Research on the Prediction Method of Gompertz Model for High Subgrade Settlement

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Abstract. According to the development law of high embankment settlement, the Gompertz curve model is established. Combined with specific engineering example, the Gompertz curve well reflects the change process of high embankment settlement, and the prediction result of the model with the measured settlement value is very accurate and reliable.

1 Introduction

The construction of expressway in the west, high filling and excavation of roadbed structure are generally occur [1, 2]. After completion construction of high fill embankment subgrade settlement often appear, even cause the destruction of pavement structure engineering accident [3]. In order to ensure the safety of the normal operation of highway, according to the law of development of high filling subgrade settlement observation data and combining site, Gompertz curve model is applied to predict the settlement of the roadbed.

2 Mechanism of High Filling Subgrade Settlement

In the early period of construction, just after loading soil is in elastic status. The soil pore water do not exhaust [4, 5]. Due to the soil of instantaneous shear de-formation, with the increase of load the settlement has approximately linear increase. In the later period of construction, with increasing load, the load imposed in high embankment are also more and more big. The pore water in the foundation soil is gradually discharged, meanwhile excess pore water pressure gradually decreases. So compaction causes more de-formation, and soil in embankment will be in elastic-plastic state. With the continuous development of the plastic zone, settlement rate increases quickly, until the load no longer increases.

After construction, due to the pore water pressure close to disappear completely, the consolidation process is not complete, settlement will continue with the passage of time, but significantly decreased sedimentation rate. If the time of settlement is infinity, settlement will be at ultimate state, and settlement rate is zero, so the settlement at that time is really the final settlement. In fact, time generally is large enough, for the highway time takes for 15 years plus filling time.

3 The Forecast Model of Gompertz Curve

At present, there are two categories in calculation method of settlement of embankment. First, according to the constitutive model of consolidation theory combined with various soil, it is settlement calculation of the application of numerical methods, such as visco-elastoplastic finite element model method [6, 7]. The method of the calculation of the finite element method is very high, generally only used to calculate the key end of large engineering. And some constitutive model and engineering practice has a large gap, and needs a large number of soil tests to determine the soil parameters, so it is difficult to use in the general engineering design. Second, based on the measured data it is calculated settlement prediction method of quantity and time relations, such as hyperbola and exponential curve method. This method is concise and practical, and can satisfy the engineering precision requirement. Method for forecasting subgrade settlement of this selection is one of the Gompertz model.

Gompertz method is proposed by statisticians and mathematicians B. Gompertz, and it is also a growth curve, which are widely used in economics, management and statistics. Gompertz forecasting model is a growth curve model, and can be used to express and describe the process of growth and development. The practice indicated prediction model can be used as a reliable, effective method to forecast the settlement prediction of roadbed.

4 Calculation of Parameters

The three parameters in the model can be obtained using the three estimation. First of all, make it into a modified exponential form, then the data is divided into three sections (if not divisible by 3, can increase and decrease the individual data which is a multiple of 3), each containing a number of segments, i.e., seek and obtain.

$$y_t = e^{(k+ab^t)} \quad (1)$$

$$y' = k + ab^t \quad (2)$$

$$y_1 = k + ab^1 \quad (3)$$

$$y_2 = k + ab^2 \quad (4)$$

$$y_3 = k + ab^3 \quad (5)$$

$$y_t = k + ab^T \quad (6)$$

$$\sum_1 y_t = \sum_{t=1}^n y_t = nk + ab(b^0 + b^1 + b^2 + \cdots + b^{n-1}) \quad (7)$$

$$\sum_2 y_t = \sum_{t=n+1}^{2n} y_t = nk + ab^{n+1}(b^0 + b^1 + b^2 + \cdots + b^{n-1}) \quad (8)$$

$$\sum_3 y_t = \sum_{t=2n+1}^{3n} y_t = nk + ab^{2n+1}(b^0 + b^1 + b^2 + \cdots + b^{n-1}) \quad (9)$$

$$(b^0 + b^1 + b^2 + \cdots + b^{n-1}) = \frac{b^n - 1}{b - 1} \quad (10)$$

$$\sum_1 y_t = nk + ab \frac{b^n - 1}{b - 1} \quad (11)$$

$$\sum_2 y_t = nk + ab^{n+1} \frac{b^n - 1}{b - 1} \quad (12)$$

$$\sum_3 y_t = nK + ab^{2n+1} \frac{b^n - 1}{b - 1} \quad (13)$$

$$b = \sqrt[n]{\frac{\sum_3 y_t - \sum_2 y_t}{\sum_2 y_t - \sum_1 y_t}} \quad (14)$$

$$a = \frac{b - 1}{(b^n - 1)^2 b} (\sum_2 y_t - \sum_1 y_t) \quad (15)$$

$$k = \frac{1}{n} (\sum_1 y_t - ab \frac{b^n - 1}{b - 1}) \quad (16)$$

$$k = \frac{1}{n} \left[\frac{\sum_1 y_t \sum_3 y_t - (\sum_2 y_t)^2}{\sum_1 y_t + \sum_3 y_t - 2 \sum_2 y_t} \right] \quad (17)$$

