



Recommender System Based on Fuzzy Reasoning and Information Systems

Martin Tabakov^(✉)

Faculty of Computer Science and Management, Department of Computational Intelligence, Wrocław University of Science and Technology, Wrocław, Poland
martin.tabakow@pwr.edu.pl

Abstract. In this research a recommender system with possible applications in e-commerce, based on rule induction mechanism and fuzzy reasoning, is presented. The theoretical concept proposed assume the application of fuzzy sets in a procedure of rule induction, as an information generalization, in purpose to predict the degree of subjective customer satisfaction with respect to his previous reviews. The innovative idea lays in the transformation of decision rules into fuzzy rules, regarding to the basic Mamdani reasoning model. The research was verified on real data, i.e. customer reviews of different products.

Keywords: E-commerce · Recommender systems · Fuzzy systems
Rule induction

1 Introduction

In recent years, e-commerce systems have developed rapidly as customers purchase regularly products from online stores. What more, there are many ways and tools to monitor the activity of potential customers, for example through social media and many others applications. As part of the business information flow, recommender systems have proven to be a successful tool to assist customers, by advising and finding the most suitable products to facilitate online decision-making [10, 15–18]. There are many successful technologies for recommender systems, such as collaborative filtering (CF) systems, which have already been applied by many commercial web sites such as Amazon.com, Netflix, and so on [1, 2, 7, 8]. The basic concept of these systems lays in the use of historical data related to user preferences or behavior to predict how new users will act [6, 9]. There is a lot of development of this concept using fuzzy sets as well [4, 5, 12, 20].

In the research proposed, a new concept combining rule induction and fuzzy reasoning is proposed. The concept introduced uses learning set, which presents customer preferences and generalize information with fuzzy sets to predict degree of customer interest with new products. What distinguishes the research proposed, is the interpretation of the information function, applied in the rule induction process, by defining it's values as fuzzy sets, which allows to generate fuzzy rule base from the data itself. The rest of the paper is organized as follows: in Sect. 2 the necessary theoretical background is briefly explained, in Sect. 3 the recommender system process flow diagram is

presented, in Sect. 4 the data set used is explained, in Sect. 5 experiments and results are given and finally conclusions are introduced.

2 Theoretical Background

In this section, the pre preliminaries of the Mamdani fuzzy model [11] and Pawlak's information systems rule induction procedure [19], are briefly explained.

2.1 Fuzzy Sets

A fuzzy set A consists of a domain X of real numbers together with a function

$$\mu_A: X \rightarrow [0, 1], [21] \text{ i.e.:}$$

$$A =_{df} \int_X \mu_A(x)/x, x \in X \quad (1)$$

here the integral denotes the collection of all points $x \in X$ with associated membership grade $\mu_A(x) \in [0, 1]$. The function μ_A is also known as the membership function of the fuzzy set A , as its values represents the grade of membership of the elements of X to the fuzzy set A . The idea is to use membership functions as characteristic functions (any crisp set is defined by its characteristic function) to describe imprecise or vague information. This possibility along with the corresponding defined mathematical apparatus, initiated a number of applications.

2.2 Fuzzy Reasoning – Mamdani Model

Figure 1 shows the schematic diagram of a type-1 fuzzy controller. The main idea is that all input information are fuzzified and then processed with respect to the assumed knowledge base, inference method and the corresponding defuzzification method.

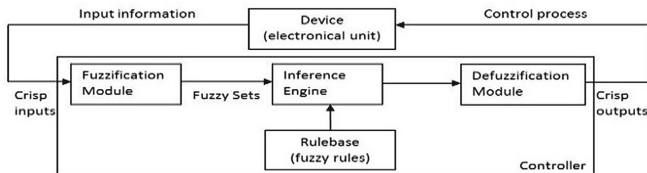


Fig. 1. Information flow within a typical type-1 fuzzy controller

Let consider the rule base of a fuzzy logic controller consisting of N rules which take the following form:

$$R^n :_{df} IF(x_1 \text{ is } X_1^n) \text{O} \dots \text{O} (x_i \text{ is } X_i^n) THEN y \text{ is } Y_n \quad (2)$$

Where X_i^n ($i = 1, \dots, I$; $n = 1, \dots, N$) are fuzzy sets defined over corresponding domains and Y_n is an output information, which in the Mamdani model [11] is assumed as fuzzy set as well, defined over some domain Y . The binary operator ‘o’ is the t - or s -norm ($o \in \{\otimes, \oplus\}$; $\otimes, \oplus: [0, 1]^2 \rightarrow [0, 1]$) which have the commutative, associative and the monotonic properties, and have the constants 1 and 0 as unit elements, respectively. In fuzzy logic, the t -norm operator provides the characteristic of the AND operator, while the s -norm provides the characteristic of the OR operator [3].

