Optics and Renaissance Art

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11.1 Introduction

An extensive visual investigation by the artist David Hockney [1] lead to the discovery of a variety of optical evidence in paintings as described in a number of technical papers [2–8]. This work demonstrated European artists began using optical devices as aids for creating their work early in the Renaissance well before the time of Galileo. These discoveries show that the incorporation of optical projections for producing certain features coincided with the dramatic increase in the realism of depictions at that time. Further, it showed that optics remained an important tool for artistic purposes continuing until today.

Our earliest evidence of the use of optical projections is in paintings of Jan van Eyck and Robert Campin in Flanders c1425, followed by artist including Bartholome Bermejo in Spain c1474, Hans Holbein in England c1530, and Caravaggio in Italy c1600, to name a few. Significantly, the optical principles of the camera obscura were described the eleventh century Arab scientist, philosopher, and mathematician, Abu Ali al-Hasan ibn al-Haytham, known in the West as Alhazen or Alhacen (b.965 Basra d.1039, Cairo). This is important for the present discussion because by the early thirteenth century al-Haytham’s writings on optics had been translated into Latin and incorporated in the manuscripts on optics of Roger Bacon (c1265), Erasmus Witelo (c1275), and John Peckham (c1280).

Concurrent with the growing theoretical understanding optics were practical developments, such as the invention of spectacles in Italy around 1276. Pilgrims carried small convex mirrors into cathedrals to use as wide-angle optics to enable a much larger area of the scene to be visualized, showing how common the uses of optics had become by this time. As described below, evidence within paintings shows that at some point during this period someone realized replacing the small opening in a *camera obscura* with a lens resulted in a projected image that was both brighter and sharper. One lens from a pair of reading spectacles allows projection of images of the size, brightness, and sharpness necessary to be useful to artists, although with the optical “artifact” of having a finite depth of field (DOF). It is important to note that concave mirrors also project images, but with the advantage for an artist that they maintain the parity of a scene. For this reason it seems likely that, at least in the initial period, artists used them rather than refractive lenses.

The earliest visual depiction of lenses and concave mirrors of which I am aware are in Tomaso da Modena’s 1352 paintings of “Hugh of Provence” and “Cardinal Nicholas of Rouen.”¹ Either the spectacles or the magnifying glass in these paintings would have projected an image useful for an artist. His “St. Jerome” and “Isnardo of Vicenza” both show concave mirrors as well. This shows that the necessary optics to project images of the size and quality needed by artists were available 75 years before the time of Jan van Eyck.

The examples in what follows are selected from several well-known European artists. As will be shown, in each case features are shown in portions of their works that are based on optical projections.

¹ These paintings are located in the Chapter House of the Seminario building of the Basilica San Nicolo in Treviso, Italy.

11.2 Analysis of Paintings

11.2.1 Jan van Eyck, *The Arnolfini Marriage*, 1434

One of the earliest examples we have found of a painting that exhibits a variety of evidence that the artist-based portions of it on optical projections is shown in Fig. 11.1. Several different types of optical analysis demonstrate the chandelier, enlarged in Fig. 11.2, is based on an optical projection.

The advantage of an optical projection of a real chandelier for an artist even of the skill of van Eyck is it would have allowed him to mark key points of the image. Even without tracing most of the image this would have enabled him to obtain the level of accuracy seen for this complex object that never had been previously achieved in any painting. The use of a lens results in an optical base for certain of the features, even though a skilled artist would not have needed to trace every detail in order to produce a work of art even as convincing as this one.

Since an optical projection only would be useful for certain features of any painting, and not for others, it is important to analyze appropriate aspects of the chandelier to determine whether or not they are based on optical projections. After establishing an optical base it would have been easier for van Eyck to “eyeball” many of the features [1]. As a result, paintings like the *Arnolfini Marriage* are collages consisting of both optical and non-optical elements, with even the optical elements containing eyeballed features as well [1]. Another important point is that all paintings of three-dimensional objects reduce those objects to two dimensions and, in doing so, lose some of the spatial information.

