

# On Balancing Delay and Cost for Routing Paths<sup>\*</sup>

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**Abstract.** The distributed adaptive routing is the typical routing algorithm that is used in the current Internet. The path cost of the least delay (LD) path is relatively more expensive than that of the least cost (LC) path, and the path delay of the LC path is relatively higher than that of the LD path. In this paper, we propose an effective parameter that is the probabilistic combination of cost and delay. It significantly contributes to identify the low cost and low delay unicasting path, and improves the path cost with the acceptable delay.

## 1 Introduction

For distributed real-time applications, the path delay should be acceptable and also its cost should be as low as possible. We call it as the delay constrained least cost (DCLC) path problem [3,5]. It has been shown to be NP-hard [2]. As you see, the DCLC is desirable to find a path that considers the cost and the delay together. Even though there is a loss for the cost, two parameters should be carefully negotiated to reduce the delay. This is because the adjustment between the cost and the delay for the balance is important. Hence, we introduce the new parameter that takes in account both the cost and the delay at the same time.

The rest of paper is organized as follows. In section 2, we describe the network model, section 3 presents details of the new parameter. Then we analyze and evaluate the performance of the proposed parameter by simulation in section 4. Section 5 concludes this paper.

## 2 Network Model

We consider a computer network represented by a directed graph  $G = (V, E)$ , where  $V$  is a set of nodes and  $E$  is a set of links. Each link  $(i, j) \in E$  is associ-

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ated with two parameters, namely available cost  $c_{(i,j)}$  and delay  $d_{(i,j)}$ . Given a network  $G$ , we define a path as sequence of nodes  $u, i, j, \dots, k, v$ , such that  $(u, i), (i, j), \dots$ , and  $(k, v)$  belong to  $E$ . Let  $P(u, v) = \{(u, i), (i, j), \dots, (k, v)\}$  denote the path from node  $u$  to node  $v$ . If all elements of the path are distinct, then we say that it is a simple path. We define the length of the path  $P(u, v)$ , denoted by  $n(P(u, v))$ , as a number of links in  $P(u, v)$ . Let  $\preceq$  be a binary relation on  $P(u, v)$  defined by  $(a, b) \preceq (c, d) \leftrightarrow n(P(u, b)) \leq n(P(u, d)), \forall (a, b), (c, d) \in P(u, v)$ .  $(P(u, v), \preceq)$  is a totally ordered set. For given a source node  $s \in V$  and a destination node  $d \in V$ ,  $(2^{s \Rightarrow d}, \infty)$  is the set of all possible paths from  $s$  to  $d$ .  $(2^{s \Rightarrow d}, \infty) = \{ P_k(s, d) \mid \text{all possible paths from } s \text{ to } d, \forall s, d \in V, \forall k \in \Lambda \}$ , where  $\Lambda$  is a index set. Both cost and delay of an arbitrary path  $P_k$  are assumed to be a function from  $(2^{s \Rightarrow d}, \infty)$  to a nonnegative real number. Since  $(P_k, \preceq)$  is a totally ordered set, if there exists a bijective function  $f_k$  then  $P_k$  is isomorphic to  $\mathcal{N}_{n(P_k)}$ .  $f_k : P_k \rightarrow \mathcal{N}_{n(P_k)}$ . We define a function of path cost  $\phi_C(P_k) = \sum_{r=1}^{n(P_k)} c_{f_k^{-1}(r)}$  and a function of delay along the path  $\phi_D(P_k) = \sum_{r=1}^{n(P_k)} d_{f_k^{-1}(r)}$ ,  $\forall P_k \in (2^{s \Rightarrow d}, \infty)$ .  $(2^{s \Rightarrow d}, supD)$  is the set of paths from  $s$  to  $d$  for which the end-to-end delay is bounded by  $supD$ . Therefore  $(2^{s \Rightarrow d}, supD) \subseteq (2^{s \Rightarrow d}, \infty)$ . The DCLC problem is to find the path that satisfies  $min\{ \phi_C(P_k) \mid P_k \in (2^{s \Rightarrow d}, supD), \forall k \in \Lambda \}$ .

### 3 Proposed Parameter for Low Cost and Low Delay

Since only link-delays are considered to compute  $P_{LD}$ ,  $\phi_C(P_{LD})$  is always greater than or equal to  $\phi_C(P_{LC})$  [1]. If the cost of the path,  $\phi_C(P_{LD})$ , is decreased by  $100(1 - \frac{\phi_C(P_{LC})}{\phi_C(P_{LD})})\%$ ,  $\phi_C(P_{LD})$  is obviously equal to  $\phi_C(P_{LC})$ . Meanwhile,  $P_{LC}$  is computed by taking into account link-cost only. Because only link-costs are considered to compute  $P_{LC}$ ,  $\phi_D(P_{LC})$  is always greater than or equal to  $\phi_D(P_{LD})$ . If  $\phi_D(P_{LC})$  is decreased by  $100(1 - \frac{\phi_D(P_{LD})}{\phi_D(P_{LC})})\%$ , then  $\phi_D(P_{LC}) = \phi_D(P_{LD})$ . The following steps explain a process for obtaining new parameter.

