Study on Methods for Predicting the Settlements of Soft Clay Roadbed

Baotong Shi and Xiangxing Kong^(✉)

The First Highway Survey and Design Institute of China Communications
Construction Company Ltd., Xi'an, China
{117021654, 103121153}@qq.com

Abstract. The soft clay has the characteristics of high compressibility, high water content, low strength and poor permeability, which will lead to large settlements under long-term loads, therefore, the prediction of settlements of soft clay roadbed has been one of the important research topics in Geotechnical Engineering. In this paper, the commonly-used methods in practice to predict the settlements of soft clay roadbed have been summarized, and the advantages and disadvantages have been evaluated.

1 Introduction

The soft clay has the characteristics of high compressibility, high water content, low strength and poor permeability, which will lead to large settlements under long-term loads. Therefore, the settlement of soft clay roadbed is one of the prominent problems during the construction of railway and highway in soft clay area, and many engineering accidents are caused by the large settlement of subgrade. Thus, the prediction of settlements of soft clay roadbed has been one of the important research topics in Geotechnical Engineering.

As the settlement of soft clay usually develops with time, the settlement vs. time curve is usually used to describe the settlement properties of soft clay [1–3]. The most common method to predict the long term settlement is fitting the settlement data measured in the field. Many traditional fitting curves have been used to predict the settlement of soft clay, such as exponential curve, hyperbolic curve, Poisson curve and Asaoka method [4, 5]. Furthermore, in recent years, more and more new methods based on new theories have been introduced to improve the prediction of settlement, such as Grey forecasting method [6], Logistic method [7], and Gompertz model [8]. In this paper, the commonly used methods mentioned above to predict the settlement of soft clay have been evaluated and compared with each other.

2 Mechanism of the Settlement of Soft Clay Roadbed

The total settlement of soft clay roadbed contains three components: immediate settlement, consolidation settlement and secondary consolidation settlement. The development of settlement can be divided into the following four stages (as shown in Fig. 1):

Fig. 1. Sketch view of settlement vs. time

- (1) at the initial status, the soil is elastic and the change of settlement with load is linear;
- (2) then with the increase of load, the settlement of soft clay and the corresponding rate increase and the nonlinear is obvious;
- (3) although the load keeps constant, the settlement continues to increase due to consolidation and rheological property of clay, but the settlement rate declines;
- (4) when the time is long enough, the settlement of clay will reach a limit state, the corresponding settlement rate is zero and the settlement arrives at its final value.

The purpose to study the properties of soft clay is to develop theoretical or empirical models to predict the settlement of soft clay at any time and any load. In recent years, many fitting curve methods based on measured field settlement data have been proposed, we will introduce these models detailedly in the next section.

3 Fitting Curve Methods

3.1 Exponential Curve

According to the consolidation theory proposed by Terzaghi, the relation between pore water pressure with time can be described by exponential expression. For soil with linear elastic property, the consolidation degree of soil defined by using stress equals to that defined by using strain. the consolidation degree can be expressed as follows:

$$U = 1 - \alpha \exp(-\beta t) \quad (1)$$

Where α and β are consolidation parameters, of which the expressions relate to drainage condition and are shown in Table 1.

Then according to the measured settlement data, the average consolidation degree can be expressed as,

Table 1. Theoretical values of α and β

Drainage condition	α	β
Consolidation with vertical drainage	$\frac{8}{\pi^2}$	$\frac{\pi^2 c_v}{4H^2}$
Drainage wells run through the soil layer	$\frac{8}{\pi^2}$	$\frac{8c_h}{F_n d_w^2} + \frac{\pi^2 c_v}{4H^2}$
Drainage wells do not run through the soil layer	$\frac{8}{\pi^2 \rho}$	$\frac{8c_h}{F_n d_w^2}$

$$U = \frac{s_t - s_d}{s_f - s_d} \quad (2a)$$

Where s_t is the measured settlement at time t , s_d and s_f are immediate consolidation settlement and final consolidation settlement.

Combined Eq. (1) with Eq. (2a), the following expression can be obtained,

$$s_t = (s_f - s_d)[1 - \alpha \exp(-\beta t)] + s_d \quad (2b)$$

In exponential curve fitting method, three data points on the settlement vs. time curve are selected and the time interval between the points must be the same, that is,

$$\Delta t = t_3 - t_2 = t_2 - t_1 \quad (3)$$

Then put the three points into Eq. (2b), three equations can be obtained and the expressions of β , s_f , and s_d are shown as follows,

$$\beta = \frac{1}{\Delta t} \ln\left(\frac{s_2 - s_1}{s_3 - s_2}\right) \quad (4a)$$

$$s_f = \frac{s_3(s_2 - s_1) - s_2(s_3 - s_1)}{(s_2 - s_1) - (s_3 - s_2)} \quad (4b)$$

$$s_d = \frac{s_t - s_f[1 - \alpha \exp(-\beta t)]}{\alpha \exp(-\beta t)} \quad (4c)$$

The above equations are the controlling equations in exponential curve.