Elsewhere, based on the size of the candle flame, we estimated the magnification of the chandelier is 0.16 [6]. This means the outer diameter of the original chandelier was approximately 1 m which is consistent with the sizes of surviving chandeliers of that period. This magnification is small enough that the DOF for a lens falling within any reasonable range of focal lengths and diameters would be over 1 m. Because of this, van Eyck would have seen the entire depth of the real chandelier in the projected image without needing to refocus. Hence, if based on an optical projection the positions of the tops of each of the six candle holders should exhibit something close to perfect hexagonal symmetry after correcting for perspective. However, even if he had carefully traced a projected image there should be deviations from ideal symmetry due to the imperfections of any such large, hand-made object. If, instead, he had painted this complex object without the aid of a projection, and without the knowledge of analytical perspective that was only developed many decades later [9], larger deviations in the positions of these candle holders would be expected.

Marked with dots in Fig. 11.3 are the positions of the tops of each of the candle holders. The six-sided shape connecting them is an ideal hexagon that has been corrected for perspective. As can be seen, the agreement of the positions of the candle holders with the points of a perfect hexagon is remarkable. The maximum deviation of any of the candle holders from a perfect hexagon is only 7° , corresponding to the end of that half-meter-long arm being bent only 6.6 cm away from its “ideal” hexagonal position. Importantly, this analysis shows the arms are bent away from their “ideal” positions, but that none of them is either longer or shorter than the others. This is just what would be expected for a real chandelier. The deviations from perfect hexagonal symmetry are all on a circle, with the root-mean-square deviation only 4.1 cm. Although we shouldn’t expect a hand-made fifteenth century chandelier to exhibit accuracy greater than this, some or all of the deviations could have resulted from slight bends during fabrication, transportation, hanging, or subsequent handling.

Fig. 11.1 Jan van Eyck, *The Arnolfini Marriage*, 1434

■ Fig. 11.2 Jan van Eyck, *The Arnolfini Marriage* (detail), 1434

Although the overall chandelier is three dimensional, the individual arms are two dimensional. We devised an analysis scheme based on this, as shown in ■ Fig. 11.4 [3, 6]. In this figure we individually corrected each of the six arms of the chandelier for perspective and overlaid them to reveal similarities and differences. Where a complete arm is not shown in the figure it is because it is partially obscured by arms in front of it. While the loss of spatial information when projecting a three-dimensional object into two dimensions introduces ambiguities, the scheme we used to analyze this chandelier avoids this limitation.

After transformation of the arms to a plan view of each the main arcs are identical to within 5 % in width and 1.5 % in length. That they are the same length is consistent with our independent analysis of the radial positions of the candle holders described above [6]. However, since it would have been easier for van Eyck to eyeball many aspects of this chandelier, rather than to trace the entire projected image, it is not surprising that there are variations in the positions of the decorative features attached to those arcs.

From this evidence and other that we published [1–3, 6] we can conclude with a high degree of confidence that van Eyck's chandelier is based on an optical projection of a real chandelier. Further, the small differences provide insight into the artistic choices van Eyck made to deviate from simply tracing the projection. However, the most important point is that the unprecedented realistic perspective of this complex object is a result of an optical projection that was made over a century earlier than previously thought possible [9].

■ Fig. 11.3 *The Arnolfini Marriage* (detail). As can be seen, a perspective-corrected hexagon fits the positions of the tops of the candle holders to a remarkable accuracy, with small deviations from ideal symmetry consistent with a large, hand-made fifteenth century object

11.2.2 Lorenzo Lotto, *Husband and Wife*, 1523–1524

“Family Portrait” by Lorenzo Lotto (1523–1524) shown in ■ Fig. 11.5 provides considerable quantitative information about the lens that optical evidence indicates Lotto used in creating this painting. ■ Figure 11.6 is a detail from *Husband and Wife* showing an octagonal pattern on an oriental carpet that appears to go out of focus at some depth into the painting. Overlaid on this painting are three segments of a perspective-corrected octagon whose overall fit to the pattern is seen to be excellent, and whose quantitative details we calculate below.