Assuming an input vector $\bar{x} = \{x'_1, x'_2, x'_3, \dots, x'_i\}$, typical computations of a fuzzy system consist of the following steps:

- (1) Compute the membership grades of x'_i on each X_i^n , $\mu_{X_i^n}(x'_i)$, $i = 1, \dots, I$; $n = 1, \dots, N$
- (2) Compute the firing value of the n^{th} rule, f^n :

$$f^n(\bar{x}) =_{df} \mu_{X_1^n}(x'_1) o \dots o \mu_{X_I^n}(x'_I) \in [0, 1] \quad (3)$$

- (3) Compute defuzzification output. The most common method is the centre of gravity (COG) method with assumed relation between the premise and the conclusion of the fuzzy rules as the *min* operator:

$$Y_{COG}(\bar{x}) =_{df} \frac{\sum_{y \in Y} \mu_{\cup_n Y'_n}(y) \cdot y}{\sum_{y \in Y} \mu_{\cup_n Y'_n}(y)} \quad (4)$$

$$\text{where : } \forall_{y \in Y} \mu_{Y'_n}(y) =_{df} \min\{f^n, \mu_{Y_n}(y)\}, n = 1, \dots, N \quad (5)$$

The output value is directly related to the control process.

2.3 Rule Induction

In the eighties and nineties Zdzislaw Pawlak introduced the fundamentals of information systems [13] and rough sets [14]. An information system (IS) is defined by the following elements:

$$IS =_{df} (U, A, V, f) \quad (6)$$

where U is a universe, A is a set of attributes, V represents attributes domains: $V =_{df} \cup_{a \in A} V_a$, where V_a is the domain of the a^{th} attribute ($a \in A$) and f is the so called information function: $f: U \times A \rightarrow V$, $\forall_{x \in U, a \in A} f(x, a) \in V_a$. Important role in the theory plays the introduced *indiscernibility binary relation*, defined over U : $\text{IND}(B) =_{df} \{(x, y) \in U \times U : \forall_{a \in B} f(x, a) = f(y, a)\}$, under which the *lower* and *upper* approximations of any subset of U can be defined, respectively:

$$B \downarrow X =_{df} \{x \in U : [x]_{\text{IND}} \subseteq X\}, B \uparrow X =_{df} \{x \in U : [x]_{\text{IND}} \cap X \neq \emptyset\}, \quad (7)$$

where $B \subseteq A$ and $X \subseteq U$.

Information systems can be interpreted as a decision tables if a decision attribute is introduced. With this assumption, a decision making approach was introduced by Skowron and Suraj [19], which generates a set of rules for any decision attribute value. A detailed explanation is omitted here, but briefly the procedure consists of the following steps:

1. Define an information system with decision attribute,
2. Eliminate object conflicts (i.e. objects with same information function values, but different decision values), applying *lower* or *upper* approximation precision analysis,
3. Provide *attribute reduct* using *discernibility matrix*,
4. Apply rule induction algorithm on the so defined new information system (completing step 2 and 3) and thus, define set of rules for each decision attribute value, which correctly cover the decision problem.

Below, an example is introduced which illustrates the input and the output of the rule induction algorithm, assuming that step 2 and 3 are completed.