#### Steps to calculate the New Parameter

1. Compute two paths  $P_{LD}$  and  $P_{LC}$
2. Compute  $\bar{C} = \frac{\phi_C(P_{LD})}{n(P_{LD})}$  and  $\bar{D} = \frac{\phi_D(P_{LC})}{n(P_{LC})}$
3. Compute  $F^{-1}(\frac{3}{2} - \frac{\phi_C(P_{LC})}{\phi_C(P_{LD})})$  and  $F^{-1}(\frac{3}{2} - \frac{\phi_D(P_{LD})}{\phi_D(P_{LC})})$  i.e.,  $z_{\alpha/2}^d$  and  $z_{\alpha/2}^c$
4. Compute  $post_{LD} = \bar{C} - z_{\alpha/2}^d \frac{S_{LD}}{\sqrt{n(P_{LD})}}$  and  $post_{LC} = \bar{D} - z_{\alpha/2}^c \frac{S_{LC}}{\sqrt{n(P_{LC})}}$   
 where  $S_{LD}$  and  $S_{LC}$  are the sample standard deviation
5. Compute  $Cfct_{(i,j)}(c_{(i,j)}) = max\{ 1, 1 + (c_{(i,j)} - post_{LD}) \}$  and  $Dfct_{(i,j)}(d_{(i,j)}) = max\{ 1, 1 + (d_{(i,j)} - post_{LC}) \}$
6. We obtain the new parameter  $Cfct_{(i,j)}(c_{(i,j)}) \times Dfct_{(i,j)}(d_{(i,j)})$ .

In order to obtain the percentile( $z_{\alpha/2}^\beta, \beta = d, c$ ), we can use the cumulative distribution function (CDF). Ideally, the CDF is a discrete function but we assume that the CDF is a continuous function in convenience through out this paper. Let the CDF be  $F(x) = \int_{-\infty}^x \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}} dy$ . Then, the percentile is a

solution of the following equation.  $F(z_{\alpha/2}^d) - \frac{1}{2} = 1 - \frac{\phi_C(P_{LC})}{\phi_C(P_{LD})}$  which means  $z_{\alpha/2}^d = F^{-1}(\frac{3}{2} - \frac{\phi_C(P_{LC})}{\phi_C(P_{LD})})$  if  $100(1 - \frac{\phi_C(P_{LC})}{\phi_C(P_{LD})})\% < 50\%$ . Table 1 shows the percentile we have calculated.

**Table 1.** The percentile

$\eta = [ 100 ( 1 - \frac{\phi_C(P_{LC})}{\phi_C(P_{LD})} ) ] \%$ where $[x]$ gives the integer closest to $x$ .													
$z_{\alpha/2} = 3.29$ if $\eta \geq 50$ and $z_{\alpha/2} = 0.00$ if $\eta = 0$ ( $z_{\alpha/2}$ is either $z_{\alpha/2}^d$ or $z_{\alpha/2}^c$ )													
$\eta$	$z_{\alpha/2}$	$\eta$	$z_{\alpha/2}$	$\eta$	$z_{\alpha/2}$	$\eta$	$z_{\alpha/2}$	$\eta$	$z_{\alpha/2}$	$\eta$	$z_{\alpha/2}$	$\eta$	$z_{\alpha/2}$
49	2.33	48	2.05	47	1.88	46	1.75	45	1.65	44	1.56	43	1.48
42	1.41	41	1.34	40	1.28	39	1.23	38	1.18	37	1.13	36	1.08
35	1.04	34	0.99	33	0.95	32	0.92	31	0.88	30	0.84	29	0.81
28	0.77	27	0.74	26	0.71	25	0.67	24	0.64	23	0.61	22	0.58
21	0.55	20	0.52	19	0.50	18	0.47	17	0.44	16	0.41	15	0.39
14	0.36	13	0.33	12	0.31	11	0.28	10	0.25	9	0.23	8	0.20
7	0.18	6	0.15	5	0.13	4	0.10	3	0.08	2	0.05	1	0.03

Once the  $Cfct_{(i,j)}(c_{(i,j)})$  and the  $Dfct_{(i,j)}(d_{(i,j)})$  are found, we compute the value  $Cfct_{(i,j)}(c_{(i,j)}) \times Dfct_{(i,j)}(d_{(i,j)})$  for each link of  $P$ . The best feasible selection is the link with the lowest cost per delay on initial  $P$ . In other words, the link with the highest  $1/\text{cost per delay}$  could be selected. So then,

$$\left( \frac{1}{Cfct_{(i,j)}(c_{(i,j)})} \right) / Dfct_{(i,j)}(d_{(i,j)}) = 1 / ( Cfct_{(i,j)}(c_{(i,j)}) \times Dfct_{(i,j)}(d_{(i,j)}) ) .$$

If the value of the formular is low, the performance should be poor. Therefore, we use Dijkstra’s technique [1] with  $Cfct_{(i,j)}(c_{(i,j)}) \times Dfct_{(i,j)}(d_{(i,j)})$ .