As we have shown elsewhere [3, 6], based on the scale of the woman in the painting the magnification is approximately $M = 0.56$. Any optical projection at such a high magnification intrinsically has a relatively shallow DOF, the value of which depends on the focal length and diameter of the lens as well as the magnification. To change the distance of sharp focus requires physically moving the lens with respect to the subject and the image plane. To refocus an image on a region further into a scene from its original plane of focus requires moving the lens further away from the scene. This movement of the lens to refocus results in a small decrease in the magnification of the projected scene, as well as in a slight change in the vanishing points. Although such effects are fundamental characteristics of images projected by lenses, they are extremely unlikely to occur in a painting if an artist had instead laid out patterns using sighting devices or

■ **Fig. 11.4** This figure contains the outlines of all six arms on the chandelier after correcting for perspective with the arms to the viewer's right flopped horizontally to overlay on the arms to the left. The main arc of all six arms is the same to within 1.5 % in length and 5 % in width. Variations are consistent with the decorative features on the main arc having been hand attached to the original chandelier as well as having been eyeballed when creating the painting

following geometrical rules first articulated in the fifteenth century [9]. Since we already have discussed several aspects of this painting elsewhere, here we summarize our previous analysis [3, 6–8].

The distance across the wife's shoulders in the painting, compared with measurements of real women, provides an internal length scale that lets us determine the magnification to be $M \approx 0.56$. This in turn allows us to determine the repeat distance of the triangular pattern on the actual carpet to be 3.63 cm. Since the first place where the image of the carpet changes character is approximately 4–5 triangular-repeats into the scene, we calculate the depth of field to be $\text{DOF} = 16 \pm 1.5$ cm. We now can use geometrical optics to extract quantitative information from this painting.

The focal length (FL) and magnification (M) are given by the following equations from geometrical optics: [3]

$$1/FL = 1/(d_{\text{lensBsubject}}) + 1/(d_{\text{lensBimage}}) \quad (11.1)$$

and

$$M = (d_{\text{lensBimage}})/(d_{\text{lensBsubject}}) \quad (11.2)$$

Fig. 11.5 Lorenzo Lotto, *Husband and Wife*, c1523–c1524

As indicated by the overlays in Fig. 11.6, there are three regions of this octagonal pattern. These regions are the result of Lotto having refocused twice as he exceeded the DOF of his lens. We label these Regions 1, 2, and 3, with Region 1 the closest to the front of the painting. Thus, for the first two Regions,

$$1/FL = 1/(d_{\text{lensBsubject1}}) + 1/(d_{\text{lensBimage1}}) \quad (11.3)$$

and

$$1/FL = 1/(d_{\text{lensBsubject2}}) + 1/(d_{\text{lensBimage2}}) \quad (11.4)$$

However, the measured DOF is 16 ± 1.5 cm, so for Region 2

$$d_{\text{lensBsubject2}} \approx d_{\text{lensBsubject1}} + 16 \text{ cm} \quad (11.5)$$

and thus

$$1/FL = 1/(d_{\text{lensBsubject1}} + 16 \text{ cm}) + 1/(d_{\text{lensBimage2}}) \quad (11.6)$$

Because Region 2 is further into the scene it is at a slightly lower magnification than is Region 1 so its DOF will be somewhat larger than 16 cm. We can calculate DOF_2 from

■ **Fig. 11.6** *Husband and Wife* (detail). The overlays are perspective-corrected sections of an octagonal pattern that we fit to the painting. As described in the text, the details of this portion of the painting are in excellent qualitative and quantitative agreement with the three-segment, perspective-corrected octagon that is predicted by the laws of geometrical optics for such a projected image

$$\text{DOF}_2 = 2 C \times f\# \times (1 + M_2) / M_2^2 \quad (11.7)$$

where C is the circle of confusion, $f\#$ is the lens diameter/focal length, and M_2 is the magnification of Region 2. Hence,

$$\text{DOF}_2 = \text{DOF}_1 \times (1 + M_2) / (1 + M_1) \times (M_1/M_2)^2 \quad (11.8)$$

Region 3 of the pattern thus starts at a depth of $16 \text{ cm} + \text{DOF}_2$ into the scene, so

$$d_{\text{lensBsubject3}} = d_{\text{lensBsubject1}} + 16 \text{ cm} + \text{DOF}_2 \quad (11.9)$$

and

$$1/\text{FL} = 1/(d_{\text{lensBsubject1}} + 16 \text{ cm} + \text{DOF}_2) + 1/(d_{\text{lensBimage3}}) \quad (11.10)$$

