Chapter 31

Improving Mathematics Pedagogy Through Student/Teacher Valuing: Lessons from Five Continents

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Abstract This chapter focuses on the construct of values/valuing, using the findings of the large-scale, ‘What I Find Important (in mathematics learning)’ [WIFI] study to explore how values/valuing promotes effective (mathematics) pedagogy. The analysis of some 16,000 questionnaires collected from 19 economies reveals the absence of any relationship between values and specific actions, suggesting that the actions that reflect what are being valued are culturally-dependent. Students in economies which perform well in the PISA assessments were also found to value connections, understanding, communication, and recall in their mathematics learning, whereas their peers at the other end of the league table appeared to value relevance and practice more. The notion of intrinsic and extrinsic valuing will be discussed. In acknowledging the presence of value differences and conflicts that arise from inter-personal interactions in mathematics lessons, teachers’ capacity to engage with values alignment is highlighted.

Keywords Values • Intrinsic/extrinsic valuing • Values alignment
Volition • Conation

31.1 Introduction

Hattie’s (2015) more than 1200 meta-analyses of some 65,000 studies involving about 250 million students has identified factors associated with students’ academic success at school. Amongst the 195 factors, 89 of these displayed effect sizes of 0.4 or more, which Hattie considered to be the hinge point above which the factors are worth employing to advance student learning. Six key findings were summarized from these 89 interventions that mattered, suggesting that the key to effective teaching lies in the valuing of just a couple of main ideas. One of these key findings

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refers to heightened impact on student learning “when teachers base their teaching on students’ prior learning” (Hattie 2015, p. 81), emphasizing the valuing of *prior learning*. Another key finding identifies teachers setting “appropriate levels of challenge” (p. 81), suggesting the importance and valuing of *challenge*. The message here is that effective teaching practices can be described in ways which are generic, focusing just on the essence that is of value. Thus, for example, for the intervention ‘appropriate levels of challenge’, *challenge* is being valued and is the focus; what constitute appropriate levels and indeed what they look like seem to be flexible and able to be defined in context.

This focus on values and valuing to account for effective or successful learning has also been evidenced in individual studies, some of which would have been analyzed by John Hattie in his meta-analysis exercise. For instance, a Nuffield Foundation-commissioned review asserted that

high attainment may be much more closely linked to cultural values than to specific mathematics teaching practices. This may be a bitter pill for those of us in mathematics education who like to think that how the subject is taught is the key to high attainment. But study after study shows that countries ranked highly on international studies – Finland, Flemish Belgium, Singapore, Korea – do not have particularly innovative teaching approaches. (Askew et al. 2010, p. 12)

This chapter focuses on these culturally-situated values, using the findings of a large-scale research study to explore the sorts of values/valuing that are associated with effective (mathematics) pedagogy. It will begin with a review of research that had been conducted on values and valuing in the context of mathematics education. This review would highlight the process of valuing as involving both cognition and affect, how its evaluation is complicated by the fact that values are invisible, implicit, and not always activated, and how it provides one with the want to embrace it. This will be achieved through reflecting on the findings of the large-scale, ‘What I Find Important (in mathematics learning)’ Study. The discussion will be presented next, emphasizing that constituent actions of values are culturally-dependent rather than absolute. The notion of intrinsic and extrinsic valuing will be discussed, with a tentative proposal of how these might be related to mathematical performance. The absence of correlation between values and constituent actions will be discussed. Lastly, the inevitable and prevalent instances of values alignment will be highlighted in the context of the data analysed.

31.2 The Nature of Values and Valuing in Mathematics Education

Although the concept of values and valuing in school education is not new (e.g. moral education programs), the acknowledgement of its role in the teaching and learning of individual school subjects is a relatively recent research activity. Values in mathematics and in mathematics education were first proposed by Bishop

(1988a, 1996) respectively. For the former, “the three value components of culture - White’s sentimental, ideological and sociological components - appear ... to have pairs of complementary values associated with mathematics” (Bishop 1988b, p. 185), namely, *rationalism* and *objectism*, *progress* and *control*, *mystery* and *openness*.

While Seah and Andersson’s (2015) conception are rather similar, it is also more explicit in highlighting two aspects of values and valuing in mathematics education. One aspect acknowledges that the values that are being espoused in mathematics education need not stem from mathematics lessons alone, but from the wider sociocultural context as well. The other aspect that is more explicitly stated is that the valuing that are inculcated through mathematics education goes beyond being in students’ memories, and they in fact ‘swing back’ to affect the quality of mathematics learning. For them, values and valuing reflect

the convictions which an individual has internalised as being the things of importance and worth. What an individual values defines for her/him a window through which s/he views the world around her/him. Valuing provides the individual with the will and determination to maintain any course of action chosen in the learning and teaching of mathematics. They regulate the ways in which a learner’s/teacher’s cognitive skills and emotional dispositions are aligned to learning/teaching in any given educational context. (p. 169)

31.2.1 Values and Valuing as Involving Both Cognition and Affect

Seah and Andersson’s (2015) definition above implies that values are neither cognitive nor affective constructs per se. Instead, valuing is regarded as being both cognitive and affective in nature (see also, Hartman, n.d.; Huitt 2004).

