

# Finite Element Analysis of Rock Slope Stability Using Shear Strength Reduction Method

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**Abstract.** Finite element analysis incorporating the shear strength reduction method was applied to study the west slope stability of an open cut mine in Australia using Mohr–Coulomb and generalized Hoek–Brown criteria. The pit of the mine had multiphase excavations and reached 180 m in depth. The study investigated three slope configurations, namely, Stage 1 inter ramp slope 43°, Stage 2 inter ramp slope 49° and optimized Stage 2 slope 54°. When implementing the generalized Hoek–Brown failure criterion, the equivalent factor of safety was 1.96, 1.87 and 1.40 under dry slope for the three configurations, respectively. However, under partly saturated conditions, the optimised slope would have a factor of safety 1.16. Furthermore, the generalised Hoek–Brown criterion generated lower factors of safety than the Mohr–Coulomb failure criterion. The difference is related to an overestimation of the shear strength parameters by the linear Mohr–Coulomb criterion under low confining stresses compared with the non-linear Hoek–Brown.

## 1 Introduction

Slopes need be designed and cut with a margin of safety for open pit mines. Economics could be improved by steepening the slope thereby reducing the amount of waste excavation; however, excessive steepening of slope could result in failure leading to loss of life and damage to property (Singh et al. 1989; Singh and Singh 1992). The factors, which mainly influence the stability of a typical open-pit slope, are the shear strength parameters of slope forming material, the presence and characteristics of discontinuities in the slope mass and the groundwater conditions (Singh and Monjezi 2000; Singh et al. 2008).

The shear strength reduction (SSR) method was used for soil slope stability analysis in 1975 (Zienkiewicz et al. 1975). Duncan (1996) defined the factor of safety of soil slopes as the ratio of actual shear strength to the minimum shear strength required to avoid failure, or the factor by which shear strength must be reduced to bring a slope to failure. This method was applied for rock masses (Shangyi et al. 2003). Yingren and Shangyi (2004) and Hammah et al. (2007) demonstrated the efficiency of SSR method for slopes of soil and rock masses. This method was also adopted in several other studies, such as Yeo and Chan (1993), Dowson et al. (1999), Hammah et al. (2004a, b),

Zheng et al. (2009) and Gupta et al. (2016). The advantages of the SSR method over limit equilibrium method (LEM) are: (1) No assumption is required of the interslice shear force distribution; (2) the critical failure surface in the slope can be found from the shear strain; and (3) this method is suitable for complex slope conditions in order to interpret details of displacements, stresses and water pressures. LEM may give lower estimates of failure volumes than SSR method in numerical modelling (Chiwaye 2010).

The SSR approach includes the search for a stress reduction factor (SRF) value that brings the slope to fail. The shear strength reduced by a factor of safety  $F$  can be determined using a series of trials to adjust the friction angle ( $\phi'$ ), and the cohesion ( $c'$ ) of slope rock mass. For example of the Mohr-Coulomb criterion (Eq. 1), the process of shear stress reduction can be expressed in Eq. 1a.

$$\tau = c' + \sigma' \tan \phi' \quad (1)$$

$$\frac{\tau}{F} = \frac{c'}{F} + \frac{\sigma' \tan \phi'}{F} \quad (1a)$$

Where, the reduced value of cohesion  $c' = \frac{c}{F}$  and the reduced value of internal friction angle  $\phi' = \arctan\left(\frac{\tan \phi}{F}\right)$ .

This paper presents a finite element (FE) analysis of slope stability using SSR method for three inter-ramp slopes at the Handlebar open pit mine, namely Stage 1 ( $43^\circ$ ), Stage 2 ( $49^\circ$ ) and an optimised Stage 2 slope ( $54^\circ$ ), under both dry and partly saturated conditions that implements the Mohr–Coulomb and the generalised Hoek–Brown failure criteria.

