, t)$ is the magnetic field gradient. Notice that $\overline{\mathbf{G}}$ is exactly the first moment of $\mathbf{G}(r, t)$ with respect to time. If the field gradient is altered between two consecutive recordings, then by subtracting the resulting phases

$$\Phi_1 - \Phi_2 = \gamma \mathbf{v} \cdot (\overline{\mathbf{G}}_1 - \overline{\mathbf{G}}_2) \ , \quad (2)$$

the velocity in the $(\overline{\mathbf{G}}_1 - \overline{\mathbf{G}}_2)$ -direction is implicitly given. In this way a desired velocity component can be calculated for every volume element simultaneously. To construct the velocity vector in 3D, the natural way is to apply appropriate gradients to produce the x-, y- and z-components respectively.

Fig. 1. Examples of the velocity vector field for a transversal slice with the heart in the center of the image. (left to right) v_x , v_y , and v_z -velocity component and velocity magnitude $|\mathbf{v}|$.

2.2 Level Set Formulation

The level set method was introduced in [5] as a tool for capturing moving interfaces. The time dependent surface $\Gamma(t)$ is implicitly represented as the zero level set of a function $\phi(\mathbf{x}, t) : R^3 \times R_+ \rightarrow R$ as

$$\Gamma(t) = \{\mathbf{x} ; \phi(\mathbf{x}, t) = 0\} \ , \quad (3)$$

where ϕ is defined such that

$$\phi(\mathbf{x}, t) \begin{cases} < 0 \text{ inside } \Gamma \\ = 0 \text{ on } \Gamma \\ > 0 \text{ outside } \Gamma \end{cases} . \quad (4)$$

Using the definition above, the outward unit normal $\mathbf{n}$ and the mean curvature κ of Γ are¹

$$\mathbf{n} = \frac{\nabla\phi}{|\nabla\phi|} \quad \text{and} \quad \kappa = \nabla \cdot \frac{\nabla\phi}{|\nabla\phi|} . \quad (5)$$

The zero set of $\phi(\mathbf{x}, t)$ represents $\Gamma(t)$ at all times t . This means that $\phi(\mathbf{x}(t), t) \equiv 0$ for a curve $\mathbf{x}(t) \in \Gamma(t)$. Differentiating with respect to t gives

$$\phi_t + \mathbf{u} \cdot \nabla\phi = 0 \Leftrightarrow \phi_t + u_n |\nabla\phi| = 0 , \quad (6)$$

where $\mathbf{u} = d\mathbf{x}(t)/dt$ and u_n is the normal component of the surface velocity. To move the surface according to some derived velocity, a PDE of the form (6) is solved. One of the advantages of this representation is that the topology of the surface is allowed to change as the surface evolves, thus making it easy to represent complex surfaces that can merge or split and also surfaces that contain holes. For a more thorough treatment of level set surfaces, see [3,4].

3 Problem Formulation

Given velocity data and basic dynamic characteristics of the flow, the problem consists of finding the boundary between the blood and the vessel walls. This boundary should be a closed surface within the domain of interest i.e. within the measured volume. The segmentation is then determined by the interior of the surface representing the vessels. The interior of Γ will be denoted Ω and the exterior Ω^c . The solution is found by maximizing the discontinuity of the flow at the surface and maximizing the flow inside the surface (or equivalently, minimizing the flow outside).

3.1 Using Velocity as a Cue

As mentioned above, phase contrast MRI provides velocity measurements in three directions (x,y,z) in an arbitrary coordinate system. The velocity can be expressed in vector form $\mathbf{v} = (v_x, v_y, v_z)^T$, and the velocity magnitude is

$$|\mathbf{v}| = \sqrt{\mathbf{v} \cdot \mathbf{v}} = \sqrt{v_x^2 + v_y^2 + v_z^2} .$$

There are two important reasons why the velocity field $\mathbf{v}$ should be used for segmentation. First, $\mathbf{v}$ is discontinuous across the vessel boundary since the fluid inside moves parallel to the boundary surface and the walls move roughly normal to the boundary. Second, it has been noted, e.g. by [7], that the velocity magnitude, $|\mathbf{v}|$, of the fluid is large compared to the magnitude of the wall motion. These observations constitute the foundation for the variational formulation introduced in this paper.

¹ Here $\nabla\phi$ denotes the gradient of ϕ and $\nabla \cdot$ denotes the divergence.

