Topic Study Group No. 51: Diversity of Theories in Mathematics Education

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The Programme

Session I

- Presentation 1: Anna Sfard: On the need of theory of mathematics learning and the promise of ‘commognition’.
- Presentation 2: Cristina Frade: The social construction of mathematics teachers’ identity: Rorty’s pragmatistic perspective.

Session II

- Presentation 3: Ricardo Cantoral: Origins and evolution of the socioepistemological program in mathematics education.
- Presentation 4: Carolina Tamayo Osorio and Antonio Miguel: Wittgensteinian ‘therapeutic couch’ and indigenous experience in (mathematics) education.
- Presentation 5: Higinio Dominguez: Reciprocal noticing in mathematics classrooms with non-dominant students.

Session III

- Presentation 6: Yasuhiro Sekiguchi: Theories and traditions: Tensions between mathematics teaching practices and a recent school reform in Japan.

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- Presentation 7: Verena Rembowski: Semiotic and philosophical-psychological aspects of concept formation.
- Presentation 8: Stefan Halverscheid: An example for interdisciplinary networking of theories for the design of modeling tasks: A case study on ethical dilemmas.

Session IV

- Presentation 9: Michèle Artigue: The challenging diversity of theories in mathematics education.
- Discussion: What have we learned?

Summary

In the closing discussion we have asked what were the implied or explicit ideas about some of the fundamental questions about mathematics, its teaching and its learning, mentioned in the abstract of TSG51? We look at some of these questions in turn.

What is Mathematics?

Anna Sfard's response was that mathematics is a collection of collections of stories about different things (shapes, numbers, sets, functions, ...) told using specialized discourses characterized by (1) keywords and their uses (e.g., number, function, limit, derivative, ...); (2) visual mediators (sign systems) intended to clarify what the particular story is about (e.g., positional number systems, functional notation, graphs, ...); (3) routine actions of the storytellers (abstraction, generalization, deduction, induction, reasoning by contradiction to prove the impossibility of some hypothetical story, testing stories for internal consistency and coherence with other stories, ...), and meta-rules (explicit definitions of technical terms, laws of logic, ...) (Sfard, 2007).

Ricardo Cantoral said that this is a philosophical question and therefore best left to philosophers. A question for mathematics educators is what are the differences between mathematics as a body of theoretical knowledge and mathematics as a school subject?

In relation to the nature of mathematics, both Anna Sfard and Ricardo Cantoral addressed a question that was not explicitly posed in the abstract of our group: Where do mathematical concepts come from? For Anna Sfard, mathematical concepts come from a feedback loop between practical activities and discourses; for Ricardo Cantoral—from cultural practices, techniques, traditions. It is an important question in mathematics education for, whenever we plan to teach a mathematical concept, we seek the sources of its meaning so that we can construct instructional situations that will help students to construct these meanings for themselves. This question underlies Davydov's concept of "object sources" and Brousseau's concept of "fundamental situations" for particular mathematical notions.

What is Mathematics as a School Subject?

Regarding mathematics as a school subject, there were differences of opinion between Anna Sfard and Ricardo Cantoral. Anna Sfard described school mathematics as a discourse obtained from mathematicians' mathematics by "customization" to the needs and capacities of young learners. She said that it differs from mathematicians' mathematics mostly by the meta-rules, which are less strict and also different in nature.

Ricardo Cantoral, on the other hand, proposed that school mathematics is not only a transposition of scholarly mathematical knowledge. For him, school mathematics is part of the culture of a given society and place; it contains cultural traditions, riddles and games known from popular culture, technology, and other things used in that culture to construct mathematical knowledge in the classroom. School mathematics is culturally situated. There is popular mathematical knowledge, technical mathematical knowledge and scientific mathematical knowledge, and all have to be taken into account when building theories of teaching and learning mathematics at school. There is no hierarchy among the three kinds of knowledge. They are all part of human wisdom (Cantoral, 2013).

What Does it Mean to Teach Mathematics in General or a Particular Mathematical Concept or Process? What Does it Mean to Teach it Well?