## 2 Site Geology

Handlebar Hill open - pit mine is located at Mt. Isa, north Queensland, Australia (Fig. 1). In the Mount Isa Valley, the rock formation is represented by Magazine Shale, Spears-Kennedy Siltstone, Urquhart Shale, Native Bee Siltstone, and Eastern Creek Volcanics (Figs. 1 and 2). The ground is oxidized and leached of sulphides. The leaching can extend to great depths aided by faults (Fig. 2). The leached zone in the rock slope is regarded as an unconfined aquifer (Rosengren and Associates 2007). Pre-mining groundwater level data indicated that groundwater levels coincided with the base of the oxidized zone at a depth of about 50 m below ground level.

Mining started in 2008 through multiple stages. Stage 1 was excavated to a depth of 77 m and Stage 2 to 180 m. There were six benches at each stage of varying bench height from 10 m to 17 m (Fig. 2). The inter-ramp angle (IRA) was  $43^\circ$  and  $49^\circ$  for Stage 1 and Stage 2, respectively, on the west wall at the 4440 N cross-section.

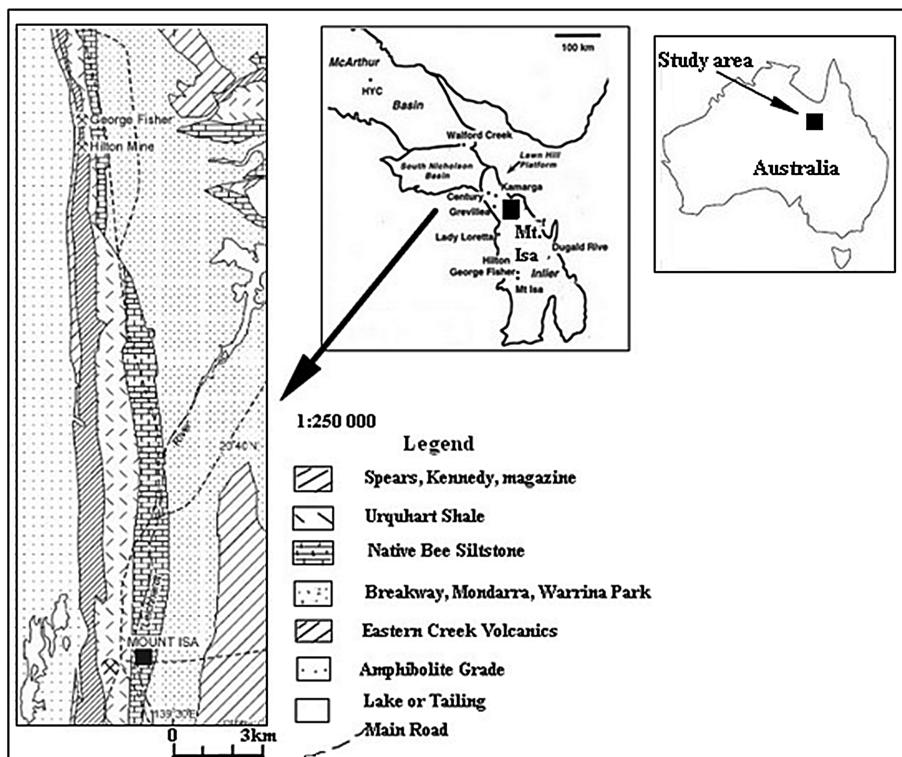


Fig. 1. Location and geological map of the study area (after Conaghan et al. 2003)

