Chapter 8
Other Types of Recursion Theoretic Unknowables

The following theorems of recursive analysis [7, 564] have some implications for theoretical physics.

- Specker's theorem of recursive analysis: There exist recursive monotone bounded sequences of rational numbers whose limit is no computable number [477].
  A concrete example of such a number is Chaitin's Omega number [103, 109, 129], also discussed in Appendix A.6, the halting probability for a computer (using prefix-free code), which can be defined by a sequence of rational numbers with no computable rate of convergence.
- Specker's other theorem of recursive analysis: There exist a recursive real function which has its maximum in the unit interval at no recursive real number [478]. This has implications for the principle of least action [320].
- Wang's theorem of recursive analysis: The predicate "there exists a real number r such that whether or not $G(r) = 0$" is recursively undecidable for $G(x)$ in a class of functions which involves polynomials and the sine function [559]. This, again, has some bearing on the principle of least action.
- Uncomputable solutions of differential equations: There exist uncomputable solutions of the wave equations for computable initial values [78, 418].
- Ubiquity and pervasiveness of undecidability: On the basis of theorems of recursive analysis [433, 442] many questions in dynamical systems theory are provable undecidable [107, 157, 280, 487].

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