Using Consistency Measures and Attribute Dependencies for Solving Conflicts in Adaptive Systems

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Abstract. In this paper we consider two problems related to consensus determining as a tool for conflict resolution. The first problem concerns the consistency measures for conflict profiles which could be useful for taking decision if the consensus should be determined or not. The second problem refers to dependencies of attributes representing the content of conflicts, which cause that one may not treat the attributes independently in consensus determining. We show that using some kind of distance functions may enable to determine consensus in the same way when the attributes are independent on each other.

1 Introduction

Conflict resolution is one of the most important aspects in distributed systems in general, and in adaptive systems in particular. The resources of conflicts in these kinds of systems come from the autonomy feature of their sites. This feature causes that each site of a distributed or multiagent system processes a task in independent way and there may arise such situation that the for same task different sites may generate different solutions. Thus, one deals with a conflict.

For a conflict one can distinguish the following three its components: *conflict body*, *conflict subject* and *conflict content*. Consensus models seem to be useful in conflict solving. The oldest consensus model was worked by such authors as Condorcet, Arrow and Kemeny [1]. This model serves to solving such conflicts in which the content may be represented by orders or rankings. Models of Barthelemy [2] and Day [4] enable to solve such conflicts for which the structures of the conflict contents are n-trees, semilattices, partitions etc. The common characteristic of these models is that they are one-attribute, it means that conflicts are considered only referring to one feature. Multi-feature conflicts are not investigated.

In works [7],[8] the author presents a consensus model, in which multi-attribute conflicts may be represented. Furthermore, in this model attributes are multi-valued, what means that for representing an opinion of an agent on some issue one may use not only one elementary value (such as +, -, or 0) [9] but a set of elementary values. Consensus determining is often required in adaptive systems. The reason is that to be adaptive a system needs to use information from different resources for determining a better way to serve users. These resources can contain inconsistent information referring to the same class of users. Thus, for generating a more adaptive model in user serving the system has to reconcile the inconsistency. One of the most used ways for this process is based on consensus calculating.

In mentioned models a consensus always is able to be determined, it means that for each profile representing a conflict one can calculate the consensus referring to some criterion. However, a question may arise: Is the chosen consensus good enough and can it be acceptable as the solution of given conflict situation? In other words, is the conflict situation susceptible to (good) consensus? We will consider the susceptibility to consensus for conflict profiles. Before defining the notion of susceptibility to consensus below we present an example. For this problem a notion of so called *susceptibility to consensus* has been defined [8], which is relied on defining a criterion allowing to assess if a conflict can have a solution. In this paper we present another approach [3],[6] to this problem. It is based on measuring the consistency of the conflict. Thus a conflict profile should have a number representing its consistency degree. A set of postulates for consistency functions and some concrete functions are presented.

Another problem is related to attribute dependencies. Consensus model presented in [7],[8] enables to process multi-feature conflicts, but attributes are mainly treated as independent. However, in many practical conflict situations some attributes are dependent on others. For example, in a meteorological system attribute *Wind_power* (with values: *weak*, *medium*, *strong*) is dependent on attribute *Wind_speed*, the values of which are measured in unit *m/s*. This dependency follows that if the value of the first attribute is known, then the value of the second attribute is also known. It is natural that if a conflict includes these attributes then in the consensus the dependency should also take place. The question is: Is it enough to determine the consensus for the conflict referring to attribute *Wind_speed*? In other words, when is it true that if some value is a consensus for the conflict referring to attribute *Wind_speed*, then its corresponding value of attribute *Wind_power* is also a consensus for the conflict referring to this attribute? In this paper we consider the answer for mentioned above question. For this aim we assume some dependencies between attributes and show their influence on consensus determining.

2 Postulates for Consistency Measures

Formally, let U denote a finite universe of objects (alternatives), and $\Pi(U)$ denote the set of subsets of U . By $\hat{\Pi}_k(U)$ we denote the set of k -element subsets (with repetitions) of the set U for $k \in \mathbb{N}$, and let $\hat{\Pi}(U) = \bigcup_{k \geq 0} \hat{\Pi}_k(U)$. Each element of set

$\hat{\Pi}(U)$ is called a *profile*. In this paper we only assume that the macrostructure of the set U is known and a distance is a function $d: U \times U \rightarrow \mathbb{R}^+$, which is: a) *Nonnegative*: $(\forall x, y \in U)[d(x, y) \geq 0]$; b) *Reflexive*: $(\forall x, y \in U)[d(x, y) = 0 \text{ iff } x = y]$ and c) *Symmetrical*: $(\forall x, y \in U)[d(x, y) = d(y, x)]$. For normalization process we can assume that values of function d belong to interval $[0, 1]$.