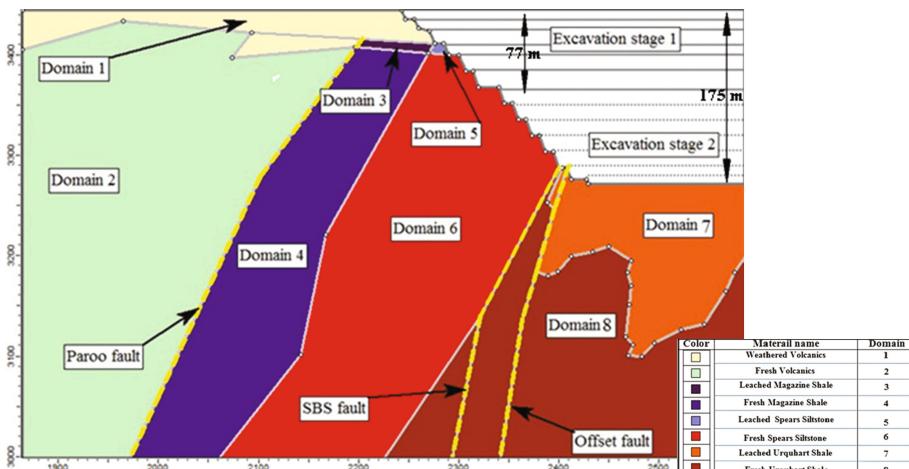


Fig. 2. Rock domains of the west slope at N4440 cross-section

### 3 Data Preparation and Finite Element Analysis

Both the Mohr–Coulomb criterion (Eq. 1) and the generalized Hoek–Brown criterion (Eq. 2) are used in this analysis.

$$\sigma'_1 = \sigma'_3 + \sigma_{ci} \left( m_b \frac{\sigma'_3}{\sigma_{ci}} + s \right)^a \quad (2)$$

Some of the parameters used in the Hoek–Brown criterion (e.g.,  $s$ ,  $m_b$ , and  $a$ ) are available from rock mass data, while other parameters used ( $m_b$ ,  $v$ ,  $\sigma_{ci}$ ) were derived using laboratory rock tests (Seville 1981; Tarrant and Lee 1984). From June 2006 to June 2007, a total of 20,189.6 m of diamond core was drilled and 3,080 m of rock chips were completed (Rosengren and Associates 2008). The geological strength index GSI was determined from the diamond core logging and in-situ surface mapping. The mean GSI value of the overall slope was reduced by 20% in this study (Table 1). For the weathered Magazine Shale, a GSI value of 24 was used because this represented a poorer rock mass quality below the base of complete oxidization in the simulations. The rock mass modulus ( $E_{rm}$ ) of all domains was estimated according to Hoek and Diederichs (2006). A disturbance factor of  $D = 0.7$  was used for small scale blasting in civil engineering slopes that results in modest rock mass damage if controlled blasting is used (Hoek et al. 2002). The parameters used in the study are detailed in Table 1.