3.2 Hyperbolic Curve

For soft clay with high compressibility, the stress vs. strain curve is non-linear, so the consolidation process of soft clay does not necessarily conform exponential curve.

When $0.6 < U < 0.9$, the relation between U and time factor T can be expressed by equilateral hyperbola. The theory background of hyperbolic curve is given as follows:

The settlement of soft clay at time t can be calculated by the following equation,

$$s = s'_0 + \frac{t - t'_0}{\alpha'_0 + \beta'(t - t'_0)} \quad (5)$$

Rewriting Eq. (5), then,

$$\frac{t - t'_0}{s - s'_0} = \alpha'_0 + \beta'(t - t'_0) \quad (6)$$

Where α'_0 and β' are the intercept and slope of vs. curve.

3.3 Asaoka Method

The one dimensional consolidation equation described by using volumetric strain can be expressed as follows,

$$c_v \frac{\partial^2 \varepsilon_v}{\partial z^2} = \frac{\partial \varepsilon_v}{\partial t} \quad (7)$$

On the basis of Eq. (7), Asaoka employed a series to approximate the expression,

$$s + \alpha_1 \frac{ds}{dt} + \alpha_2 \frac{d^2s}{dt^2} + \dots + \alpha_n \frac{d^ns}{dt^n} = b \quad (8)$$

Where s is consolidation settlement, $\alpha_i (i = 1, 2, 3, \dots, n)$ is consolidation coefficient, b is a constant accounting for boundary condition.

According to Asaoka's suggestion, the parameters in Eq. (8) can be determined by using field measured data. Furthermore, Eq. (7) can also be simplified as recursive relation as follows,

$$s_j = \beta_0 + \sum_{i=1}^n \beta_i s_{j-1} \quad (9)$$

Taking the fixed boundary condition, the solution for the equation is given by,

$$s(t) = s_\infty - (s_\infty - s_0) \exp\left(-\frac{t}{\alpha_1}\right) \quad (10)$$

where s_0 and s_∞ are the initial and final settlement of soft clay, respectively, $\alpha_1 = 5h^2/(12c_v)$.

3.4 Poisson Curve

Poisson curve is the logistic curve, which is suitable for incremental or decayed “S” shaped curve. The expression of the commonly-used Poisson curve is given by,

$$s_t = \frac{k}{1 + ae^{-bt}} \quad (11)$$

where a, b and k are parameters that should be determined by measure data.

The parameters in the expression of Poisson curve can be fitted by using the measured s_t curve, the main procedures are shown as follows:

Taking a time interval of and the total number of time series is $3n$, then divide the time series into three sub-series and each sub-series has a number of n data points, that is,

$$\begin{cases} \text{sub-series 1} & t = 1, 2, 3, \dots, n \\ \text{sub-series 2} & t = n+1, n+2, n+3, \dots, 2n \\ \text{sub-series 3} & t = 2n+1, 2n+2, 2n+3, \dots, 3n \end{cases} \quad (12)$$

Then assuming that S_1 , S_2 and S_3 are the sum of the reciprocal of settlements corresponding to each time sub-series, the expressions to determine the parameters a, b and k are given as follows,

$$\begin{cases} a = \frac{(S_1 - S_2)^2 (1 - e^{-b}) k}{[(S_1 - S_2) - (S_2 - S_3)] e^{-b} (1 - e^{-nb})} \\ b = \frac{\ln(S_1 - S_2) / (S_2 - S_3)}{n} \\ k = \frac{n}{S_1 - \frac{(S_1 - S_2)^2}{(S_1 - S_2) - (S_2 - S_3)}} \end{cases} \quad (13)$$