By using the method of analogy, Gompertz prediction parameters can be calculated as below:

$$a = \frac{b - 1}{(b^n - 1)^2 b} (\sum_2 \ln y_t - \sum_1 \ln y_t) \quad (18)$$

$$b = \sqrt[n]{\frac{\sum_3 \ln y_t - \sum_2 \ln y_t}{\sum_2 \ln y_t - \sum_1 \ln y_t}} \quad (19)$$

$$k = \frac{1}{n} \left[\frac{\sum_1 \ln y_t \sum_3 \ln y_t - (\sum_2 \ln y_t)^2}{\sum_1 \ln y_t + \sum_3 \ln y_t - 2 \sum_2 \ln y_t} \right] \quad (20)$$

5 Engineering Examples

Combined with the subgrade settlement observation data in Hunan province Changde-Zhangjiajie expressway, the K103+380, K123+560 are chosen as the case study. The high fill subgrade filler is typical, where is filled with low liquid limit clay and shale, an embankment height is about 13 m (Fig. 1).

Fig. 1. Results of Gompertz and forecasting and measured in K123+560

Fig. 2. Results of Pearl and forecasting and measured in K103+380

In Fig. 2, the time-scattered settlement is “S” shape. The first 12 points (20–240 days) data are selected as samples, Matlab6.5 is used to calculate related parameters of Gompertz curve model (as shown in Table 1), then the results of Gompertz curve model for settlement prediction of roadbed (as shown in Table 2) are obtained, as well as the sum squared error (prediction precision) 168.13 (Table 3).

Table 1. Results of Gompertz and forecasting and measured in K123+560

Time (d)	Gompertz forecasting (mm)	Measured in site (mm)	Absolute error (mm)
20	23.6187	23.9	0.3
40	32.1777	32.9	0.7
60	41.7554	46.3	4.6
80	52.0064	52.1	0.1
100	62.5736	60.6	2.0
120	73.1273	65.7	7.5
140	83.3896	80.9	2.5
160	93.1464	94.4	1.3
180	102.2482	106.3	4.1
200	110.6040	111.6	1.0
220	118.1724	114.9	3.2
240	124.9498	124.6	0.3
260	130.9612	132.7	1.7
280	136.2502	137.0	0.7
300	140.8721	142.3	1.4
320	144.8878	145.2	0.3
340	148.3601	147.1	1.3
360	151.3503	148.0	3.3
380	153.9166	150.2	3.7
400	156.1125	151.4	4.8

Table 2. Results of Pearl and forecasting and measured in K103+380

Time (d)	Pearl forecasting (mm)	Measured in site (mm)	Absolute error (mm)
20	25.32749	22.944	2.383488
40	31.62893	31.584	0.044928
60	39.0169	44.448	5.4311
80	47.4527	50.016	2.5633
100	56.79898	58.176	1.37702
120	66.81466	63.072	3.742656
140	77.17133	77.664	0.49267
160	87.49296	90.624	3.13104

(continued)

Table 2. (continued)

Time (d)	Pearl forecasting (mm)	Measured in site (mm)	Absolute error (mm)
180	97.40851	102.048	4.63949
200	106.6032	107.136	0.5328
220	114.8543	110.304	4.550304
240	122.0436	119.616	2.427648
260	128.1486	127.392	0.756576
280	133.2204	131.52	1.700352
300	137.3577	136.608	0.749664
320	140.6829	139.392	1.290912
340	143.3236	141.216	2.107584
360	145.4007	142.08	3.320736
380	147.0224	144.192	2.830368
400	148.281	145.344	2.937024

Table 3. Parameters of Gompertz method

K	a	b
5.1264	-2.3314	0.8426

6 Conclusion

In the forecasting of road foundation settlement, different methods can often provide valuable information.

- (1) The settlement of high embankment has its own law of development, Gompertz model can express S shape relationship between settlement and time of the whole process of subgrade. The settlement reflects very good, but also can be accurately applied in prediction of high embankment.
- (2) Using the Gompertz model to forecast the settlement of roadbed, in order to obtain good prediction effect enough measured data is basis. The measured settlement value should be checked, and delete obvious error.
- (3) In order to improve the prediction accuracy, the selection of measured sample points should be increased, and with the passage of time, the settlement of recent is added as the original sample, so that the prediction effect is better.

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