**Table 1.** Input data used in the slope stability analysis

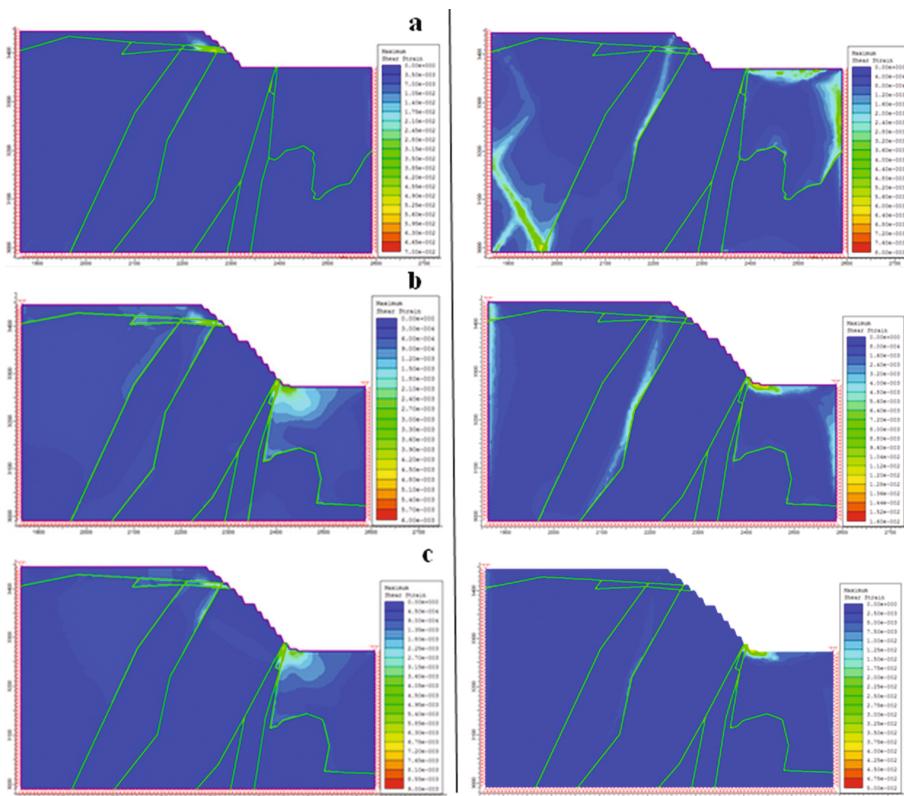
Parameter	Unit	Domain							
		1	2	3	4	5	6	7	8
$\gamma$	kN/m <sup>3</sup>	27.4	28.3	26.5	26.5	23.5	27.2	26.9	31.1
$E_{rm}$	MPa	3000	10000	4500	4500	1500	8500	2000	12000
$\varphi$	(°)	23	32	13	25	20	35	22	44
$c$	kPa	350	800	50	410	240	990	300	1250
$\sigma_t$	MPa	0.1	0.3	0.0	0.1	0.0	0.4	0.0	0.2
$\sigma_{ci}$	MPa	36.4	55.5	32.0	32.0	26.3	111.1	32.7	108
$v$	—	0.3	0.2	0.25	0.2	0.25	0.2	0.25	0.2
$m_b$	—	0.184	0.42	0.271	0.271	0.148	0.319	0.165	1.036
$m_i$	—	4	4	4	4	4	4	4	11
$s$	—	0.0003	0.0026	0.0008	0.0008	0.0002	0.0013	0.0002	0.002
$a$	—	0.509	0.503	0.505	0.505	0.511	0.504	0.510	0.504
$D$	—	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7
80% GSI	—	36	48	24	41	32	43	34	46
Dilation	(°)	5	8	4	5	7	12	4	12

SSR technique is often used with FEM to solve sophisticated problems such as estimating stability of slope (Ng et al. 2000). In this study, Phase<sup>2</sup> (RocScience 2014) FE program is used for the slope stability simulations under different scenarios. The model comprises an area of 750 m in width and 450 m in depth (Fig. 2).

The pit bottom is located at an elevation of 3,368 m (175 m deep), and the top of the west slope is at an elevation of 3,543 m (ground-surface level). The ramp crosses the west slope section at an elevation of 3,463 m (77 m deep).

Phase<sup>2</sup> is a powerful FE program, which can be used for a wide range of engineering projects including slope stability, groundwater seepage and probabilistic analysis. It can simulate and analyze a complex multi-stage model. Progressive failures and explicit modelling of discontinuities can be simulated to gain further insight into the rock mass behavior of the slope. When setting up the project, the multi-stage model is available to simulate the stresses resulting from different excavations. The program was designed to calculate the last excavation stage for the critical SRF, or factor of safety. Therefore, the strength reduction factor analysis has to be run many times for the two stages of excavations. The FE simulations were carried out to further improve the overall slope stability analysis, namely to optimize the Stage 2 IRA.

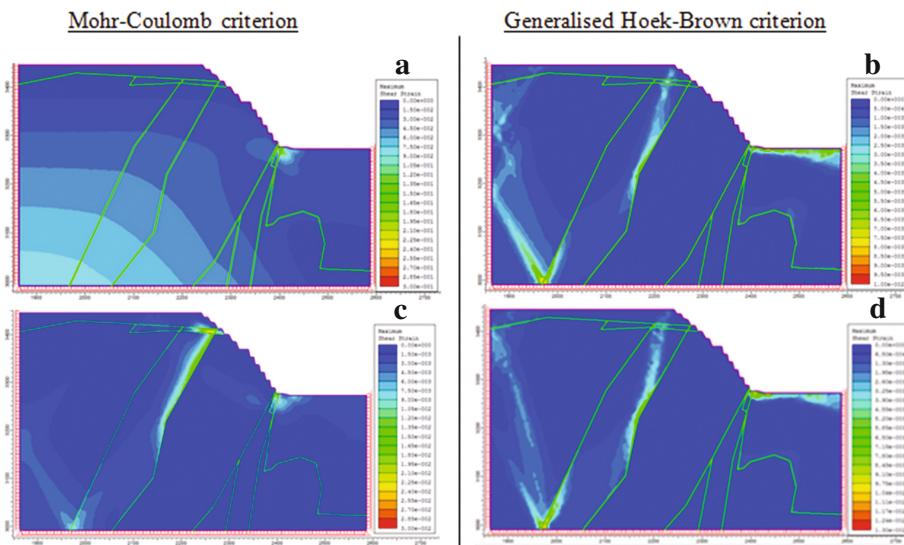
Figure 3 shows maximum shear strains simulated for IRA 43° and 49° using the Mohr–Coulomb criterion and the generalized Hoek–Brown criterion. At the upper IRA 43° (Stage 1, excavation depth 77 m), there is a localized high shear strain zone in