Combining Eq. (13) with Eq. (12), the settlement of clay at any time s_t can be calculated, and the final settlement equals to the ultimate value of s_t , when the time t trends to infinity, that is,

$$s_\infty = \lim_{t \rightarrow \infty} s_t = k \quad (14)$$

3.5 Grey Forecasting Method

The Grey forecasting method is always referred to as GM model, and the most common GM Model has isometric time sequence, that is, GM(1, 1) model, the expression of which is given by the following differential equation,

$$\frac{dx^{(1)}}{dt} + cx^{(1)} = u \quad (15)$$

The corresponding time response equation is,

$$\hat{x}^{(1)}(t) = [x^{(1)}(0) - \frac{u}{c}]e^{-ct} + \frac{u}{c} \quad (16)$$

The discrete response equation is,

$$\hat{x}^{(1)}(k) = [x^{(1)}(0) - \frac{u}{c}]e^{-c(k-1)} + \frac{u}{c} \quad (17)$$

3.6 Logistic Model

Logistic model is a kind of growth model that is widely used in the fields of ecology and demography. The differential form of logistic model is as follows,

$$\frac{ds}{dt} = rs(1 - \frac{s}{K}) \quad (18)$$

According to Eq. (18), Logistic curve contains three parameters that should be determined. If the measured settlement data can be obtained, there are three methods used to determine these three parameters: three section method, grey model and non-linear regression method. The main processes of the three methods can be found in other literatures.

3.7 Gompertz Model

Gompertz model is also a kind of growth curve, the mathematical expression of which shows as follows,

Fig. 2. Typical Gompertz curve

$$y = m \exp[-e^{(n-lt)}] \quad (19)$$

where m , n and l are constants, t is time series.

The typical Gompertz curve is shown in Fig. 2. From the figure, it is found that the theoretical curve agrees well with measured settlement curve. Therefore, it is feasible to use Gompertz model to predict the settlement of soft clay.

4 Discussions

In Sect. 3, seven methods to predict the settlement of soft clay have been reviewed detailedly. Although all of them have been applied to predict the settlements of some highway subgrade successfully, each method has its own application scope.

The exponential curve conforms to the one-dimensional consolidation theory proposed by Terzaghi and the parameter β has definite physical meaning. However, the method to determine the parameters α and β based on three data points is subjective, which will influence the accuracy.

According to Terzaghi's one dimensional consolidation theory, the hyperbolic method will make the relation between consolidation degree U and time factor T simpler than the exponential curve. Therefore, the hyperbolic method is more suitable for engineers in the field. However, the disadvantage of this method is that the consolidation parameters can not be reflected.

Compared with the consolidation of Terzaghi, it is found that Asaoka method predicts the settlement trends once the degree of consolidation exceeds 0.6. Furthermore, the final settlement of the soft clay can also be predicted well.

The Poisson curve can describe well the S-shaped relationship between settlement and time. The applicability of Poisson curve for other settlement vs. time curves is questionable and should be given more attentions.

The four methods mentioned above are traditional methods used for predicting the settlement of soft clay, while the Grey forecasting method, Logistic model and Gompertz model are new methods suitable for special kind of soft clay or different boundary conditions.

5 Conclusion

In this paper, seven different methods for predicting the settlement of soft clay roadbed have been reviewed detailedly based on the presentation of the settlement mechanism of soft clay. The applicability of each method has been analyzed. Although all the methods introduced in this paper have been applied into practices, the accuracy of each method should be studied in further researches.

References

1. Zhao, M., Liu, Y., Cao, W.: The developing regularity and forecasting of settlement in soft clay roadbed. *J. Cent. South University (Nat. Sci.)* **35**(1), 157–161 (2004)
2. Chen, S., Wang, X., Xu, X., Yu, F., Qin, S.: Three-point modified exponential curve method for predicting subgrade settlements. *Rock Soil Mech.* **32**(11), 3355–3360 (2011)
3. Zai, J., Mei, G.: Forecast method of settlement during the complete process of construction and operation. *Rock Soil Mech.* **21**(4), 322–325 (2000)
4. Tan, S.A.: Hyperbolic method for settlement in clays with vertical drains. *Can. Geotech. J.* **31**(1), 125–131 (1994)
5. Asaoka, A.: Observational procedure of settlement prediction. *Soil Found.* **18**(4), 87–101 (1978)
6. Lei, X., Bai, S., Meng, Q.: The application of Grey forecasting to analyzing soft foundation settlements. *Rock Soil Mech.* **21**(2), 145–147 (2000)
7. Xu, H., Shi, B., Li, X.: Logistic growth model and its applicability for predicting settlement during the whole process. *Rock Soil Mech.* **26**(3), 387–391 (2005)
8. Yu, C., Liu, S.: A study on prediction of embankment settlement with the Gompertz model. *Rock Soil Mech.* **26**(4), 82–86 (2005)