Modelling of Interaction between Surface Waves and Mud Layer

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Abstract. The analytical and numerical modelling of interaction between the mud layer at the sea bottom and the surface waves have been presented. In the simulations the theory for linear water wave movement in a two-layer viscous fluid system has been considered. The upper layer represents water and the lower layer represents fluid-mud. The type of the bottom material over which waves are propagating is assumed to be similar to a viscous fluid, characterized by a viscosity and density greater than the overlying fluid. It is assumed that the two fluids are incompressible and isotropic, and the rigid strata is smooth and impermeable. At the surface the height of the surface wave is specified and the forced interfacial wave is determined. Developed model solves the equations of motion for an incompressible fluid by composite finite difference-finite element approximations on a staggered scheme. Results of analytical and numerical solutions are compared with the experimental results and favourable agreement has been obtained.

1 Introduction

All over the world, at a great number of coastal areas, especially near the river mouths carrying large quantities of sediments, the sea bottom is covered by the mud layers. Surface waves start to interact with the mud layer at the sea bottom when they enter to the intermediate or shallow wave regions. This interaction generally causes the dissipation of surface waves step by step and suspension and transportation of mud due to the pressure generated at the sea bed. The dissipation caused by the mud layer is greater than the dissipation caused by the friction of rigid sea bottom. Under surface wave action, mass transport occurs in the mud layer. Gade[1] who is one of the former reasearchers that investigated the interaction of surface waves with the mud layer at the bottom, modelled the mud layer as a fluid with a high viscosity. Dalrymple and Liu[2], analytically investigated the interaction of surface waves with the viscous mud layer for variable layer thicknesses. Jiang and Zhao[3], investigated analytically the propagation of waves over fluid mud layer and worked with nonlinear surface waves. Wen and Liu[4], searched for the effects of viscous dissipation on mass transport for two dimensional small amplitude interfacial waves in a two-layered fluid system. In this study, the interaction of surface waves propagating over the mud layer at the sea bottom has been investigated analytically and numerically.

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2 Theory

In this study, the water layer with a thickness of h and underlying mud layer with a thickness of d are modelled as a two layered fluid system. In the modelling, it is assumed that surface waves follow the linear wave theory; liquid mud layer exhibits viscous liquid properties; there is no mixture at the interface of water layer and mud layer; liquids are incompressible; system lies on a smooth, horizontal, impervious and rigid bottom; at the interface of the system normal and shear stresses are continuous. The density of the underlying mud layer is greater than the density of the water layer. The developed numerical model solves fluid particle velocities by Navier-Stokes equations. In the numerical solution of the equations central finite difference approximations are applied. In the vertical Galerkin finite element linear shape functions are used. Water depths are divided into same number of layers at each point in the solution domain. The model hydrodynamic equations in the Cartesian coordinate system is as follows:

$$\frac{\partial u_j}{\partial x}+\frac{\partial w_j}{\partial z}=0 \tag{1}$$

$$\frac{\partial u_j}{\partial t}+u_j\frac{\partial u_j}{\partial x}+w_j\frac{\partial u_j}{\partial z}=-\frac{1}{\rho_j}\frac{\partial P_j}{\partial x}+\upsilon_j\left(\frac{\partial^2 u_j}{\partial x^2}+\frac{\partial^2 u_j}{\partial z^2}\right) \tag{2}$$

$$\frac{\partial w_j}{\partial t}+u_j\frac{\partial w_j}{\partial x}+w_j\frac{\partial w_j}{\partial z}=-\frac{1}{\rho_j}\frac{\partial P_j}{\partial y}+\upsilon_j\left(\frac{\partial^2 w_j}{\partial x^2}+\frac{\partial^2 w_j}{\partial y^2}\right) \tag{3}$$

Where, x is horizontal coordinate, z is vertical coordinate; t is time; u, w are the fluid particule velocities in x,z directions respectively; υ is kinematic viscosity; p is pressure, j=1,2 are upper and lower layer indixes.

The analytical solution of the model has been performed as well. In the analytical solution the boundary layer approximations applied by Dalrymple and Liu[2] are considered. The numerical solution method is a composite finite element-finite difference method. The governing equations are solved by the Galerkin Weighted Residual Method in the vertical plane and by finite difference approximations in the horizontal plane. The water depths are divided into the same number of layers following the bottom topography. In the depth following coordinate system, the layer thickness is proportional to the local water depth. To increase the vetical resolution, wherever necessary, grid clustering can be applied in the vertical plane. The system of nonlinear equations are solved by the Crank-Nicholson Method which has second order accuracy in time.