Input: information system $(x_1, x_2, \dots, x_5 \in U; \{a, b, c\} \subseteq A; a^* - \text{decision attribute}; V_a = V_b = V_c = V_{a^*} = \{0, 1, 2\})$

	a	b	c	a*	Output:
x_1	1	0	1	0	$Rule_1$ (concern decision value 0): $f(x_1, a) \vee f(x_3, a) \vee (f(x_5, a) \vee f(x_5, b)) \Rightarrow (decision: 0)$, $Rule_2$: $f(x_2, c) \Rightarrow (decision: 1)$, $Rule_3$: $f(x_4, a) \wedge f(x_4, c) \Rightarrow (decision: 2)$.
x_2	0	0	0	1	
x_3	2	0	1	0	
x_4	0	0	1	2	
x_5	1	1	1	0	

The only disadvantage with this decision making approach is that the induced rules are very crisp, i.e. $rule_2$ is interpreted as follows: *if object of x_2 type has information function value for the attribute 'c' exactly equal to '0' then make decision '1'*.

2.4 Fuzzy Information System

The above mentioned disadvantage of the rule induction process, lays in the basic of the research proposed. It is enough to involve fuzzy sets in the process of rule induction and thus, to achieve information generalisation. Therefore, the new proposal is to modify the information function introduced, by defining all attribute values as fuzzy sets. What more, for any numerical attribute, the basic fuzzy sets *low*, *medium* and *high* can be defined by using directly the data, assuming normal distributions.

So, for any attribute $A_i =_{df} \{a_1^i, a_2^i, \dots, a_n^i\}$ with respect to all considered objects, a normal distribution of attribute values may be considered in purpose to define the fuzzy set *medium*, as follows:

$$\mu_{medium}(a) =_{df} e^{\frac{-(a-a_0)}{2\sigma^2}} \quad (8)$$

determining the expected value (a_0) and the standard deviation (σ) directly from the set A_i . Next, the fuzzy sets *low* and *high* attribute values can be easily defined as well:

$$\mu_{low}(a) =_{df} \left\{ \begin{array}{l} 1 - e^{\frac{-(a-a_0)}{2\sigma^2}} : a < a_0 \\ 0 : a \geq a_0 \end{array} \right\}; \mu_{high}(a) =_{df} \left\{ \begin{array}{l} 0 : a < a_0 \\ 1 - e^{\frac{-(a-a_0)}{2\sigma^2}} : a \geq a_0 \end{array} \right\} \quad (9)$$

Therefore, an exemplary fuzzy information system could take the following form:

	attribute ₁	attribute ₂	decision attribute
object ₁	<i>low</i>	<i>medium</i>	D_1
object ₂	<i>medium</i>	<i>low</i>	D_1
object ₃	<i>high</i>	<i>low</i>	D_2

with the interpretation of the information function for example pair object/attribute (object₁ and attribute₁), as follows: $f_{fuzzy}(\text{object}_1, \text{attribute}_1) = \text{low}$, meaning $f(\text{object}_1, \text{attribute}_1)$ has the highest degree of membership to the fuzzy set *low*, defined over the domain of the attribute₁.

The decision attribute values can be defined as fuzzy sets as well, representing the degree of decision. So, under this interpretation of a fuzzy information system, the rule induced are very simple to be transformed into fuzzy rules. For example, if a rule is induced as follows:

$$f(\text{object}_1, \text{attribute}_1) \wedge f(\text{object}_2, \text{attribute}_1) \Rightarrow \text{decision} : D_1,$$

it can be naturally transformed into the fuzzy rule:

$$\text{IF } (f(\text{object}_1, \text{attribute}_1) \text{ is 'small'}) \otimes (f(\text{object}_2, \text{attribute}_1) \text{ is 'medium'}) \text{ THEN} \\ (\text{decision is } D_1).$$

In the provided experiments in this research, this concept of fuzzy information system and rule induction mechanism, was applied.

3 Recommender System Proposal

The aim of the system proposed is to apply fuzzy sets during the assumed rule induction procedure with fuzzification defined directly from the data used and therefore, to induce in automatic manner fuzzy rules suitable for further reasoning process with respect to the Mamdani model. The chart below shows the process flow diagram of the recommender mechanism proposed considering the learning phase and the system use as well.

