Chapter 37

Digital Pedagogy in Mathematical Learning

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Abstract Digital pedagogy is a learning paradigm that can allow learners to be active partners in discovering and developing their own mathematical knowledge. In this sense, Piaget’s constructivist principles lay the foundation for developing digital pedagogy. In the paper that follows, we present a novel, intuitive, digital mathematical learning model. The model is focused on problem solving through computational thinking and is targeted to empower teenagers. More features and outcomes of this model will be discussed as well. As a foundation moving forward, the “use-modify-create” framework offers a helpful progression for developing computational thinking over time. It illustrates the benefits arising from engaging youth with progressively more complex tasks and giving them increasing ownership of their learning. The gained knowledge and skills of this cognitive learning both empower learners and enhance creativity. In its essence, we aim to develop the utopia of digital pedagogy in mathematical learning.

Keywords Digital pedagogy • Computational thinking • Gamification of education • Project-based learning • Problem solving

37.1 Introduction

We review an effective digital pedagogy for mathematical learning. We intend to present a way to create a cognitive-learning digital environment in today’s ubiquitously connected world and align with the surrounding and dynamic cultural

In memory of **Seymour Papert** (1928–2016).

Digital pedagogy is a legacy of Seymour Papert. I developed this work based on his work “Mindstorms” in the last four years. I delivered a presentation on digital pedagogy on July 29, 2016 at ICME-13. He passed away on July 31, 2016.

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trends of the mobile computational device. Smart computational devices compose a medium for digital pedagogy and affect in the way people think and learn.

To enhance creativity, connecting the developmental psychology of Piaget (1966) to the digital pedagogy in mathematics learning is key to developing an innovative learning model. In the modern approach towards teaching and learning mathematics, students should be partners and active agents in the learning process and problem solving (Boaler 2008). This modern approach in mathematics learning is more experimental and collaborative based on “learning by doing” (Dewey 1897), and students’ learning gain is organic as they are partners in the learning process. They learn and develop their own knowledge step by step through innovative and creative thinking, experiences and discoveries, and collaboration and teamwork. Access to information and online resources has the potential to change the means of learning, allowing for a personalized and collaborative learning environment that is no longer restricted to schools and classes. The gained knowledge and skills of such cognitive learning empower students for everyday activities such as data analysis, reasoning, and problem solving. Digital pedagogy is a way to create such an environment for cognitive learning that requires rich toolkits rather than force-fed knowledge. Nevertheless, an interactive learning platform and digital resources are needed to develop the new paradigm for cognitive learning.

The growth mindset is also another important aspect of the digital pedagogy. In a growth mindset, students understand that their talents and abilities can be developed and expanded through effort, good learning, and persistence. In contrast, a fixed mindset proposes that students’ basic abilities, intelligence, and talents are inherent characteristics and thus are not expandable: They have a certain amount of “smartness” and that’s that. In the growth mindset, students do not necessarily believe everyone possesses the same intelligence, but they suppose anyone can get smarter if they work at it (Dweck 2006; Boaler 2016). Digital pedagogy is the ultimate paradigm to attract learners and support a growth mindset. Problem solving ability or “smartness” grows with experience on the digital pedagogic platform.

We intend to present a digital platform for mathematical learning through problem solving that enables creative engagement, develops mathematical skills, and supports a growth mindset. Briefly, we go through the background of digital pedagogy, introducing a theoretical model for the digital platform, and finally we discuss a case study and some experimental results.

37.2 Background

Digital pedagogy of mathematics learning is a legacy of Seymour Papert (Blikstein 2013); we summarize his work to touch on how his ideas have affected mathematics education. We take the opportunity to adapt his approach to the advancements of computational technology. We will also describe the Piagetian learning path, but in the context of Papert, who connected it to digital pedagogy.

We proceed to see how Piagetian developmental psychology has been connected to digital learning through Papert's work, reflected in *Mindstorms* (Papert 1980), but we discuss technological advancements as well.

37.2.1 *Mindstorms*

Innovative learning models must reflect on what is happening in the surrounding culture and use dynamic cultural trends as media to carry educational interventions. It has become commonplace to say that today's culture is marked by a ubiquitous smart computational device. Smart computational devices can contribute to mental processes, not only as instruments we improve at using (e.g., touch screens), but also in more essential conceptual ways, influencing how we may think. Smart computational devices will enter the private worlds of learners everywhere and can create a new paradigm to form new relationships with knowledge in a personalized way. In this regard, the whole process of mathematics learning should be involved in a dialectical interaction between new technologies and new ways of learning mathematics.

Digital pedagogy is essential in reflection of the Piagetian learning path, and in this sense we should create an environment in which learners surf and interact with their environment to learn how to "talk mathematically" as a means to capture mathematical concepts and ideas implicitly. Smart computational devices are the best tools to create such a new paradigm for a learning environment that can help learners learn and develop their own mathematical knowledge organically. Computational devices can address personalized learning; they are unique in providing us with means to counteract what Piaget saw as an obstacle.