According to Ricardo Cantoral, to teach a fundamental mathematical concept (e.g., derivative), we need to find a "cultural basis" for it and help students anchor their understanding of the concept in this cultural basis (e.g., the idea of taste—sweetness, salinity—can be used as a cultural basis for the concept of rate and hence of slope of a linear function).

Higinio Dominguez looked at a more social-interactive aspect of teaching and proposed that to teach well it is necessary to engage in "reciprocal noticing" with students: "In reciprocal noticing, what is noticed is not individual reasoning but rather the emerging and continuous influence of people's reasoning WITH (not *for*) one another"

Cristina Frade's perspective was focused on the person of the teacher, but it was not psychological: the question was how a person constructs her or his identity as a teacher. This construction is an important element of becoming a teacher—and therefore, a condition of "teaching well". It is social in nature, by means of language: the individual develops a vocabulary with which to justify their actions, compare their past and present behaviors, and generally narrate their live stories.

A theory of teaching well was the main concern of Yasuhiro Sekiguchi's presentation. The theory of "School as Learning Community" (SLC) was proposed in the 1980s by Manabu Sato and has become the foundation of a school reform in Japan (Saito et al., 2015). Preparation for teaching well is sought through a well-developed system of "lesson study" conferences and developing collaboration between children, teachers, parents, and people in the region.

What Does it Mean to Know Something or a Given Particular Thing in Mathematics (e.g., Fractions)?

According to Anna Sfard, to know something in mathematics means to “extend one’s discursive repertoire by individualizing” the discourse in which a particular mathematical story or a collection of such stories is told in school mathematics. “To individualize a discourse means to be able to communicate according to its rules” with others and with oneself. Carolina Tamayo Osorio pointed to a subtle transitional aspect of the process of individualization of a discourse: based on her observations of the Gunadule indigenous Community of Alto Caiman (Colombia) and Wittgenstein’s notion of grammar, she proposed that to learn something in mathematics is to extend one’s repertoire of language games by constructing a third “border” grammar between one’s native grammar and the grammar of school mathematics.

Ricardo Cantoral’s take on “knowing something in mathematics” can be summarized as follows: To be able to participate in a certain social (cultural) practice, not only to be able to perform a certain individual intellectual act. This practice may take place in school, but also out of school (e.g., building a log cottage requires knowledge that is technical mathematical knowledge).

Yasuhiro Sekiguchi stressed that in the SLC theory, learning is a collaborative endeavor. For an individual to know something, a whole community must know it; school must be a learning community.

The question, *What does it mean to know something well?* did not raise much discussion, but it is worth mentioning Anna Sfard’s thesis that success and failure in mathematics are elements of a discourse. An individual who participates in that discourse constructs his or her identity as a “success” or a “failure” in mathematics.

Meta-Questions

In her presentation in the last session, Michèle Artigue criticized the questions asked in the abstract from the perspective of the French approach to mathematics education. Some of the questions contained the verb “should” and thus suggested value judgements and a normative point of view that the French didactic culture strongly rejects. Such questions are not scientific: answers to them are not verifiable by scientific means. A scientific question would be: what are the consequences (for the practice of teaching and learning, for example) of such and such normative perspective? Other participants pointed out that in other cultures the construction of mathematics education research as “science” is not necessarily a priority, and that choices made in posing research questions and selecting aspects to consider are inevitably value-laden and guided by more or less explicit answers to the fundamental questions asked in the abstract for our group. So it is useful, for understanding the motives and aims of research, to know the researchers’ position on these questions.

The last question debated in the group was,

Why Do We Have So Many Theories in Mathematics Education?

Several hypotheses appeared.

A theory is created or borrowed because it is useful. But usefulness is relative to values and needs (problems?). Since the latter are diverse in mathematics education, a diversity of theories is needed to respond to them. (Anna Sfard)

Teaching and learning of mathematics is culturally situated. Cultures of mathematics education in different populations must be studied empirically. To explain differences between these cultures, existing theories may not be enough. (Ricardo Cantoral)

In Japan, “theory” is always a theory of some practice. A practice develops, somebody notices it, reflects upon it and constructs a theory of this practice. Practices evolve, change; new practices emerge. Hence many theories. (Takeshi Miyakawa)

References

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