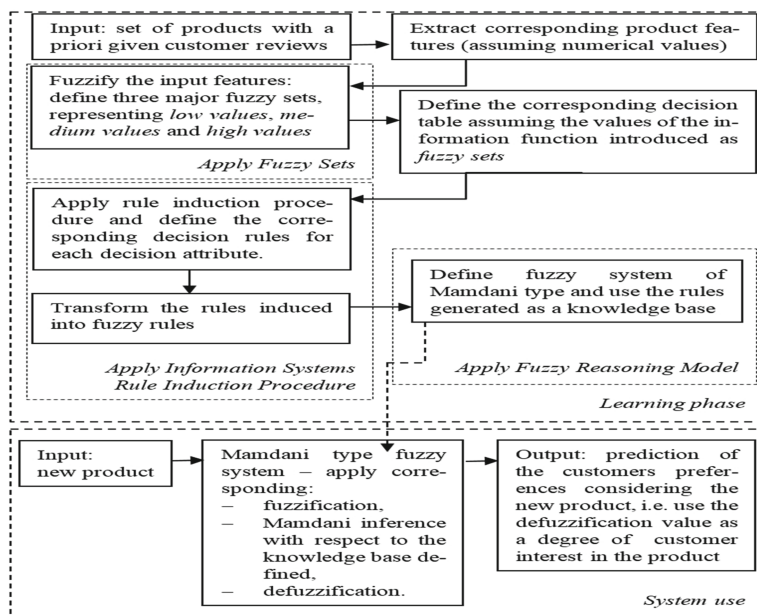


Fig. 2. Process flow diagram

4 The Data Set Used

The experiments provided, were carried out by polling 45 potential customers (IT students) of electronic products, such as: *laptops*, *cell phones* and *tablets*. Each of the customer was given the opportunity to review a list of product items during a survey. A customer had the possibility to review a product and to mark it with one of four numerical values related to a certain level of interest with the item: 0 – ‘not interested’, 1 – ‘very little interested’, 2 – ‘interested’ and 3 – ‘very interested’. Degree of interest 0 and 1 could be interpreted as negative and degree of interest 2 and 3 as positive. These four possible customer evaluation values are defined by corresponding fuzzy sets, as they determine the conclusions in the Mamdani reasoning model proposed. The fuzzy sets were defined as Gaussian distributions (but without the factor before the exponential function to ensure the achievement of 1), representing the degree of customer interest, with expected values = 10, 20, 30 and 40 and equal standard deviation $\sigma = 25$ (see Fig. 3).

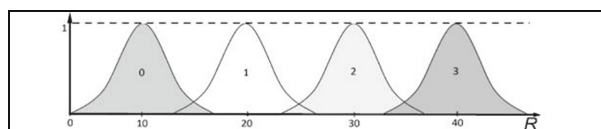


Fig. 3. Representation of the degree of customer interest with fuzzy sets

Below, an exemplary evaluation table is presented with customer review of cell phones with eight randomly chosen items¹. It should be noted, however, that only numerical attributes were considered (in the table below, attribute OS was ignored) (Table 1).

Table 1. Exemplary evaluation table.

	Screen diagonal	Screen resolution	Built – in memory	Battery	Weight	RAM	Camera (Mpix)	Thickness (mm)	Clock speed	OS	Number of cores	Price	Customer evaluation
Apple iPhone 5S	4"	1080 × 1920	32 GB	1560 mAh	112 g	1 GB	8 Mpix	7,6	1.3 GHz	iOS 7	2	3 149 zł	0
Samsung Galaxy S5	5.1"	1080 × 1920	16 GB	2800 mAh	146 g	2 GB	16 Mpix	8.3 mm	2.45 GHz	Android	4	2 999 zł	3
HTC One Max	5.9"	1080 × 1920	16 GB	3300 mAh	217 g	2 GB	2.1 Mpix	10,28	1.7 GHz	Android	4	2 976 zł	2
Sony Xperia Z2	5.2"	1080 × 1920	16 GB	3200 mAh	163 g	3 GB	20.7 Mpix	8,2	2.36 GHz	Android	4	2 799 zł	2
Nokia Lumia 1020	4.5"	768 × 1280	32 GB	2000 mAh	160 g	2 GB	41.0 Mpix	10,5	1.5 GHz	Windows Phone 8	2	2 099 zł	0
Samsung Galaxy S4	5"	1080 × 1920	16 GB	2600 mAh	130 g	2 GB	13 Mpix	7,9	1,9	Android	4	1 899 zł	1

Summarizing the data used, 45 potential customers were asked to provide reviews of three groups of electronic products: *laptops*, *cell phones* and *tablets*, with 25 item in each product group. Therefore, each customer has reviewed 25×3 : 75 product items, with total number of reviews: 45×75 : 3375 items. The set of attributes describing the product items considered is listed in Table 2.