37.2.2 *Piagetian Learning Path*

To touch and understand how computational technology and a digital learning environment can be a medium for knowledge development, we should look at the Piagetian learning path.

Piaget is the leading theorist of learning without deliberate teaching. This, however, does not imply a spontaneous atmosphere that leaves the learner alone; rather, it means supporting learners to build their own intellectual structures. In this sense, we are looking for an environment where mathematics can become a natural vocabulary, a learning environment with the proper emotional and cultural support where learners can learn not only that they can excel at mathematics but also that they can share the joy of mathematical experiences. This concept shows how to use computational devices as vehicles to develop digital pedagogy in a new environment.

We are focused on the Piagetian learning path as the natural, spontaneous learning of people in interaction with their environment. Piaget's thoughts have been underplayed because they offered no possibilities for action in the world of traditional education. But in the learning environment of the digital pedagogy, enriched by smart computational devices and supported by artificial intelligence, Piaget's principles can come to fruition. For many years his ideas could not be expanded due to lack of means of implementation, but digital pedagogy is going to make it available.

37.3 A Model

Our dream is to create a digital learning environment in which the task is not to learn a set of formal rules but to develop sufficient insight for mathematical concepts. We look for a digital environment to grow learners' mathematical mindsets through experience and a flourishing joy of mathematics. We expect an empowering platform to enhance creativity and develop mathematical skills and naturally explore domains of mathematical ideas in the sense of Piagetian learning path. We would like to present a model for such a learning paradigm as an online interactive "learning-by-doing" environment. We consider problem solving and algorithmic thinking as the means of exploring mathematical concepts on a platform that can expose computational thinking.

37.3.1 *Characteristics*

We consider a gamified learning environment that utilizes the computational thinking process and computational mathematical skills. It should be a personalized and collaborative learning platform targeted towards teenagers.

Computational thinking concepts were envisioned by Papert (1980, 1996) and involve how to use computation to enhance thinking, create new knowledge, and change patterns of access to knowledge. More recently, however, Wing (2006) brought a different approach and new attention to computational thinking. She considered the topic a fundamental skill for everyone's analytical ability, along with reading, writing, and arithmetic, and as a process to formulate a complicated problem and algorithmically solve it. We consider computational thinking based on Papert's enhancement of thinking, but specifically with a problem-solving approach in the sense of algorithmic thinking. In brief, computational thinking combines critical thinking with the computing power as the foundation for innovating solutions to real-life problems.

We also consider the gamification of mathematical concepts as a framework. Games bring a new approach to pedagogy (Gee 2007; Devlin 2011) and possess the potential to create interaction and insert motivation; players are driven to their

virtual goals and learn by doing. Allowing players to make mistakes through experimentation in a risk-free environment brings about learning by doing implicitly through mistakes: Players “feel” their way around games, and, by receiving instant feedback to their actions, they can adjust their problem-solving strategies accordingly. Put simply, games bridge the gap between formal knowledge and intuitive understanding. Another crucial aspect of games is the immense amount data generated by players that can be used as feedback for assessment of the learning process. The basic idea is to implicitly ease learners into the world of mathematics while they are enjoying themselves.

37.3.2 Playground

We consider computational thinking as the process for problem solving on the proposed platform; we also go for functional programming as a tool to formalize intuition about the problem-solving process.

Computational thinking is a four-stage problem-solving framework consisting of decomposition, pattern recognition, abstraction, and algorithm design, as shown in Fig. 37.1. We have enriched and connected the stages with a “playground” as a place for experimental problem solving.

In this model, the playground is an easily accessible place where learners can tackle problems through experimentation.

We also consider functional programming as a toolbox on the playground for problem solving. The functional programming paradigm explicitly supports a pure functional approach to problem solving, which involves composing the problem as a set of functions to be executed. Functional programming is a style that avoids changing state, so it is a powerful tool that can be used in a modular form for problem solving. Such modularity is key, as it specifically empowers learners to utilize what they have built in the past for future solutions.

Gained knowledge in this model empowers learners in reasoning, problem solving, and algorithmic thinking in a gamified fashion. We can gather users’ data and analyze them through the design-based research method (Brown 1992), which

Fig. 37.1 Four steps around playground

should be embedded in the platform. Results of the analyzed data can be used to improve the platform and also bring recommendation and feedback to the learners.

37.4 A Case Study

Piaget (1966) showed how learners construct a world out of materials in their environment. Papert (1980) has also mentioned that experience with games is a bridge between formal knowledge and intuitive understanding. In this sense, we have developed and considered a gamified digital learning platform as a case study.

To develop a case study for the first approach, we went through digital mathematical puzzle games in an interactive fashion, but we found they only attracted students who showed a proclivity for mathematical thought rather than the general population. But, in the revised version we considered that the following objectives should be achieved by the platform:

- Enable creative engagement
- Develop mathematical skills
- Support a growth mathematical mindset
- Be collaborative and social.