The magnifications M of the three regions are given by:

$$0.56 = d_{\text{lensBimage1}} / d_{\text{lensBsubject1}} \quad (11.11)$$

$$M_2 = d_{\text{lensBimage2}} / (d_{\text{lensBsubject1}} + 16 \text{ cm}) \quad (11.12)$$

$$M_3 = d_{\text{lensBimage3}} / (d_{\text{lensBsubject1}} + 16 \text{ cm} + \text{DOF}_2) \quad (11.13)$$

This analysis gives us seven Eqs. (11.3), (11.6), (11.8), (11.10), (11.11), (11.12), and (11.13) and eight unknowns: FL, $d_{\text{lensBsubject1}}$, $d_{\text{lensBimage1,2,3}}$, DOF_2 , $M_{1,2}$. If we make a single assumption about any one of these unknowns we can then solve these equations uniquely for the other seven unknowns using simple algebra. Assuming that the distance from the lens to the carpet was at least 1.5 m, but not greater than 2.0 m (i.e., $d_{\text{lensBsubject1}} = 175 \pm 25$ cm) we find

$$\text{focal length} = 62.8 \pm 9.0 \text{ cm}$$

$$M_2 = 0.489 \pm 0.9$$

$$M_3 = 0.423 \pm 1.5$$

The magnification when moving from Region 1 to Region 2, as measured from our fit of a perspective-corrected octagon, decreases by 13.1 % from the original 0.56 of the painting, in excellent agreement with the -12.6 ± 1.5 % calculated from the above equations. Similarly, the measured magnification decreases by a further 13.3 % when going to Region 3, again in excellent agreement with the calculated value of -13.5 ± 1.6 %.

From Eq. (11.7),

$$f\# = [\text{DOF}_1 \times M_1^2] / [2 C (1 + M_1)]$$

If we assume the simple lens available to Lotto resulted in a circle of confusion on the painting of 2 mm, we find $f\# \approx 22$, and hence a diameter of 2.9 ± 0.4 cm. As we have confirmed with our own experiments, a lens or concave mirror with these properties projects a quite useful image of a subject that is illuminated by daylight.

To summarize, using only the measured magnification of this painting (0.56, i.e., roughly half life size, as determined from the size of the wife), and making a reasonable assumption about the distance Lotto would have positioned his lens from the carpet (175 ± 25 cm), equations from geometrical optics uniquely determine both changes in magnification, -13.1 and -13.3 %, of the central octagonal pattern, as well as the focal length and diameter of the lens, 62.8 ± 9.0 cm and ~ 3 cm, respectively, used to project this image. The three sets of vanishing points exhibited by the octagonal pattern, as well as the depths into the painting where they occur, are a direct consequence of the use of a lens to project this portion of the painting. Other quantitative information extracted from this painting is discussed elsewhere [3, 6–8].

Recently we developed a portable high-resolution digital camera that allows us to acquire important information about paintings without needing to remove them from museums for detailed study [7]. Since infrared light penetrates many pigments further than does visible light it often can be used to reveal “underdrawings” or other features not apparent in the visible [10, 11]. ■ Figure 11.7 is an infrared (IR) “reflectogram” of the Lotto painting captured in situ where it was located on the wall of the Hermitage State Museum in St. Petersburg. Although many features are revealed in this image, one immediate observation is we can see that Lotto used a different pigment for the woman’s dark dress than he used for the man’s jacket. This provides us with previously unknown information about the artist’s working technique.

■ Figure 11.8 shows the octagonal pattern of the table covering in greater detail. As can be seen by comparison with ■ Fig. 11.5, the red and yellow pigments Lotto used are largely transparent in the IR so this image provides an uncluttered view of the black lines he used to create this feature on the painting.

Three distinct types of markings can be clearly seen for the lines making up the triangular pattern of the border of this feature. Well-defined lines are in the region nearest the front of the image, consistent with tracing a projected image. These “traced” lines abruptly change to tentative lines in the middle region, at just the

■ Fig. 11.7 Lorenzo Lotto, *Husband and Wife*, c1523–c1524. Infrared (IR) reflectogram

depth into the scene where our previous analysis showed the magnification was reduced by $12.6 \pm 1.5\%$ due to having to refocus because of exceeding the depth-of-field. Because of this, Lotto faced significant difficulty to create a plausible match for this geometrical pattern after refocusing. His abrupt change to tentative lines reflects this difficulty. After re-establishing a plausible freehand sketch form of the geometrical pattern by the rear of this central region, the quality of the lines again abruptly changes to only short dashes in the region farthest into the scene, where our previous analysis shows the magnification was reduced by an additional $13.5 \pm 1.6\%$ due to having to refocus a second time after again reaching the limit of the depth-of-field. These results from IR reflectography provide important insights into the actual working practices of an artist, revealing quite specific details about how he made use of projected images 75 years prior to the time of Galileo.