## 4 Performance Evaluation

We compare our new parameter to only link-delays and only link-costs. Two performance measures -  $\phi_C(P)$  and  $\phi_D(P)$  - are combined our concern and investigated here. We now describe some numerical results with which we compare the performance for the new parameter. The proposed one is implemented in C+++. We consider networks with number of nodes which is equal to 25, 50, 100, and 200. We generate 10 different networks for each size given above. The random networks used in our experiments are directed, symmetric, and connected, where each node in networks has the probability of links ( $P_e$ ) equal to 0.3 [4]. Randomly selected source and destination nodes are picked uniformly. We simulate 1000 times ( $10 \times 100 = 1000$ ) for each  $n$  and  $P_e$ . Fig. 1 shows the average  $\phi_C(P)$  and  $\phi_D(P)$ , where each path  $P$  is  $P_{LC}$ ,  $P_{LD}$ , and  $P_{New}$ . As a result, the proposed new parameter ascertains that  $\phi_C(P_{LC}) \leq \phi_C(P_{New}) \leq \phi_C(P_{LD})$  and  $\phi_D(P_{LD}) \leq \phi_D(P_{New}) \leq \phi_D(P_{LC})$ . For details on analyzing performance for the new parameter, refer to Fig. 1 (d). The path cost  $\phi_C(P_{LC}) = 3.04$  is far superior, and  $\phi_C(P_{LD}) = 13.51$  is the worst. Likewise the path delay  $\phi_D(P_{LD}) = 3.03$

is far better, and  $\phi_D(P_{LC}) = 13.53$  is the highest. Let us consider path  $P_{New}$  which is measured by the probabilistic combination of cost and delay at the same time. Because the  $\phi_C(P_{New})$  occupies  $\frac{5.92-3.04}{13.51-3.04} \times 100 = 27.5\%$  between  $\phi_C(P_{LC})$  and  $\phi_C(P_{LD})$ ,  $\phi_C(P_{New})$  is somewhat expensive than  $\phi_C(P_{LC})$  but becomes more superior than  $\phi_C(P_{LD})$ . In the same manner, the  $\phi_D(P_{New})$  occupies  $\frac{6.21-3.03}{13.53-3.03} \times 100 = 30.3\%$  between  $\phi_D(P_{LD})$  and  $\phi_D(P_{LC})$ . In other words, the new parameter takes into account both cost and delay at the same time.

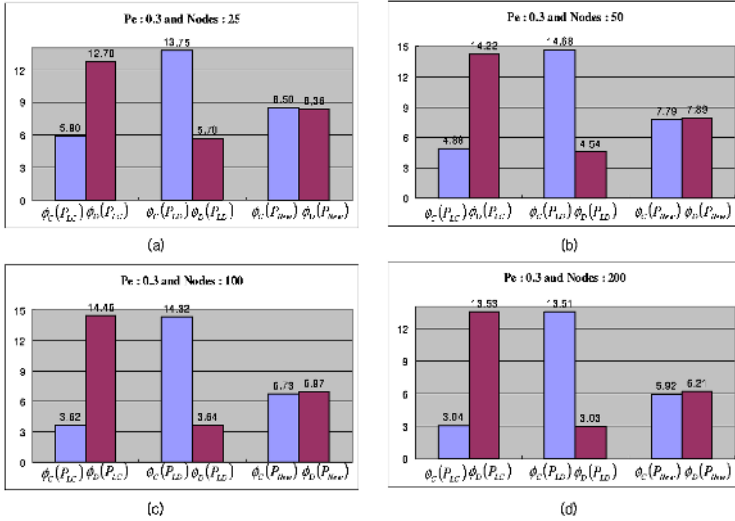


Fig. 1. Performance comparison for each  $P_e$  and  $n$

## 5 Conclusion

In this paper, we have formulated the new parameter for DCLC path problem, which is known to be NP-hard [2]. Because the DCLC must consider together cost and delay at the same time,  $P_{LC}$  and  $P_{LD}$  are unsuitable to the DCLC problem. Hence the new parameter takes into consideration both cost and delay at the same time. We would like to extend the new parameter to the weighted parameter that can regulate as desirable  $\phi_C(P)$  and  $\phi_D(P)$ .

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