**Fig. 3.** Maximum shear strain contours, (a) Stage 1, IRA 43°, (b) Stage 2, IRA 49° and (c) IRA 49° with enhanced slope cohesion strength, dry slope conditions. Right: Generalized Hoek–Brown criterion, left: Mohr–Coulomb criterion

Domains 3 and 5, which is structurally controlled leached Magazine shale and Spears siltstone (Fig. 3a, left). At the lower IRA  $49^\circ$  (Stage 2, excavation depth 180 m), the shear strain concentration at toe is prominent (Fig. 3b, c). In comparison, there is less deformation in the slope using the Mohr–Coulomb criterion than using the generalized Hoek–Brown failure criterion (Fig. 3). These results indicate that an increase in maximum shear stress occurs at the final excavation of stage 2.

In the third case of the study, the Stage 2 IRA was increased  $5^\circ$  to optimize the open-pit design to be as steep as possible. Figure 4 shows the maximum shear strain contours for the IRA  $54^\circ$  under dry and partly saturated conditions. As the effective normal stress decreases under partly saturated slope conditions, larger maximum shear strains develop. There is a clear increase in shear strain in Domains 3 and 5 and at the toe of the slope (Fig. 4b, c).



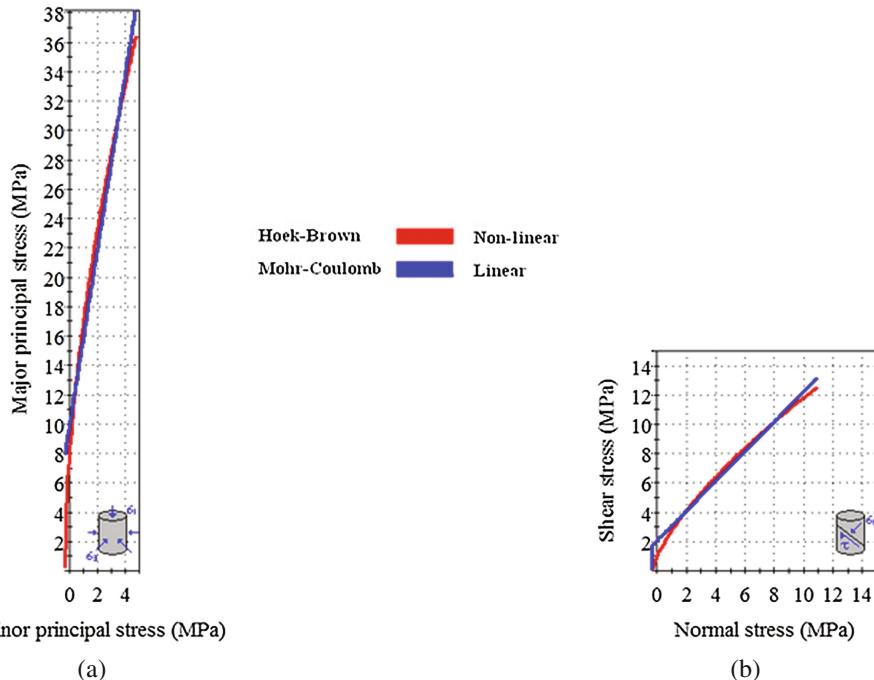
**Fig. 4.** Maximum shear strain contours for optimized slope after two excavation stages, for lower IRA of  $54^\circ$ , (a, b) dry slope, (c, d) partly saturated slope