3.2 Measure for Discontinuities

To encode the information in the vector field $\mathbf{v}$ we define a field of matrices $\mathbf{M}_{\mathbf{v}} = \mathbf{M}_{\mathbf{v}}(\mathbf{x})$, $\mathbf{x} \in R^3$ where $\mathbf{M}_{\mathbf{v}}$ is the positive semi-definite, symmetric matrix defined by

$$\mathbf{M}_{\mathbf{v}} = \mathbf{v}\mathbf{v}^T = \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} \begin{bmatrix} v_x & v_y & v_z \end{bmatrix} = \begin{bmatrix} v_x^2 & v_x v_y & v_x v_z \\ v_x v_y & v_y^2 & v_y v_z \\ v_x v_z & v_y v_z & v_z^2 \end{bmatrix}. \quad (7)$$

$\mathbf{M}_{\mathbf{v}}$ has rank one with eigenvalues $\lambda_1 = |\mathbf{v}|^2$ and $\lambda_2 = \lambda_3 = 0$. Let $W \geq 0$ be a weight function (typically a Gaussian filter G_σ) and define the average matrix field to be the convolution

$$\overline{\mathbf{M}}_{\mathbf{v}} = W * \mathbf{M}_{\mathbf{v}},$$

taken componentwise. We denote this matrix field the *density matrix field*. Depending on the values of $\mathbf{v}$ in the region of interest there are three cases.

- i) **1 dominant direction.** If $\mathbf{v} \approx \text{constant}$ in a region we will have a case of $\overline{\mathbf{M}}_{\mathbf{v}}$ similar to (7) with eigenvalues $\lambda_1 \approx |\mathbf{v}|^2$ and $\lambda_2 \approx \lambda_3 \approx 0$.
- ii) **2 dominant directions.** This is the interesting case since we want to detect discontinuities in the vector field originating from the motion of the blood and the motion of the walls. Basically, the motion of the blood is orthogonal to the normal of the wall surface and the motion of the wall is approximately in the normal direction. With two dominant directions, $\overline{\mathbf{M}}_{\mathbf{v}}$ will have approximately rank two and only one small eigenvalue, $\lambda_3 \approx 0$.
- iii) **3 dominant directions.** $\overline{\mathbf{M}}_{\mathbf{v}}$ will have full rank and this is the case where there are three prominent directions of $\mathbf{v}$ or when there is no prominent direction at all, e.g. where there is high noise.

To discriminate i) from ii), the following real valued function is introduced, inspired by [11] and [12],

$$R = \frac{4\lambda_1\lambda_2}{(\lambda_1 + \lambda_2)^2}, \quad (8)$$

where $\lambda_1 \geq \lambda_2 \geq 0$ are the two largest eigenvalues of $\overline{\mathbf{M}}_{\mathbf{v}}$ and $0 \leq R \leq 1$. For case i) $R \approx 0$ and for ii) R will be large, i.e. $R \approx 1$. It turns out that this measure is an excellent detector for discontinuities in $\mathbf{v}$. This will be used in the next section.

4 Variational Formulation

In this section the segmentation problem is formulated as a variational problem. A functional is introduced and minimized using the level set framework. The desired surface should enclose as much of the flow in $\mathbf{v}$ as possible and the flow

should be discontinuous over the surface. This leads to the minimization of the following energy functional

$$E(\Gamma) = \underbrace{\int_{\Omega^c} \chi(\mathbf{v}) d\mathbf{x}}_{\text{flow outside the surface}} - \underbrace{\int_{\Gamma} R(\mathbf{x}) dS}_{\text{discontinuities at the surface}} , \quad (9)$$

where Γ is the surface, Ω^c the exterior of the surface, $R(\mathbf{x})$ is the measure for discontinuities from Section 3.2 and $\chi(\mathbf{v})$ is a C^2 approximation to the translated Heaviside function defined by

$$\chi(\mathbf{v}) = \chi(|\mathbf{v}|) = H(|\mathbf{v}| - \delta) = H_{\delta}(|\mathbf{v}|) , \quad (10)$$

where $\delta \in R_+$. Here H is defined as in [13]

$$H(x) = \begin{cases} 1 & x > \epsilon \\ 0 & x < -\epsilon \\ \frac{1}{2} \left[1 + \frac{x}{\epsilon} + \frac{1}{\pi} \sin\left(\frac{\pi x}{\epsilon}\right) \right] & |x| \leq \epsilon \end{cases} . \quad (11)$$

This is commonly used in level set based segmentation, cf. [14]. The definition above will make $\chi(\mathbf{v})$ equal to zero for very low velocities and one otherwise. This makes it an approximate characteristic function for $\mathbf{v}$ and the measure of the first integral in (9) is then in volume units.

Representing the surface using the zero level set of a function ϕ , the energy (9) becomes

$$E(\phi) = \int \chi(\mathbf{v}) H(\phi) d\mathbf{x} - \int R(\mathbf{x}) |\nabla \phi| \delta(\phi) d\mathbf{x} , \quad (12)$$

where $H(\phi)$ is again the approximation to the Heaviside function and therefore a characteristic function for Ω^c . The term $|\nabla \phi| \delta(\phi) d\mathbf{x}$ in the second integral is the surface area element and $\delta(\cdot)$ is the one dimensional delta function. From the Euler-Lagrange equation for (12), the motion PDE for the surface, as a gradient descent, is then

$$\phi_t = (-\chi + \kappa R + \nabla R \cdot \mathbf{n}) |\nabla \phi| , \quad (13)$$

where ϕ_t denotes derivative with respect to the evolution time and κ is the mean curvature of the surface. This acts as a natural regularizer of the boundary.