Rather than being an affective construct as it was generally known (e.g. Bishop 1996; Krathwohl et al. 1964), the process and act of valuing invariably involve reasoning and thinking. Even though Krathwohl et al.’s (1964) taxonomy of educational objectives might refer to the affective domain, the ‘organization’ phase involves the individual relating the values s/he subscribes to amongst themselves such that these values co-exist, which is a task that involves thinking and reasoning.

Similarly, Raths et al. (1987) conception regards successful attainment of a value as involving all seven criteria, namely, choosing freely, choosing from alternatives, choosing after thoughtful consideration of the consequences of each element, prizing and cherishing, prizing through affirming to others, acting with the choice, and acting repeatedly in some pattern of life. Clearly, the choosing components involve reasoning, whilst the prizing components involve affect.

31.2.2 Values and Valuing as Being Socio-cultural

Values and valuing is also socio-cultural in nature. What we value reflect years of learning and influence from our historical experiences and social interactions as members of the cultures we belong. Indeed, the notion of cultures has been regarded as “an organised system of values which are transmitted to its members both formally and informally” (McConatha and Schnell 1995, p. 81).

The discussion thus far has signaled a perspective to learning where the learner’s objectivised actions are culturally and symbolically mediated by values, and which can be examined through activity theory. Activity theory provides a useful theoretical framework also in that it explains how the mediation gets internalised within cultures, giving the learner a particular identity that characterises him or her in culturally unique ways. In particular, the Cultural Historical Activity Theory [CHAT] embodies the construct of values very well. CHAT represents the third generation of the activity theory approach to understanding learning and education. While the first and second generations were associated with Vygotsky’s sociocultural theory of teaching and learning, in which students participate in negotiation and co-construction of knowledge (Haenen et al. 2003), and Leontiev’s activity theory, the set of specific notions, claims, and arguments that consider the relationship between a subject (typically an individual human) and the object (Kaptelinin and Nardi 2012). In this third-generation interpretation of the activity theory, Engeström’s activity system model extended Leontiev’s original concept of subject-object interaction to become a three-way interaction between ‘subject’, ‘object’, and ‘community’. The new theory went beyond a focus on activity systems to emphasise the interactions between and amongst activity systems, so that learning is meaningful through a process of multi-voicedness, difference, and conflict negotiation. Gummesson (2006) had argued that the main outcome of this process is value co-creation. In the classroom, for example, pedagogical activities take place through the interaction of what students, teachers, and indirectly, the wider community value. The interactions have brought together the different things that teachers and their students value similarly and differently, and the co-creation of values can be perceived as the agreed-upon, aligned values that facilitate the continued functioning of the activity systems in interaction. Importantly, while the first generation of the activity theory focuses on the individual learner, and the second generation directs the attention to the community within which learning takes place, CHAT considers as the unit of analysis joint activity amongst individuals in the learning environment. In relation to values, thus, we can imagine values not only as being acquired over time, but that the negotiations of values between and amongst activity systems would also lead to values being challenged and refined on an ongoing basis, depending on the opportunities for one’s values to come into contact with values from other activity systems.

31.2.3 Values and Valuing Driving Performance

Research evidence has also supported the belief that mathematics performance is related to students' valuing. In addition to the Nuffield Foundation-commissioned report (Askew et al. 2010) mentioned above which highlighted the role of cultural values, there is also more recent research by Jerrim (2014), who sub-divided the Australian dataset for PISA 2012 by broad student ethnicity, specifically, high-performing East Asian, low-performing East Asian, Indian, British, and native Australian. Given that the students in the sample were second-generation immigrants experiencing an Australian mathematics education with their native Australian peers, it can be assumed that the factors underlying the differences existed beyond the school level, with their different emphases and valuing on different aspects of school education.

Schukajlow's (2017) study with 192 Years 9/10 students in Germany demonstrated a similar relationship between student valuing and mathematics performance. Differences were found, however, between performance on problems related to real-life scenarios and problems which were not. Schukajlow had flagged this for further investigations, and it represents existing research effort into understanding how values might be used to further enhance the mathematics learning experience of young children.