Table 2. The attributes used.

Laptops	Cell phones	Tablets
– screen diagonal (inches)	– screen diagonal (inches)	– screen diagonal (inches)
– screen resolution (pixels)	– screen resolution (pixels)	– screen resolution (pixels)
– hard drive (GB)	– built-in memory (GB)	– built-in memory (GB)
– number of USB 3.0 connectors	– battery (mAh)	– battery (mAh)
– weight (kg)	– weight (g)	– weight (g)
– RAM	– RAM (GB)	– RAM (GB)
– video graphic card memory (MB)	– camera (MPix)	– camera (MPix)
– clock speed (GHz)	– thickness (mm)	– clock speed (GHz)
– number of cores	– clock speed (GHz)	– number of cores
– price (PLN)	– number of cores	– price (PLN)
	– price (PLN)	

¹ The price attribute may not be up-to-date, but it doesn't change the concept proposed.

5 Experiments and Results

The main assumption of the conducted experiments was to investigate the accuracy of the recommender concept proposed. As the considered case is not a classification problem, but rather requires a certain interpretation of the results achieved as they are very subjective, the following two accuracy tests were proposed:

- 1. *Test one* – all product from each customer list were processed by the fuzzy system proposed and sorted with respect to the system outputs.
- 2. *Test two* – products from each customer list were divided into two parts: learning set (19 items) and test set (6 randomly selected items) for validation purpose.

In both cases, for each customer, a knowledge base was generated with respect to his specific product review under the concept proposed (see Fig. 2). Therefore, for each customer a fuzzy reasoning system under the recommender concept proposed was developed which provides personalization.

Test one – results

The results are shown in the infographics below, with corresponding explanation (Fig. 4).

The results are shown in the infographics below, with corresponding explanation.

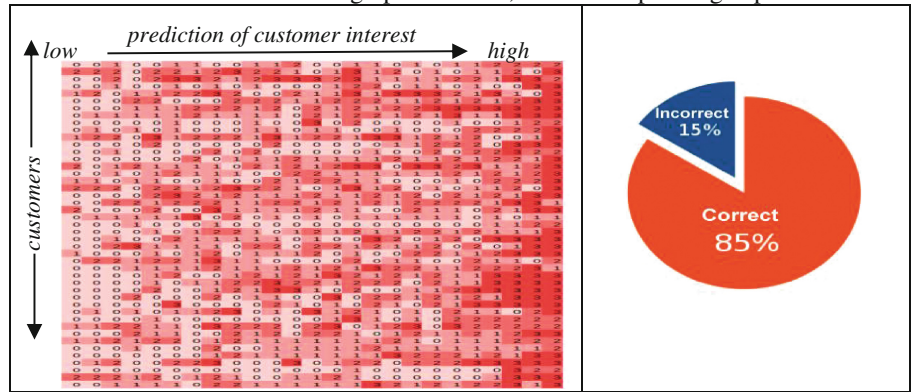


Fig. 4. Infographic presenting the results achieved for product *laptops* with corresponding classification

To explain the infographic above, let consider the first row. It contains the first customer product reviews highlighted with increasing colour saturation with respect to the review values. The first customer marked the chosen products with values: 0, 1 and 2. The values in the columns were sorted in ascending order with respect to the fuzzy recommender system output. Therefore, high system accuracy is related to more intense red colours on the right part of the infographic, see Fig. 5 below.

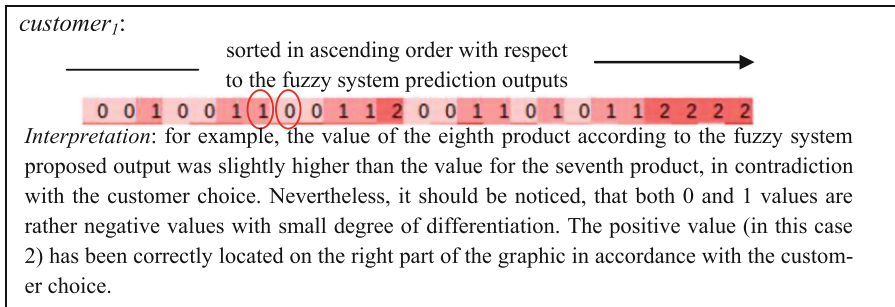


Fig. 5. Infographic interpretation. (Color figure online)