5 Experiments

The performance of the response function for vector field discontinuities R was tested on both real and synthetic phase contrast MRI data. The value of R is shown for a transversal slice of a human heart, see Fig 2. The results are very satisfactory. The heart walls are clearly shown in white ($R=1$), and the heart cavities in black, indicating only one prominent flow direction.

Fig. 2. The value for R , shown for a transversal cross section of the heart.

Fig. 3. (top row) The surface evolution at initial value, 300 iterations, 500 iterations, 700 iterations and final surface. (bottom row) Different views of the resulting surface and value of R for a cross section with the resulting boundary marked in black.

Experiments to determine the viability of the proposed variational method were also performed. Unfortunately we are not allowed to use the data for publication. We therefore present results on synthetic data here. We intend to show results on clinical data in the near future.

The synthetic data consists of a double T-junction in a volume of resolution $60 \times 60 \times 60$. The velocity of the walls was 30% of the maximum blood velocity and noise with amplitude $\pm 20\%$ of the maximum blood velocity was added to the whole volume.

6 Conclusions

The experiments in the previous section confirm that velocity data is a valuable cue for this segmentation problem. We also find the performance of the flow discontinuity measure introduced in Section 3.2 to be quite satisfactory.

Future work will include: incorporating other cues such as the gradient of intensity MRI images as boundary information, integrating prior information using statistical shape models [15,16] and analysis of methods for initialization.

Acknowledgements. The authors would like to thank the Department of Clinical Physiology, Lund University, especially Erik Bergvall and Karin Markenroth at Philips Medical Systems, for interesting discussions regarding MRI.

References

1. Frangi, A., Niessen, W., Viergever, M.: Three-dimensional modeling for functional analysis of cardiac images: A review. *IEEE Trans. on Med. Imag.* **20** (2001) 2–25
2. Kass, M., Witkin, A., Terzopoulos, D.: Snakes: Active contour models. *Int. J. Computer Vision* **1** (1987) 321–331
3. Osher, S.J., Fedkiw, R.P.: *Level Set Methods and Dynamic Implicit Surfaces*. Springer Verlag (2002)
4. Sethian, J.: *Level Set Methods and Fast Marching Methods Evolving Interfaces in Computational Geometry, Fluid Mechanics, Computer Vision, and Materials Science*. Cambridge University Press (1999)
5. Osher, S., Sethian, J.A.: Fronts propagating with curvature-dependent speed: Algorithms based on Hamilton-Jacobi formulations. *Journal of Computational Physics* **79** (1988) 12–49
6. Ebberts, T.: *Cardiovascular Fluid Dynamics*. PhD thesis, Departments of Biomedical Engineering & Medicine and Care, Linköping University, Sweden (2001)
7. Wong, A., Liu, H., Shi, P.: Segmentation of myocardium using velocity field constrained front propagation. In: *IEEE Applications in Computer Vision*, Orlando, FL (2002)
8. Lorigo, L., Faugeras, O., Grimson, W., Keriven, R., Kikinis, R., Nabavi, A., Westin, C.F.: Curves: Curve evolution for vessel segmentation. *IEEE Transactions on Medical Image Analysis* **5** (2001) 195–206
9. Vasilevskiy, A., Siddiqi, K.: Flux maximizing geometric flows. In: *Proc. Int. Conf. on Computer Vision*, Vancouver, Canada (2001)
10. Pelc, N.J., Herfkens, R.J., Shimakawa, A., Enzmann, D.: Phase contrast cine magnetic resonance imaging. *Magnetic Resonance Quarterly* **4** (1991) 229–254
11. Harris, C., Stephens, M.: A combined corner and edge detector. In: *Proc. Alvey Conf.* (1988) 189–192
12. Knutsson, H.: Representing local structure using tensors. In: *The 6th Scandinavian Conference on Image Analysis*, Oulu, Finland (1989) 244–251 Report LiTH-ISY-I-1019, Computer Vision Laboratory, Linköping University, Sweden, 1989.
13. Zhao, H., Chan, T., Merriman, B., Osher, S.: A variational level set approach to multiphase motion. *J. Computational Physics* **127** (1996) 179–195
14. Paragios, N.: A level set approach for shape-driven segmentation and tracking of the left ventricle. *IEEE Transactions on Medical Imaging* **22** (2003) 773–776
15. Rousson, M., Paragios, N.: Shape priors for level set representations. In: *Proc. European Conf. on Computer Vision*. Volume 2351 of LNCS., Springer (2002)
16. Cremers, D.: *Statistical Shape Knowledge in Variational Image Segmentation*. PhD thesis, Dept. of Mathematics and Computer Science, University of Mannheim, Germany (2002)