Such an association between valuing and mathematics performance is important, and even more so given that what are being valued also affect the cognitive processes and affective states that in turn influence the quality of mathematics learning. As such,

the extent to which the educational aspirations of students and parents are the result of cultural values or determinants of these, and how such aspirations interact with education policies and practices is an important subject that merits further study. (OECD 2014, p. 20)

In responding to this call, the guiding assumption is that students' possession or acquisition of relevant valuing allows each of them to apply appropriate cognitive skills and to develop positive affective states which promote desirable outcomes in mathematics learning, whether these be related to measurable performance or to relational understanding.

In addition to being culturally-referenced, what is being valued is also invisible and implicit. Due to the inevitable presence of competing and overriding values (Seah 2005), what one values is not articulated in all situations. Indeed, Takuya Baba had likened values and valuing to the underground roots of a tree, which are not only invisible and implicit, but also crucial to supporting and nurturing the healthy growth of what is visible of the tree above the ground, such as student results.

Herein lies one of the most important aspect of values and valuing, that is, how it supports the development of cognitive functioning and nurturing of affective states. It is as if attending to the cognitive and affective development of mathematics learners alone is not sufficient to bring about meaningful learning. The learner should want to engage, to understand, to learn, and perhaps to achieve as well in the

first place. As the saying goes, ‘you can bring a horse to the water, but you can’t make it drink’. That is, facilitating the valuing of relevant attributes in mathematics learning by the learners themselves is a crucial—and often forgotten—component of mathematics pedagogy, for this in turn supports the development of cognitive functioning and nurturing of affective states that would more directly impact on the quality of learning.

31.3 The ‘What I Find Important (in Mathematics Learning)’ [WIFI] Study

In fact, the ‘What I Find Important (in mathematics learning)’ [WIFI] study was conceptualised with this guiding assumption in mind. The objective of the WIFI study has been to find out what students in the last two years of primary schooling and what 15-year-old students value in their mathematics learning experiences.

The desire to facilitate a ‘mapping of the scene’ has necessitated a large-scale study, which also highlighted the need for a methodology that allowed for the assessment of student values in time-efficient ways. Thus, instead of adopting the qualitative approaches such as in Chin and Lin (2000) or in Clarkson et al. (2000), the WIFI study made use of the questionnaire method. This way, a large number of students could be surveyed for the attributes of mathematics education which they personally find important, and also so that the data collected could be interpreted efficiently using the SPSS software.

The WIFI questionnaire is divided into four sections. Section A is made up of 64 items, each of which being a mathematics classroom activity (e.g. ‘outdoor mathematics activities’, ‘explaining my solutions to the class’) or a pedagogical norm (e.g. ‘shortcuts to solving a problem’). Student respondents were expected to rate on the 5-point Likert scale the extent to which an activity or norm was important to each of them. Section B is made up of 10 continuum dimension items, in which opposing values are located on both ends of each continuum dimension (e.g. ‘how the answer to a problem is obtained’ vs. ‘what the answer to a problem is’) and student respondents needed to indicate where s/he stood in relation to the two opposing values. Section C is an open-ended, scenario-stimulated responses section, providing for another means of identifying what students valued in their mathematics learning. Students’ demographic and personal information were collected in Section D. Examples of the questionnaire items and of the layout can be seen in Seah et al. (2016). The WIFI questionnaire can also be accessed online at: https://www.surveymonkey.com/r/WIFI_maths.

The questionnaire was administered in class, that is, student participants filled in the questionnaire individually in their own classroom setting, with the exercise facilitated by their mathematics teachers. The questionnaire is available in hardcopy and online versions, and any participating school will select one of the two possible formats for all its student participants.

To date, some 20,000 questionnaires have been completed (with more than 16,000 analysed) across the 19 different economies. Only the findings from the analysis of responses to Section A items are reported in this chapter. These items are listed in the Appendix.

Each participating economy is represented by a team of local researchers. Each team was responsible for administering the questionnaire in its own context. The quantitative analysis of the questionnaire data was conducted centrally by the Australian research team, however. The results generated by SPSSwin[®] were then returned to the respective research teams, the intention being that the sense-making could be done in a culturally-meaningful manner by their own researchers.

Upon receipt of the raw data from each participating economy, initial data screening was carried out to test for univariate normality, multivariate outliers (using Mahalanobis' distance criterion), and homogeneity of variance-covariance matrices (using Box's M tests). A Principal Component Analysis (PCA) with Varimax rotation was used to examine the questionnaire items. The significance level was set at 0.05, while a cut-off criterion for component loadings of at least 0.45 was used in interpreting the solution. Items that did not meet the criteria were eliminated. For each economy's data, the Kaiser-Meyer-Olkin (KMO) measure of sampling adequacy was noted, and Bartlett's test of sphericity (BTS) (Bartlett 1950) was also checked for significance at the 0.001 level, so that factorability of the correlation matrix could be assumed, which demonstrated that the identity matrix instrument was reliable and confirmed the usefulness of the PCA. According to the cut-off criterion, the number of items that were removed from the original 64 was understandably different between economies.