3 Model Application

Numerical model predictions and analytical solution have been compared with the experimental studies published in the literature. Firstly the results of laboratory experiments of De Wit and Kranenburg[5] are compared with the simulations. In the experiments, various wave amplitudes (< 0,1 m.) are tested. Wave period is 1,5 s. and initial water depth is 0,30 m. dir. Experimental flume has a length of 40 m. and width of 0.8 m. Velocity amplitudes in and over the fluid mud layer are presented in Figure

(1-a) for China clay and in Figure (1-b) for Westwald Clay. Experimental values of H(m.),d(m), ρ_1 [kg.m^{-3}], ρ_2[kg.m^{-3}], Q[m^{2}s^{-1}] and a[m] are 0.325, 0.115, 1000, 1316, 2,7.10^{-3} and 0.02 respectively for china clay and 0.35, 0.15, 1000, 1186, 5,9.10^{-3} and 0.02 respectively for Westwald clay.

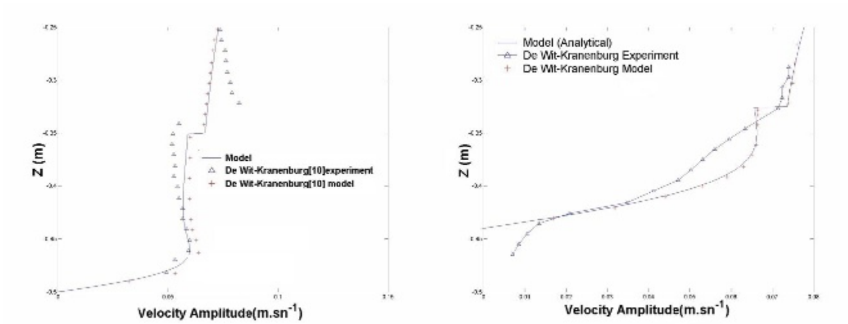

Fig. 1. Velocity amplitudes, (a) China Clay, (b) Westwald Clay

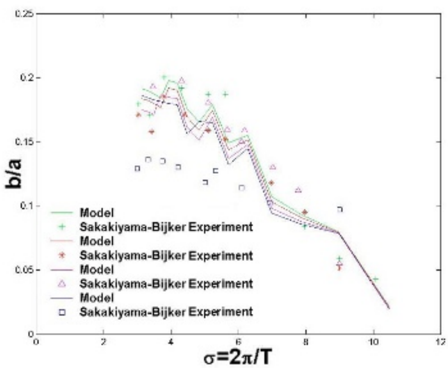

Fig. 2. Ratio of interfacial and surface wave amplitudes.

It has been observed that analytical and numerical model predictions well agree with the measurements except from the little deviations just at the interface of the mud and water layers. Secondly, the experimental results of Sakakiyama and Bijker[6] are compared with the simulations. They performed the experiment in a wave flume of 24,5 m. in length and 0.5 m. in width. For different wave periods, Sakakiyama and Bijker[6], observed the ratios of b/a (ratio of interfacial to surface wave amplitude). Simulated b/a ratios by the model are compared with the measurements and presented in Figure (2). It is seen that as the value of σ increases, the velocities in the mud layer decreases and accordingly b/a ratio decreases as well. Model simulations are favorable compared with the measurements. Calculated mean error is 0.2008% in A (□) where ρ_m is 1370 kg/m^3 and υ is 1,5 10^{-2} (m^2/s); it is 0,1971% in B (Δ) where ρ_m is 1300 kg/m^3 and υ is 1,0.10^{-2} (m^2/s); it is 0,1489% in C (*) where ρ_m is 1210 kg/m^3 and υ is 4,0.10^{-3} (m^2/s) and it is 0,1799% in D (+) where ρ_m is 1140 kg/m^3 and υ is 1,0.10^{-3} (m^2/s). Errors are at acceptable levels. Closer to the rigid bottom, difference between the simulations and measurements increases as also observed in the comparisons with the results of experiment done by De Wit – Kranenburg [5]. These discrepancies are basically due to the application of linear wave theory in the models.

4 Conclusions

The analytical and numerical modelling of interaction between the mud layer at the sea bottom and the surface waves have been presented. Variations in the mud layer due to the effect of waves and changes in the surface waves as a result of these variations are investigated. The numerical solution method is a composite finite difference-finite element method. The governing equations written in the Cartesian coordinates are solved by the Galerkin Weighted Residual Method in the vertical plane and by finite difference approximations in the horizontal plane. At each grid points in the solution domain, the water depth is divided into same number of layers following the bottom topography. In the analytical solution boundary layer approximation has been used. Results obtained from analytical and numerical solutions are compared with the measurements obtained from experimental studies, and comparisons have shown an encouraging agreement. The model that simulates the mud motion under wave action can be used as a practical tool in diverse coastal engineering applications.

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