## 4 Results and Analyses

The critical SRF or factor of safety of the FE analyses is tabulated in Table 2 for IRAs  $43^\circ$ ,  $49^\circ$  and optimized IRA  $54^\circ$  under dry and partly saturated conditions using both Mohr–Coulomb and generalized Hoek–Brown failure criteria. As can be expected, rising slope angle leads to lower critical SRF. Furthermore, the critical SRF is lower in partly saturated slope than the dry one. The modelling predicted a higher factor of safety using the Mohr–Coulomb criterion than using the generalized Hoek–Brown criterion (Table 2). This may attribute to the linear failure envelope in the Mohr–Coulomb criterion (Fig. 5). Figure 5 is plotted based on the shear strength analysis of Urquhart Shale rock using RocLab 1.0 (RocScience 2014). Rock mass strength in the

**Table 2.** Critical SRF from FE analyses for different slopes

IRA	Dry slope		Partly saturated slope	
	Mohr–Coulomb	Generalized Hoek–Brown	Mohr–Coulomb	Generalized Hoek–Brown
43°	2.76	1.96	2.70	1.71
49°	2.00	1.87	1.86	1.59
54°	1.98	1.40	1.85	1.16

**Fig. 5.** Comparison between the Mohr–Coulomb criterion and the generalized Hoek–Brown failure criterion, (a) in term of major and minor stresses, (b) in term of shear and normal stresses

area is a non-linear stress function. The linear Mohr–Coulomb failure criterion may not agree well with the rock mass failure envelope where the low confining stresses in an open-pit slope make the non-linearity of rock behaviors more possible (Shen 2013). On the other hand, the Mohr–Coulomb strength parameters may overestimate the shear strength of a rock mass at a high stress level because it was derived using a straight line fitted over the Hoek–Brown curve (Lin et al. 2014). Hammah et al. (2004a, b) stated that the generalized Hoek–Brown criterion is the most suitable strength model for predicting the failure of rock masses, especially in low normal stress ranges.

If the Stage 2 slope increases 5° to IRA 54°, the critical SRF is 1.16 under partly saturated condition using the generalized Hoek–Brown criterion, which is less than 1.3. This will have an influence on the stability of the west slope if the shear strength failure

mechanisms of the rock masses are considered to dominate the slope stability in wet conditions. The results for slopes under the other conditions show a stable slope with SRF > 1.3 using both failure criteria. For the long term operations and after heavy rain-fall, bench scale instabilities can be expected. Given that a GSI reduction of 20% was assumed and the  $E_{rm}$  was estimated as a worst case scenario for the rock mass strength parameters, the slope is expected to be stable. The groundwater has influence on the slope stability, in particular, in the case of steeper slope with an IRA of 54°.

## 5 Conclusions

Slope stability analysis of the west slope of Handlebar Hill open-pit mine was conducted using non-directional strength of rock mass properties through finite element analyses implemented the shear strength reduction technique. The generalized Hoek–Brown criterion did not generate any SRF greater than the results predicted by the Mohr–Coulomb failure criterion. The difference is related to an overestimation of the shear strength parameters by linear fit under low confining stresses compared with the non-linear Hoek–Brown. The stability of the west slope would be overestimated when directly using the shear strength parameters for the analysis. Consequently, the disturbance factor value ( $D$ ),  $E_{rm}$  and the GSI must be significantly calibrated and considered as main inputs to model the slope.

Presuming that a minimum factor of safety 1.3 is adopted for long term excavations, the Stage 2 IRA could be optimized and increased to 54°, in which SRF was 1.4. However, this IRA would have a SRF of 1.16 in case of partly saturated slope conditions. Inter-ramp stability can be controlled by both geological structure and rock mass strength, however, the combined failure mechanism is not the scope of this study.

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