The research team in each participating economy then interpreted these components, assigning a value label to each. This is a distinguishing feature of this study, in that the cultural-situatedness of valuing has meant that the researchers from each participating economy analysed and interpreted their own PCA components, and no attempt was made for each group's criteria for interpretations to be shared or made consistent across all participating economies.

31.4 What the Top Performers Value

The key research question guiding the conduct of the WIFI Study is: What do students value in their respective mathematics learning? As we saw above, it is expected that the students' valuing is shaped in context, that is, influenced by societal, ethnic, religious, family, school and other institutional cultures.

This chapter, however, focuses on what students in the top performing PISA2012 economies valued in mathematics learning. Given the relationship between student valuing and mathematics performance (see above), and given the interest in many countries across the world to understand how the top performing economies consistently lead the pack in different ranking exercises, it is hoped that the findings reported in this chapter can inform researchers on the valuing that

Table 31.1 What students in top performing economies valued

HKG [3/65]	TWN [4/65]	KOR [5/65]	MAC [6/65]
Understanding	Connections	Understanding	Achievement
Control	Recall	Connections	Humanism
Effort	Effort	Fun	Practice
Ideas	Exploration	Fluency	Technology
Recall	Openness	Accuracy	Communication
ICT	Communication	Collaborative reflection	Mathematical development
Feedback		Efficiency	
Connections		Communication	
Learning approach		Mystery	

HKG Hong Kong, *TWN* Taiwan, *KOR* Korea, *MAC* Macau

might account for students' mathematics achievement at the national level, thereby deepening what we understand about excellence in mathematics learning. Table 31.1 shows these valuing for Hong Kong, Taiwan, Korea and Macau, which were placed third, fourth, fifth and sixth by student performance in PISA2012. These economies continue to lead the world in subsequent PISA tests. For example, in PISA2015, they were ranked second, fourth, seventh, and third respectively (Thomson et al. 2016).

The number of attributes listed for each economy is different from that of another economy, since these are associated with the number of components that were elicited from the respective PCA. The attributes had been named independently by the respective research teams, based on each team's cultural interpretation of the questionnaire items which had loaded onto the components. The order of listing of the attributes in Table 31.1 reflects, for each economy, the order of the components that were derived from the PCA exercises. As shown in Table 31.1, it may be said that generally, the top performing economies have students which valued *connections*, *understanding*, *communication*, *recall*, and *ICT*.

It is also necessary to check if students in the economies which did not perform as well might have been valuing the same aspects in mathematics education. Accordingly, the students' valuing for Turkey and Thailand—ranked 44th and 50th respectively amongst the 65 surveyed economies—were referred to, as shown in Table 31.2.

Given that students in Turkey and Thailand valued *ICT* as well, it is unlikely that information and communication technology in general would have contributed to student achievement in mathematics. It might well be that certain aspects within ICT use would enhance or promote student learning and achievement, whereas other aspects of ICT use might have the opposite effect, such that its valuing was nominated by different groups of students. While this may be a possibility (which will be briefly explored below)—and indeed, this signals further research about how different groups of students might value different aspects of ICT—it is also reasonable to remove it from the list of attributes associated with top performing

Table 31.2 What students in Turkey and Thailand valued

TUR [44/65]	THA [50/65]
Relevance	Process
Practice	Fact
ICT	Practice
Collectivism	Relevance
Objectism	ICT
	Learning from others

TUR Turkey, *THA* Thailand

students’ valuing. In other words, it can be deduced that students from top performing countries in the PISA2012 assessment valued *connections*, *understanding*, *communication*, and *recall*.

It is reassuring that many of these values are being explicitly promoted in many current-day mathematics curriculum documents. For example, amongst the 5 process standards identified for the NCTM ‘Principles and Standards for School Mathematics’ (2000) are *connections* and *communication*. Incidentally, the other two values—that is, *understanding* and *recall*—are being reflected in the current Australian Curriculum (ACARA 2016) and Victorian Curriculum (VCAA 2017) for Mathematics, if we associate *recall* with being an aspect of *fluency*.