Let us notice, that the above conditions are only a part of metric conditions. Metric is a good measure of distance, but its conditions are too strong. A space (U, d) defined in this way does not need to be a metric space. Therefore we will call it a *distance space* [7]. A profile X is called *homogeneous* if all its elements are identical, that is $X = \{n * x\}$ for some $x \in U$. A profile is *heterogeneous* if it is not *homogeneous*. A

profile is called *distinguishable* if all its elements are different from each other. A profile X is *multiple* referring to a profile Y (or X is a *multiple* of Y), if $Y = \{x_1, \dots, x_k\}$ and $X = \{n \cdot x_1, \dots, n \cdot x_k\}$ for some $n \in \mathbb{N}$. A profile X is *regular* if it is a multiple of some distinguishable profile.

By symbol C we denote the consistency function of profiles. This function has the following signature:

$$C: \hat{\Pi}(U) \rightarrow [0,1].$$

where $[0,1]$ is the closed interval of real numbers between 0 and 1.

The idea of this function relies on the measuring the consistency degree of profiles' elements. The requirements for consistency are expressed in the following postulates. Those postulates are result of the intuition, because this is the only one way to translate of the empirical knowledge into the rule-based knowledge. Of course, founded function has not to grant all of those postulates. Mostly it depends on the desired appliance. The following postulates for consistency functions have been defined: **P1a.** *Postulate for maximal consistency*; **P1b.** *Extended postulate for maximal consistency*; **P2a.** *Postulate for minimal consistency*; **P2b.** *Extended postulate for minimal consistency*; **P2c.** *Alternative postulate for minimal consistency*; **P3.** *Postulate for non-zero consistency*; **P4.** *Postulate for heterogeneous profiles*; **P5.** *Postulate for multiple profiles*; **P6.** *Postulate for greater consistency*; **P7a.** *Postulate for consistency improvement*; **P7b.** *Second postulate for consistency improvement*. The content of these postulates can be found in work [6].

3 Consistency Functions and Their Analysis

Let $X = \{x_1, \dots, x_M\}$ be a profile. We define the following parameters:

- The matrix of distances between the elements of profile X :

$$D^X = [d_{ij}^X] \text{ for } i, j = 1, \dots, M.$$

- The vector of average distances of knowledge states of following objects to the rest:

$$W^X = [w_i^X] = \left(\frac{1}{M-1} \sum_{j=1}^M d_{j1}^X, \frac{1}{M-1} \sum_{j=1}^M d_{j2}^X, \dots, \frac{1}{M-1} \sum_{j=1}^M d_{jM}^X \right),$$

- Diameters of sets X and U $Diam(X) = \max_{x, y \in X} d(x, y)$, and the maximal element

$$\text{of vector } W^X: Diam(W^X) = \max_{1 \leq i \leq M} w_i^X,$$

- The average distance of objects' knowledge states:

$$\bar{d}(X) = \frac{1}{M(M-1)} \sum_{i=1}^M \sum_{j=1}^M d_{ij}^X = \frac{1}{M} \sum_{i=1}^M w_i^X,$$

- The sum of distances between an element x of universe U and the elements of set X : $d(x, X) = \sum_{y \in X} d(x, y)$,

- The minimal sum of distances from an object to the elements of profile X :

$$d_{\min}(X) = \min \{d(x, X): x \in U\},$$

- The maximal sum of distances from an object from X to its elements:

$$d_{\max}(X) = \max \{d(x, X): x \in U\}.$$

These parameters are applied for the defining the following consistency functions:

$$c_1(X) = 1 - Diam(X),$$

$$c_2(X) = 1 - Diam(W^X),$$

$$c_3(X) = 1 - \bar{d}(X),$$

$$c_4(X) = 1 - \frac{1}{M} d_{\min}(X).$$

$$c_5(X) = 1 - \frac{1}{M} d_{\max}(X).$$

Values of functions c_1 , c_2 , c_3 and c_4 reflect accordingly:

- $c_1(X)$ – the maximal distance between two elements of profile.
- $c_2(X)$ – the maximal average distance between an element of profile X and other elements of this profile.
- $c_3(X)$ – the average distance between elements of X .
- $c_4(X)$ – the minimal average distance between an element of universe U and elements of X .
- $c_5(X)$ – the maximal average distance between an element of profile X and elements of this profile.