So, it is expected that more saturated colours will be located on the right part of the graphic. Furthermore, a classification of the results in terms of correct system prediction was introduced, assuming that if the average of customer review values of the half of the products from the right site of the row is higher than the average for all products, which indicates that the most of the products with higher degree of customer interest are on the right side of the graphic, it is classified as correct, else it is classified as incorrect. For example, the above system result regarding to *customer₁* reviews, is classified as correct system answer, as the average of the half products on the right side (12 product) of the row is equal to ≈ 1.08 , which is higher than the average value for all products: 0.8. For example, the second and the third rows (*customer₂* and *customer₃* results) are recognized as incorrect. Below in Fig. 6, results for products *tablets* and *cell phones* are presented in the same manner.

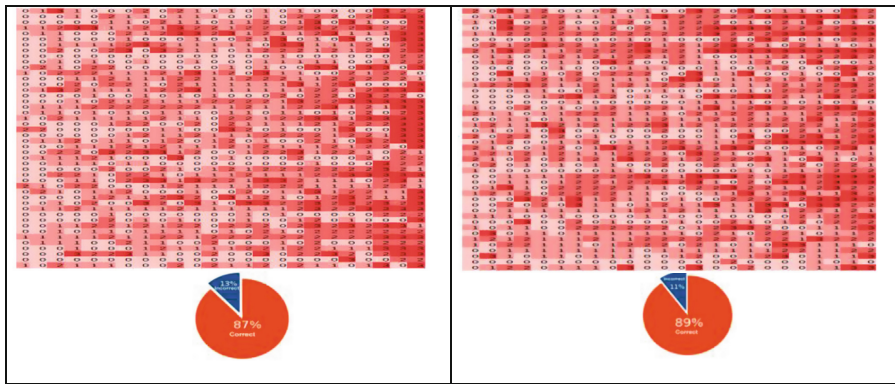


Fig. 6. Cell phones and tablets infographics and classification results, respectively.

As it can be noticed, the most saturated colours are mostly located on the right side of the infographics, which indicates correct system predictions.

Test two – results

The idea of the second test was more classic, i.e. for each customer a learning set and a test set was provided, and next a fuzzy system with respect to the concept proposed (see Fig. 2) was designed using the learning set. The system was applied on the test data set with the idea that products with higher customer interest should have higher system output values. A test for a certain customer was recognized as correct, if the sum of the customer reviews of the half of the products with higher system scores was more or equal to the sum of customer reviews with lowest system scores (Fig. 7).

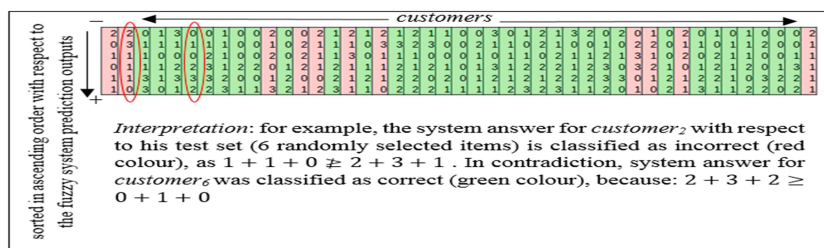


Fig. 7. Results for product: *laptops*

Below in Fig. 8, results for products *tablets* and *cell phones* are presented in the same manner.

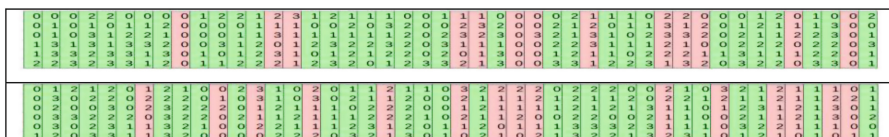


Fig. 8. Results for products: *cell phones* and *tablets* respectively

As it can be noticed, for all considered products, satisfactory results were achieved, which proves good system accuracy.