These attributes are observed to be different in nature from those which were valued by countries like Turkey and Thailand (see Table 31.2), which did not perform as well relative to the other participating countries. The valuing of *connections*, *understanding*, *communication*, and *recall* was concerned with paying attention to the attributes of the nature or structure of the mathematics discipline. These values might thus be considered to be intrinsic in nature. On the other hand, the valuing of *relevance* and *practice* by students in Turkey and Thailand highlights the importance given to what can be done with mathematical knowledge and skills, or what can be done externally to the discipline itself to acquire the knowledge and skills. These values can thus be considered to be more extrinsic. It appears that students’ mathematical performance might be related to not just valuing of individual attributes, but also, to the extent to which the valuing is related to intrinsic characteristics.

31.5 Valuing and Constituent Actions

As suggested by the questionnaire format, the invisible nature of valuing is compensated for in this study by focussing on the observable actions that are expressed by the valuing associated to it. Given the cultural nature of valuing, it was assumed that the same valuing can take different forms in different settings. This was investigated by examining the questionnaire items that loaded onto the same

Table 31.3 How *ICT* was valued across three economies

HKG	JPN	GHA
Using the calculator to check the answer	Learning mathematics with the computer	Using the calculator to check the answer
Learning mathematics with the computer	Learning mathematics with the internet	Using the calculator to calculate
Using the calculator to calculate	Using the calculator to check the answer	
Learning mathematics with the internet	Mathematics games	

HKG Hong Kong, *JPN* Japan, *GHA* Ghana

valuing across different economies. Table 31.3 shows the constituent actions corresponding to the valuing of *ICT* in Hong Kong, Japan and Ghana, for example.

Although it may initially look as if the valuing of *ICT* can be described by a common set of classroom actions, it is important to note that no one action can be found across all three economies. The interplay between valuing and its constituent actions is indeed complicated. While the valuing of *ICT* is associated with the learning of mathematics content with the computer and with internet in Hong Kong and Japan, these actions could not be found amongst the Ghana data, even though Ghanaian students also valued *ICT*. At the same time, Japanese students’ valuing of *ICT* was uniquely associated with mathematics games.

It needs to be noted that the similarity of activities emphasised in different education systems does not necessarily increase the chance of these cultures valuing the same attribute. In the example above, the classroom activities embraced by the Japanese students were similar to what their peers in Korea seemed to be preferring too. Yet, the cultural interpretations of these classroom activities in Japan and in Korea were such that the sets of activities were seen to reflect different valuing; while they referred to a valuing of *ICT* in Japan, it was a valuing of *fun* in Korea.

It is worthy to consider the impact on the effectiveness of mathematics pedagogy in different education systems when the same valuing is emphasised differently across the institutions. We can see this in the three economies’ valuing of *ICT* above, where it was speculated that an economy’s infrastructure might be a contributing factor to this difference. On the other hand, the differences might come about through culturally different conceptions of pedagogy and of education. Here we may consider Macao and Ghanaian students’ valuing of *achievement* through their preferences for 15 and 16 classroom activities respectively. Although 6 (e.g. ‘understanding concepts/processes’, ‘working out the mathematics by myself’) of these preferred activities were common between the two economies, there were still up to 10 activities which were regarded by students to be important in their respective education systems. In Macao, the students’ valuing of *achievement* through understanding and working out the mathematics individually was supported by activities such as ‘shortcuts to solving a problem’ and ‘practising how to use mathematics formulae’, which both pointed to means of achieving in

Table 31.4 How students in Korea and Hong Kong valued *connections*

KOR	HKG
Relating mathematics to other subjects in schools	Relating mathematics to other subjects in schools
Appreciating the beauty of mathematics	Appreciating the beauty of mathematics
Connecting mathematics to real life	Connecting mathematics to real life
Stories about recent developments in maths	Learning the proofs
Stories about mathematics	
Explaining my solutions to the class	
Students posing maths problems	
Looking out for maths in real life	
Making up my own maths questions	
Mathematics puzzles	
Mathematics debates	
Looking for different possible answers	
Investigations	

KOR Korea, HKG Hong Kong

mathematics through efficient and fluent working-out. Students in Ghana also aimed to achieve in mathematics through efficient and fluent practices, though these appeared to take on different forms. There, the preferred means were ‘teacher asking us questions’ and ‘remembering the work we have done’.

Even amongst the top performers which are generally perceived to be of similar culture (i.e. East Asian), the same attribute being valued is portrayed through different classroom actions. Table 31.4 provides an example for Korea and Hong Kong’s valuing of *connections*. Although there are three actions which were in common across the two economies, that is, ‘relating mathematics to other subjects in schools’, ‘connecting mathematics to real life’, and ‘appreciating the beauty of mathematics’, in each economy the students were also demonstrating their valuing of *connections* through other actions. For example, students in Korea appeared to regard their explanations of their solutions ‘in public’ in class as a means of valuing *connections*, possibly through the need for the presenting students to be able to establish how concepts and knowledge are interconnected in their respective solutions. However, this classroom action was not identified with the valuing of *connections* by their peers in Hong Kong classrooms, even though such a classroom activity is also commonly found there.