The table presented below shows result of functions analysis. The columns denote postulates and rows denote functions. Plus means that presented function complies the postulate and minus means that presented function doesn't comply the postulate.

Table 1. Results of functions analysis

	P1a	P1b	P2a	P2b	P2c	P3	P4	P5	P6	P7a	P7b
c_1	+	–	+	–	+	–	+	+	–	–	–
c_2	+	–	+	–	–	–	+	+	–	+	+
c_3	+	+	+	–	–	+	+	–	–	+	+
c_4	+	+	–	–	–	+	+	+	+	–	–
c_5	+	+	–	–	–	+	+	+	+	–	–

4 The Outline of Consensus Model

The consensus model which enables processing multi-attribute and multi-valued conflicts has been discussed in detail in works [7],[8]. In this section we present only some of its elements needed for the consideration of attribute dependencies. We

assume that some real world is commonly considered by a set of agents that are placed in different sites of a distributed system. The interest of the agents consists of events which occur (or are to occur) in this world. The task of the agents is based on determining the values of attributes describing these events. If several agents consider the same event then they may generate different descriptions (which consist of, for example, scenarios, timestamps etc.) for this event. Thus we say that a conflict takes place. For representing potential conflicts we use a finite set A of attributes and a set V of attribute elementary values, where $V = \bigcup_{a \in A} V_a$ (V_a is the domain of attribute a). Let $\Pi(V_a)$ denote the set of subsets of set V_a and $\Pi(V_B) = \bigcup_{b \in B} \Pi(V_b)$. Let $B \subseteq A$, a tuple r_B of type B is a function $r_B: B \rightarrow \Pi(V_B)$ where $r_B(b) \subseteq V_b$ for each $b \in B$. Empty tuple is denoted by symbol ϕ . The set of all tuples of type B is denoted by $TYPE(B)$. The consensus system is defined as a quadruple

$$Consensus_Sys = (A, X, P, Z),$$

where:

- A is a finite set of attributes, which includes a special attribute *Agent*; a value of attribute a where $a \neq Agent$ is a subset of V_a ; values of attribute *Agent* are singletons which identify the agents;
- $X = \{\Pi(V_a): a \in A\}$ is a finite set of consensus carriers;
- P is a finite set of relations on carriers from X , each relation $P \in P$ is of some type T_p (for $T_p \subseteq A$ and $Agent \in T_p$). Relations belonging to set P are classified into 2 groups identified by symbols "+" and "-" as the upper index to the relation names.
- At least Z is a set of logical formulas for which the relation system (X, P) is a model. These formulas are the conditions which should be satisfied by consensus as the solution of the conflict.

The structures of the consensus carriers are defined by means of a distance function between tuples of the same type. This function can be defined on the basis of one of distance functions δ and ρ between sets of elementary values [8]. These functions are defined as follows: Function δ measures the distance between 2 sets X and Y ($X, Y \subseteq V$) as the minimal costs of the operation which transforms set X into set Y ; the distance measured by function ρ is equal to the sum of shares of elements from V in this distance. Functions δ and ρ are called propositional (denoted by δ^p and ρ^p respectively) if the condition $(X \div Y \supseteq X' \div Y') \Rightarrow (\kappa(X, Y) \geq \kappa(X', Y'))$ is satisfied for any $X, X', Y, Y' \subseteq V_a$ and $\kappa \in \{\rho, \delta\}$. The distance $\partial(r, r')$ between 2 tuples r and r' of type A is equal to the number $\frac{1}{\text{card}(A)} \sum_{a \in A} \kappa(r_a, r'_a)$ where $\kappa \in \{\rho^p, \delta^p\}$.

A consensus is considered within a conflict situation, which is defined as a pair $s = \langle \{P^+, P^-\}, A \rightarrow B \rangle$ where $A, B \subseteq A$, $A \cap B = \emptyset$ and $r_A \neq \phi$ holds for any tuple $r \in P^+ \cup P^-$.