6 Conclusions

In this research a new recommender system was introduced. The theoretical concept presented combines two major approaches: rule induction procedure based on information systems and fuzzy reasoning. The involvement of fuzzy sets into the rule induction process allowed information generalization which represents subjective observations and opinions very accurate. The experiments provided, illustrated with corresponding infographics, proved good system accuracy and possibility of further development.

Acknowledgement. I would like to thank to my student Jakub Salamon for the experiments provided.

References

1. du Boucher-Ryan, P., Bridge, D.: Collaborative recommending using formal concept analysis. *Knowl.-Based Syst.* **19**(5), 309–315 (2006)
2. Breese, J., Heckerman, D., Kadie, C.: Empirical analysis of predictive algorithms for collaborative filtering. In: *UAI98*, pp. 43–52 (1998)
3. Bronstein, I.N., Semendjajew, K.A., Musiol, G., Mühlig, H.: *Taschenbuch der Mathematik*, p. 1258. Harri Deutsch (2001)
4. Cao, Y., Li, Y.: An intelligent fuzzy-based recommendation system for consumer electronic products. *Expert Syst. Appl.* **33**(1), 230–240 (2007)
5. Cheng, L.-C., Wang, H.-A.: A fuzzy recommender system based on the integration of subjective preferences and objective information. *Appl. Soft Comput.* **18**, 290–301 (2014)
6. Gediminas, A.: Toward the next generation of recommender systems: a survey of the state-of-the-art and possible extensions. *IEEE Trans. Knowl. Data Eng.* **17**(6), 734–749 (2005)
7. Greg, L., Brent, S., Jeremy, Y.: Amazon.com recommendations: item-to-item collaborative filtering. *IEEE Int. Comput.* **7**(1), 76–80 (2003)
8. Herlocker, J.L., Konstan, J.A., Borchers, A., Riedll, J.: An algorithmic framework for performing collaborative filtering. In: *Proceedings of the 1999 Conference on Research and Development in Information Retrieval* (1999)
9. Hofmann, T.: Latent semantic models for collaborative filtering. *ACM Trans. Inf. Syst.* **22**(1), 89–115 (2004)
10. Liu, D.R., Lai, C.H., Lee, W.J.: A hybrid of sequential rules and collaborative filtering for product recommendation. *Inf. Sci.* **179**(2), 3505–3519 (2009)
11. Mamdani, E.H., Assilian, S.: An experiment in linguistic synthesis with a fuzzy logic controller. *Int. J. Man-Mach. Stud.* **7**(1), 1–13 (1975)
12. Morawski, J., et al.: A fuzzy recommender system for public library catalogs. *Int. J. Intell.* **32**, 1062–1084 (2017)
13. Pawlak, Z.: Information systems theoretical foundations. *Inf. Syst.* **6**, 205–218 (1981)
14. Pawlak, Z.: *Rough Sets: Theoretical Aspects of Reasoning About Data*. Kluwer Academic Publishers Group, Dordrecht (1991)
15. Porcel, C., Herrera-Viedma, E.: Dealing with incomplete information in a fuzzy linguistic recommender system to disseminate information in university digital libraries. *Knowl. Based Syst.* **23**(1), 32–39 (2010)
16. Resnick, P., Iacovou, N., Suchak, M., Bergstrom, P., Riedl, J.: GroupLens: an open architecture for collaborative filtering of netnews. In: *Proceedings of ACM Conference on Computer-Supported Cooperative Work*, pp. 175–186 (1994)
17. Sarwar, B., Karypis, G., Konstan, J., Reidl, J.: Item-based collaborative filtering recommendation algorithms. In: *Proceedings of the 10th International Conference on World Wide Web, Hong Kong*, pp. 285–295 (2001)
18. Schafer, J.B., Konstan, J.A., Riedl, J.: E-commerce recommendation applications. *Data Min. Knowl. Disc.* **5**(1), 115–153 (2001)

19. Skowron, A., Suraj, Z.: A rough set approach to real-time state identification for decision making. Institute of computer science research, Report 18/93, Warsaw University of Technology (1993)
20. Year, R., Martínez, L.: Fuzzy tools in recommender systems: a survey. *Int. J. Comput. Intell. Syst.* **10**, 776–803 (2017)
21. Zadeh, L.: Fuzzy sets. *Inf. Control* **8**(3), 338–353 (1965)