31.6 Determining Valuing from Particular Actions

In the same way that any attribute of mathematics learning and teaching can be valued through different classroom activities, any activity needs not point to any one particular valuing. Rather, the implementation of any activity in a mathematics

classroom can point to the valuing of one or more of several possible valuing. What this means for the assessment and identification of valuing is that some kind of triangulation is needed through the observation of multiple supporting activities.

This other aspect of the absence of a one-to-one correspondence between valuing and classroom activities was observed earlier on in the study, when the different research teams were aligning individual questionnaire items of classroom activities with the valuing that was assumed to being reflected. For example, the emphasis given by students in Turkey for small-group discussions would reflect the valuing of one or more of the following attributes of mathematics education: *collaboration, communication, efficiency, fun, humanism, openness, question posing, practice, and representation*. Thus, whatever a teacher's intention or valuing is when small-group discussions is part of his/her professional practice in the mathematics classroom, students may not be able to understand what teacher valuing is being espoused through it. However, to the extent that the teacher is able to express whatever is being valued through a variety of classroom activities, students will be able to triangulate these to understand this valuing. There are implications here for research designs involving data collection through lesson observations: Multiple observations might be needed for the teachers (and students) to display a range of actions and activities, so that the underlying intentions, philosophies and valuing can be 'sieved out' from amongst the possible attributes valued. If repeated observations is not possible, then post-lesson interviews or discussions with the participants involved would be necessary to clarify these underlying valuing.

Elsewhere, Clarke's (2004) documentation of the Japanese classroom practice of teacher between-desk instruction—which the Japanese educators called 'kikan-shido'—might lead the Western academic community to associate it with teacher elicitation of student difficulties and subsequent teacher individualised explanation. However, this classroom practice has been noticed in Shanghai (Lopez-Real et al. 2004) and German mathematics lessons too. Of importance is how this similar act of teacher between-desk instruction actually expresses different valuing amongst the three economies. In Shanghai, teachers made use of their monitoring of student work to encourage students to think further. In Germany, however, the monitoring and correction of student work seemed to be absent, where the teachers appeared to be using the opportunities to ask questions for the purpose of stimulating students' mathematical thinking. Thus, even though kikan-shido might be observed in German, Japanese and Shanghai mathematics lessons, the teachers across these three cultures were portraying different valuing with regards to mathematics pedagogy.

Similarly, analysed data from the TIMSS Video Study (Hiebert et al. 2003) have suggested that even though the classroom activity of problem-solving may be embraced in many mathematics education systems, this same form should not be taken to imply that the same attributes of mathematics pedagogy are being valued. Indeed, it is instructive to note that the high performing mathematics education systems emphasise *connections* that are facilitated through the problem-solving tasks, whereas many of the other mathematics education systems emphasise *procedure*. Thus, doing what effective mathematics education systems do does not

imply that the same benefits will be gained. Rather, culturally-appropriate classroom activities are means through which the features of mathematics learning that matter are valued, expressed and operationalised.

31.7 Implications for Teacher Practice

The analysed data from the top performing PISA2012 economies suggest that student performance in mathematics was related to students' valuing of *connections*, *understanding*, *communication*, and *recall*. Given that PISA items assess students' ability to apply their mathematical knowledge in novel problems—which would require students to demonstrate knowledge, skill and application—the four attributes being valued do cover the various aspects of being able to excel in the assessment. Many of Hattie's (2015) top classroom interventions (which refer to school education generally and which also include background variables such as 'home environment' and 'ethnicity') are related to these four attributes, such as classroom discussion (effect size = 0.82), feedback (effect size = 0.73), formative evaluation (effect size = 0.68), concept mapping (0.64), and mastery learning (effect size = 0.57). Significantly, none of the last 50 interventions in the list of 195 appeared to relate to these four valuing.

This student valuing of *connections*, *understanding*, *communication*, and *recall* reflect intrinsic valuing, as opposed to extrinsic valuing which would emphasise such valuing as *application* and *relevance*. There are implications here for professional practice in the mathematics classroom, even though curriculum documents might emphasise both these categories of attributes. This is important, not least because the inculcation of extrinsic valuing can be more appealing to students and can also be easier to convey to them. On the other hand, it is likely that teachers' efforts to prompt students' appreciation and subsequent valuing of intrinsic valuing can actively be derailed by students routinely asking questions such as, "when are we ever going to use this?" It thus appears that students need to appreciate the utilitarian aspects embedded within intrinsic valuing.

