Using Braids to Introduce Groups: From an Informal to a Formal Approach

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The workshop was based on an outreach activity on braids given several times to high school students, from Grade 8 advanced students up to Grade 12, to give a taste of actively “doing” mathematics.

We reviewed and discussed some of the motivations to expose high school students to mathematics enrichment such as reinforcing motivation to learn, getting a broader view of mathematics, meeting professional mathematicians, and developing independent thinking and collaboration.

We then demonstrated a part of the activities have given to high school students: Starting from a concrete, physical object, turn it into an abstract and formal one. In particular, the participants, working in pairs, were asked to use strings to realize a braid given by a drawing, describe it in plain English “as one would on the phone”, and finally to use symbols to shorten the description, “as one would in a text message”. This process involves some notational choices and naturally leads to a description of a braid in levels, as in the picture. The symbols in the picture are just one of the many different notations proposed by students and by participants in the workshop.

We discussed the didactic importance of accepting different choices for the notation and stressing the motivations behind them. This approach gives an idea of mathematics as a creative discipline, in contrast with the usual perception of its nature as static, where conventions are imposed by teachers or books and motivations are considered too abstract and too difficult to be understood.

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We then introduced the formal counterpart of the intuitive process: Dividing the braid into levels and assigning a symbol to each of them corresponds to the theoretical process of finding generators for the braid group and building braids via an operation called composition. We noted the power of this specific example, where students naturally construct a correct model. This is just a small example, but it is representative of the general mathematical activity. Relating the informal description to a formal one can help the students appreciate the importance of formalism.

Finally, we explored the axioms of groups using the specific cases of braids under the operation of composition and integer numbers under addition, drawing a parallel between the two examples and finding one difference. We discussed the importance of the idea of structure in mathematics, which is often not recognized at all. The idea that the subject of mathematics is computations has been widespread, but the justification as to why computations are possible has been overlooked: Operations, and ultimately structure, make computations and their rules correct and applicable. Braids are a simple example where this can be demonstrated: Some properties hold which are similar to those for numbers, yet there are some differences. Finding rules that hold for braids is an appealing task, because it is in the zone of proximal development for high school students.

Participants in the workshop had different backgrounds: Some were high school teachers, some researchers in mathematics education, and some scholars doing research in mathematics and teaching. The discussion was enriched by the differences in background, teaching experiences, and goals. The contributions of the participants also included considerations about teaching group theory at the college level and about braids in ethnomathematics.

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