The first element of a conflict situation (i.e. set of relations $\{P^+, P^-\}$) includes the domain from which consensus should be chosen, and the second element (i.e. relationship $A \rightarrow B$) presents the schemas of consensus subjects of the consensus content, such that for a subject e (as a tuple of type A , included in P^+ or P^-) there should be assigned only one tuple of type B . A conflict situation yields a set $Subject(s)$ of conflict subjects which are represented by tuples of type A . For each

subject e two conflict profiles, i.e. $profile(e)^+$ and $profile(e)^-$, as relations of $TYPE(\{Agent\} \cup B)$ should be determined. Profile $profile(e)^+$ contains the positive opinions of the agents on the subject e , while profile $profile(e)^-$ contains agents' negative opinions on this subject.

Definition 1. Consensus on a subject $e \in Subject(s)$ is a pair $(C(s,e)^+, C(s,e)^-)$ of 2 tuples of type $A \cup B$ which fulfill the following conditions:

- a) $C(s,e)_A^+ = C(s,e)_A^- = e$ and $C(s,e)_B^+ \cap C(s,e)_B^- = \emptyset$,
 b) The sums $\sum_{r \in profile(e)^+} \partial(r_B, C(s,e)_B^+)$ and $\sum_{r \in profile(e)^-} \partial(r_B, C(s,e)_B^-)$ are minimal.

Any tuples $C(s,e)^+$ and $C(s,e)^-$ satisfying the conditions of Definition 1 are called consensuses of profiles $profile(e)^+$ and $profile(e)^-$ respectively.

5 Attribute Dependencies and Consensus Determining

In Definition 1 the most important is condition b) which requires the tuples $C(s,e)_B^+$ and $C(s,e)_B^-$ to be determined in such way thus the sums $\sum_{r \in profile(e)^+} \partial(r_B, C(s,e)_B^+)$ and

$\sum_{r \in profile(e)^-} \partial(r_B, C(s,e)_B^-)$ are minimal, respectively. These tuples could be calculated in

the following way: For each attribute $b \in B$ one can determine sets $C(s,e)_b^+$ and $C(s,e)_b^-$, which minimize sums $\sum_{r \in profile(e)^+} \partial(r_b, C(s,e)_b^+)$ and $\sum_{r \in profile(e)^-} \partial(r_b, C(s,e)_b^-)$ respectively.

Such way is an effective one, but it is correct only if the attributes from set B are independent. In this section we consider consensus choice assuming that some attributes from set B are dependent. The definition of attribute dependency given below is consistent with those given in the information system model [10]:

Definition 2. Attribute b is dependent on attribute a if and only if there exists a function $f_b^a: V_a \rightarrow V_b$ such that in any consensus system (A, X, P, Z) for each relation $P \in \mathbf{P}$ of type T_p and $a, b \in T_p$ the formula $(\forall r \in P)(r_b = \bigcup_{x \in r_a} \{f_b^a(x)\})$ is true.

The dependency of attribute b on attribute a means that in the real world if for some object the value of a is known then the value of b is also known. In practice, owing to this property for determining the values of attribute b it is enough to know the value of attribute a . Instead of $\bigcup_{x \in Y} \{f_b^a(x)\}$ we will write shortly $f_b^a(Y)$.

Consider now a conflict situation $s = \langle \{P^+, P^-\}, A \rightarrow B \rangle$, in which attribute b is dependent on attribute a where $a, b \in B$. Let $profile(e)^+$ be the positive profile for given conflict subject $e \in Subject(s)$. The problem relies on determining consensus for this profile. We can solve this problem using two approaches:

1. Notice that $profile(e)^+$ is a relation of type $B \cup \{Agent\}$. The dependency of attribute b on attribute a implies that there exists a function from set $TYPE(B \cup \{Agent\})$ to set $TYPE(B \cup \{Agent\} \setminus \{b\})$ such that for each profile $profile(e)^+$ one can assign exactly one set $profile'(e)^+ = \{r_{B \cup \{Agent\} \setminus \{b\}} : r \in profile(e)^+\}$.

Set $profile'(e)^+$ can be treated as a profile for subject e in the following conflict situation $s' = \langle \{P^+, P^-\}, A \rightarrow B \setminus \{b\} \rangle$.