Mathematics pedagogical approaches or strategies can be defined by what they value with regards to the teaching and learning of mathematics. At the level of the intended curriculum, the valuing that is embedded in these pedagogical approaches or strategies may not be explicitly linked to the specific approaches or strategies, but are rather merely mentioned in the introductory or rationale sections only. As a result, too, these valuing are not explicitly stated in the text or by the teacher. To the extent that this valuing can be considered the heart and soul of the particular pedagogical approach or strategy, it is important that preservice or in-service teachers who are being introduced to it is made aware of the underlying valuing.

The data collected from different economies have indicated that merely 'transplanting' a new pedagogical approach or strategy in one's classroom might not make clear to students the underlying valuing that is being advocated or taught. This is why expensive projects which attempted to introduce Japanese classrooms

in the USA and Chinese classes in the UK have largely failed to achieve their respective objectives. Teaching students with the new approach or strategy alone is likely not able to realise its intended benefits to mathematics learning. The professional discourse might need to change from ‘we are learning skill ABC or technique DEF’ or similar, to one of ‘through this skill ABC or technique DEF, we are learning to value attribute XYZ’ or similar.

In this way, it adds another dimension to how values and valuing play a key role in (mathematics) lesson planning. Not only is a focus on valuing in lesson planning expected to promote students’ cognitive and affective engagement, it also allows teachers to adopt/adapt and reap maximal potential out of teaching approaches or strategies they are introduced to.

31.8 Shaping Student’s Valuing

The discussion above assumes that students’ valuing can be and are being shaped in the mathematics education process. After all, that values are internalised and stable variables does not imply that they cannot be modified. Furthermore, modification and (re-)shaping may be easier when the individual is still young.

From a practical perspective, the data collected from around the world have also provided empirical evidence that value change takes place between the primary and secondary school years. In Japan, primary school students surveyed valued *process*, but their secondary school peers were valuing *product*, which can be viewed as the opposing attribute to *process* when considering the engagement with and completion of mathematical tasks. On the other hand, in Hong Kong, it was found that when students progressed from primary to secondary schools, they experienced a drop in their valuing of *understanding*, *recall* and *control*. Of course, given Hong Kong’s excellent performance in international assessments and this relationship to the students’ valuing of *understanding* and *recall*, it is also reasonable to assert that the reduced valuing was still significant enough to be highly regarded by 15-year old Hong Kong students who aspire to work excellently.

Teachers thus play the role of value agents even as they engage in mathematics teaching. In many ways, one can argue that this has always been a role that is played out by teachers everywhere. Teacher agency in shaping and modifying students’ valuing is very real indeed, although it can be more explicit than it normally is. In fact, this teacher role is also often advocated in curriculum statements.

Teacher teaching, shaping and modification of student valuing can take on different forms. One of the more innovative forms is the introduction of role-play activities in mathematics lessons, either through students taking on roles which correspond to particular nominated valuing (e.g. being a student who values *progress*), or taking on the role of teacher or peer tutor, which would necessitate student evaluation of the valuing that underlies effective teaching/tutoring.

31.9 Values Alignment

Interactions between teachers and students, as well as teachers' pedagogical tasks and activities in class, both bring to the fore what teachers and their students valued similarly and differently. Teacher effectiveness can depend on the extent to which teachers are able to negotiate these inevitable value differences, so as to bring about a learning environment in which everyone's valuing are aligned and inter-personal relationships are in harmony. After all, "all relationships ... are claimed to be strengthened by aligned values" (Branson 2008, p 381). Value alignment thus involves teachers making in-the-moment decisions, acknowledging that teacher practice is situated in a socially co-constructed setting. Several teacher strategies of value alignment have been reported by Seah and Andersson (2015), and further research is being conducted in this area to empower teachers to recognise, align and shape the valuing that underlies cognitive and affective functionings of mathematics teaching and learning.

Thus, values alignment should be regarded as already being part of day-to-day interactions. When individuals come together with their own value systems, they will always need to negotiate about different preferences and intentions to ensure that the interaction is successful. This calls for values alignment to take place, and it does not mean that one party needs necessarily to impose his/her/their values to the rest. There can always be middle-path compromises, for example. At the same time, it is important to note that any consideration of values being aligned (or not) is mutual between and amongst the individuals involved, both teachers and their students. In this manner, student agency is acknowledged.