We went through the next version (Polyup 2016) to look for the above objectives, and we received positive responses to the prototype from a variety of teenagers in focus groups. We will present the platform and have a brief look at the results of test cases.

37.4.1 Platform

The developed platform (Polyup 2016) for mobile computational devices is about problem solving on a functional programming platform through computational thinking. Functional programming is achieved with lambda calculus (Revesz 1998) and provides a theoretical framework for describing functions and their evaluation. Functional programming is a style of building structures and elements in a modular form that treats computation as the evaluation of mathematical functions and avoids changing-state and mutable data. To develop computation through functions, we are using a postfix, or Reverse Polish Notation, to avoid parentheses in expressions and computation (McCarthy 1960).

The platform is a user friendly environment where the user is equipped with numbers, operations, and basic functions. The user can do computation in a functional modular form; computation simply goes top to bottom with postfix. Users can drag and drop numbers and operations on stacks to script a program and

Fig. 37.2 Computational thinking playground

run it to calculate the output of the desired function. The platform and an example are shown in Fig. 37.2.

The platform is a collaborative and social playground for problem solving through personal experiences and also supports growth of a mathematical mindset. Many puzzles are preloaded in a step-by-step fashion for users to develop their own knowledge in a gamified interactive platform. To make the platform social, users can develop their own puzzles and share them with friends.

Despite its simplicity, the platform is Turing complete. It also features a chatbot as a sidekick, or a mentor, to help problem solvers. Users’ data are gathered on the platform to be used through a smart system for analytics, which provides puzzle recommendations for users and is also used for advancement of the platform.

Advanced functional techniques such as recursion also are a central part of the learning environment and are shown visually; an example script to compute triangular numbers, as well as its visual running form, is shown in Fig. 37.3. Developed functions in a modular form can be reused to address more complex problems.

37.4.2 Feedback

We have tested the platform in a variety of classes and schools, from middle schools to high schools. A summary of students’ feedback from seven different classes is shown in Table 37.1; the figures shown are the mean of students’ responses on a scale of 1–10.

Fig. 37.3 Triangular numbers on the playground

Table 37.1 Feedback of students

Grade level	8	9	8	9	10	8	11
Number of students	17	12	19	16	13	18	18
How much did you like the platform?	6.94	7.33	7.40	6.93	7.38	7.22	8.13
How much do you like the script language?	6.76	6.08	7.05	6.5	6.46	7.11	6.88
How did you like the training puzzles?	6.82	6.42	6.00	5.85	6.85	6.50	6.61
How would you like ability to control digital art with the functions that you can develop on the platform?	7.18	7.50	8.10	5.62	8.23	8.17	7.50
How would you like the ability to control robots with the functions that you can develop on the platform?	6.19	7.17	8.10	6.18	7.77	7.83	7.63
How much do you like working with another player on a shared playground?	6.71	6.92	6.95	5.5	6.62	7.00	6.75
How likely would you be to recommend the platform to your friends?	6.71	7.17	7.30	6.62	6.62	6.78	7.38

Tests consisted of one-hour sessions, starting with introducing the platform to the students and then having them solve 10–15 selected puzzles while learning implicitly about functional programming. In the last 10 min, they had a chance to develop their own puzzles, in which significant achievements were observed.

Feedback was generally positive. As observed in the live sessions, students were very well engaged and would also recommend the platform to their friends, as supported by the first and last questions of the survey. Another important observation lies in how students got involved in the technicalities of scripts and functional programming, a learning path we observed to be very natural and organic.

Another key observation lies in the questions that asked if they were interested in controlling robots or developing digital arts through functional programming. Their very high levels of interest show how important it is to reconcile mathematical problem solving with applicable skills. With a connected functional programming environment, users can become creatively engaged with the software and hardware tools in their daily lives and change these objects' functionality to better suit their interests and needs.

37.5 Conclusion

We studied significant works of Seymour Papert as a pioneer in developing digital pedagogy, and these works provided the base for our adaptation to the recent advancement of mobile computational technology. We found the opportunity to develop a digital learning environment that can engage learners in an experiential and growth-mindset fashion such that they can develop their own knowledge.

We also received positive feedback from the users, which provides motivation for further development of digital pedagogy in mathematical learning. The interest of users to develop applicable skills reveals the deficiency that current platforms have in connecting problem-solving ability to real-life applications such as digital arts and robotics. With the modularity of functional programming and the creativity of computational thinking, modification of various objects is natural. Once these tools are available, the process of “use-modify-create” will bring opportunities for endless creativity among the youth.

The translation of problem solving to allow it to have a tangible impact outside the educational environment is a novel approach that will attract and motivate a greater general audience to become engaged in computational thought. Through digital experiences, learners will develop their own mathematical ability and ultimately spread the joy of mathematics.

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