Notice that the difference between profiles $profile(e)^+$ and $profile'(e)^+$ relies only on the lack of attribute b and its values in profile $profile(e)^+$. Thus one can expect that the consensus $C(s, e)^+$ for profile $profile(e)^+$ can be determined from the consensus $C(s, e)^+$ for profile $profile'(e)^+$ after adding to tuple $C(s, e)^+$ attribute b and its value which is equal to $f_b^a(C(s, e)^+)$. In the similar way one can determine the consensus for profile $profile(e)^-$.

2. In the second approach attributes a and b are treated equivalently. That means they play the same role in consensus determining for profiles $profile(e)^+$ and $profile(e)^-$. The consensus for profiles $profile(e)^+$ and $profile(e)^-$ are defined as follows:

Definition 3. The consensus for subject $e \in \text{Subject}(s)$ in situation $s = \langle \{P^+, P^-\}, A \rightarrow B \rangle$ is a pair of tuples $(C(s, e)^+, C(s, e)^-)$ of type $A \cup B$, which satisfy the following conditions:

- a) $C(s, e)_A^+ = C(s, e)_A^- = e$ and $C(s, e)_B^+ \cap C(s, e)_B^- = \emptyset$,
- b) $C(s, e)_b^+ = f_b^a(C(s, e)_a^+)$ and $C(s, e)_b^- = f_b^a(C(s, e)_a^-)$,
- c) The sums $\sum_{r \in profile(e)^+} \partial(r_B, C(s, e)_B^+)$ and $\sum_{r \in profile(e)^-} \partial(r_B, C(s, e)_B^-)$ are minimal.

We are interested in the cases when conditions b) and c) of Definition 3 can be satisfied simultaneously. The question is: Is it true that if set $C(s, e)_a^+$ is a consensus for profile $profile(e)^+$ (as the projection of profile $profile(e)^+$ on attribute a) then set $f_b^a(C(s, e)_a^+)$ will be a consensus for profile $profile(e)^+_b$ (as the projection of profile $profile(e)^+$ on attribute)? The following theorem was proven [8]:

Theorem 1. With using distance function ρ^p or δ^p if a set $C(s, e)_a^+$ is a consensus for profile $profile(e)^+_a$ then there exists a set $C(s, e)_b^+$ being a consensus for profile $profile(e)^+_b$ such that $f_b^a(C(s, e)_a^+) \subseteq C(s, e)_b^+$.

A similar theorem may be formulated and proved for consensus of profiles $profile(e)^-_a$ and $profile(e)^-_b$.

Example 1. [5],[8] Let profiles $profile(e)^+_a$ and $profile(e)^+_b$ and function f be given:

$profile(e)^+_a$	$profile(e)^+_b$
$\{a_1, a_3\}$	$\{b_2\}$
$\{a_2, a_3\}$	$\{b_1, b_2\}$
$\{a_1, a_4\}$	$\{b_1, b_2\}$
$\{a_1, a_4, a_5\}$	$\{b_1, b_2\}$

Function $f_b^a : V_a \rightarrow V_b$	
a_1	b_2
a_2	b_1
a_3	b_2
a_4	b_1
a_5	b_1

the consensus for profile $profile(e)_a^+$ is the set $\{a_1, a_3\}$ and the consensus for profile $profile(e)_b^+$ is the set $\{b_1, b_2\}$. Notice that $f_b^a(\{a_1, a_3\}) = \{b_2\} \subseteq \{b_1, b_2\}$.

6 Conclusions

In this paper we consider 2 problems: The first refers to measuring consistency of conflict profiles. We present its definition, conditions for its determining and some concrete consistency functions. The future works should concern the deep analysis of proposed postulates and considering if a function satisfying all of them exists. The second problem is related to how dependencies of attributes influence the possibilities of consensus determining. The limitations of distance functions between attribute values were shown, guarantying determining a correct consensus despite of treating attributes independently. Using such functions provides the following profits. First of all, they enable determining a consensus for only a part of the attributes (the rest may be calculated using dependency functions). Secondly, they prevent from determining an incorrect consensus, which does not fulfill some of the dependencies of attributes. The results of these works should be useful in adaptive systems where reconciling processes are often required, for example, in user interface design [11].

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