31.10 Summarising Ideas

Valuing refers to an individual's embrace of convictions which are considered to be of importance and worth. It provides the individual with the will and grit to maintain any 'I want to' mindset in the learning and teaching of mathematics. In the process, this conative variable shapes the manner in which the individual's reasoning, emotions and actions relating to mathematics pedagogy develop and establish. The argument in this chapter is that more effective teaching and learning can only take place by paying attention to what are being valued by teachers and students respectively, and through teachers' purposeful shaping of students' valuing and alignment of the diverse values that are enacted upon by these teachers and their respective students.

Data collected and analysed from the various economies participating in the WIFI Study have demonstrated that any particular valuing is manifested in one of many possible classroom practices. Similarly, any one classroom practice is not reflective of any one valuing only.

The quantitative WIFI Study has allowed us to identify what students valued in mathematics learning across 19 different economies. It was found that students in 4 top performing mathematics education systems in PISA2012 (i.e. ranked third to sixth) generally valued *connections*, *understanding*, *communication*, and *recall*. On the other hand, students in two of the education systems which did not perform well were valuing *relevance* and *practice*. A distinction between intrinsic and extrinsic valuing is proposed; further studies are recommended to explore the extent to which intrinsic valuing fosters greater mathematical performance. In addition, *ICT* was valued across both types of mathematics education systems. Thus, there is a need to further examine the effectiveness in student valuing of *relevance*, *practice* and *ICT*.

School-aged students are in the process of defining and internalising what they each value in life and in mathematics learning. Teachers' awareness of what they themselves value, and purposeful and explicit portrayal of these valuing, are expected to facilitate students' development of what they value in mathematics and in mathematics learning. Additionally, given the inevitable opportunities for differences in what teachers and their students value, teachers assume an important task of aligning the different and potentially conflicting valuing, such that meaningful mathematics learning is facilitated.

Appendix 1: WIFI Questionnaire (Section A Only)

Note that the questionnaire layout has been altered here, to suit the publication guidelines.

The Third Wave Project

Study 3: What I Find Important (in Maths Learning)

Student Questionnaire

Section A

For each of the items below, tick a box to tell us how important it is to you when you learn mathematics.

	Absolutely important	Important	Neither important nor unimportant	Unimportant	Absolutely unimportant
1. Investigations					
2. Problem-solving					
3. Small-group discussions					
4. Using the calculator to calculate					
5. Explaining by the teacher					
6. Working step-by-step					
7. Whole-class discussions					
8. Learning the proofs					
9. Mathematics debates					
10. Relating mathematics to other subjects in school					
11. Appreciating the beauty of maths					
12. Connecting maths to real life					
13. Practising how to use maths formulae					
14. Memorising facts (e.g. Area of a rectangle = length X breadth)					
15. Looking for different ways to find the answer					
16. Looking for different possible answers					
17. Stories about mathematics					
18. Stories about recent developments in mathematics					
19. Explaining my solutions to the class					
20. Mathematics puzzles					
21. Students posing maths problems					
22. Using the calculator to check the answer					
23. Learning maths with the computer					
24. Learning maths with the internet					
25. Mathematics games					
26. Relationships between maths concepts					
27. Being lucky at getting the correct answer					
28. Knowing the times tables					
29. Making up my own maths questions					
30. Alternative solutions					
31. Verifying theorems/hypotheses					
32. Using mathematical words (e.g. angle)					
33. Writing the solutions step-by-step					
34. Outdoor mathematics activities					
35. Teacher asking us questions					
36. Practising with lots of questions					
37. Doing a lot of mathematics work					
38. Given a formula to use					
39. Looking out for maths in real life					
40. Explaining where rules/formulae came from					
41. Teacher helping me individually					
42. Working out the maths by myself					
43. Mathematics tests/examinations					
44. Feedback from my teacher					
45. Feedback from my friends					
46. Me asking questions					

(continued)

(continued)

	Absolutely important	Important	Neither important nor unimportant	Unimportant	Absolutely unimportant
47. Using diagrams to understand maths					
48. Using concrete materials to understand mathematics					
49. Examples to help me understand					
50. Getting the right answer					
51. Learning through mistakes					
52. Hands-on activities					
53. Teacher use of keywords (e.g. 'share' to signal division; contrasting 'solve' and 'simplify')					
54. Understanding concepts/processes					
55. Shortcuts to solving a problem					
56. Knowing the steps of the solution					
57. Mathematics homework					
58. Knowing which formula to use					
59. Knowing the theoretical aspects of mathematics (e.g. proof, definitions of triangles)					
60. Mystery of maths (example: $111\ 111\ 111 \times 111\ 111$ $111 = 12\ 345\ 678\ 987\ 654\ 321$)					
61. Stories about mathematicians					
62. Completing mathematics work					
63. Understanding why my solution is incorrect or correct					
64. Remembering the work we have done					
65. Comments (if any)					

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