Our analysis of this painting found a change in the vanishing point that takes place part way back in the pattern in the border of the carpet to the right, quantitatively consistent with the change that is caused by the shift in position of a lens as it is refocused. ■ Figure 11.9 shows the IR reflectogram of this portion of the painting. Overlaid to the left are seven units of a perfectly repeating structure that replicates the geometrical pattern of the border. As can be seen, after

■ Fig. 11.8 Shows the octagonal pattern of the table covering in greater detail. As can be seen by comparison with ■ Fig. 11.5, the red and yellow pigments Lotto used are largely transparent in the IR, providing us with a clear view of the black lines he used to create this feature on the painting

correcting for perspective, this structure is an excellent fit to the repeating pattern near the front of the carpet. The maximum deviation from a “perfect” fit is consistent with the degree of perfection found in the hand-made carpets of this type. Although an eighth unit of the structure does not fit at all, a small change in optical perspective makes the same repeating structure fit at the rear, again to within better than 2 mm. This change in perspective occurs at the same depth into the painting where our previous analysis found a shift in vanishing point, as

■ **Fig. 11.9** IR reflectogram of the border pattern of ■ Fig. 11.5. Overlay at left is seven segments of a repeating structure. When corrected for perspective, this is seen to be an excellent fit to the pattern at the front of the table covering. Changing the perspective, as happens when a lens is moved to refocus, gives an excellent fit to the pattern at the back. The maximum deviation of the perfect repeating structure from the pattern on the painting is 2 mm

happens when a lens is repositioned to focus further into a scene. Further, not only does the perspective change where a lens would have had to have been moved to refocus, the painting is missing a half-segment of the repeating pattern at this location. This is consistent with Lotto attempting to create a plausible match between two segments of a repeating structure after refocusing had caused the magnification and perspective to change. All of these detailed findings from IR reflectography are consistent with our other work showing this portion of the painting is based on the optical projection of an actual hand-made carpet [2, 3, 6, 7].

■ Figure 11.5 is the full image of this painting in the visible captured in situ using a standard digital camera with a 35 mm f/2 lens. This image reveals some of the difficulties with in situ image capture in a museum environment. The painting was illuminated by a combination of indirect sunlight from windows to the left, and overhead tungsten lights, each having its own color temperature. The shadows visible along the left and top borders were cast by the ornate frame in which the painting is mounted. The roughly equal darkness of these shadows indicates that the level of illumination from both types of sources was approximately equal. However, closer inspection shows that the illumination across the surface of the painting is not uniform. This can be most easily seen in the region of the man's

chest, which is too bright due to a partial specular reflection of one of the light sources that could not be eliminated by repositioning the camera within the constraints of the room.

■ Figure 11.7 is an IR reflectogram of the full 96×116 cm painting, captured under the less than ideal lighting conditions described in the previous paragraph. Although many features are revealed by this IR reflectogram, one immediate observation is that Lotto used a different pigment for the woman's dress than he used for the man's jacket, providing us with previously unknown information about the artist's working technique.

Again, all of these detailed findings from IR reflectography are consistent with our earlier work that showed this portion of the painting is based on the optical projection of an actual hand-made carpet. I note that we have used fourteenth century optical technology (i.e., one lens of a pair of reading spectacles, as well as a metal concave mirror we fabricated following descriptions in texts of the time) to accurately reproduce all of the effects we have found in this carpet, as well as in all of the other paintings we have shown to contain elements based on optical projections, including projecting such patterns directly on a screen of the same shade of red used in this painting. Even on such a colored screen, the projected images are quite distinct and easy to trace.

11.2.3 Hans Holbein the Younger, *The French Ambassadors to the English Court*, 1532

A prominent feature of *The French Ambassadors to the English Court* by Hans Holbein is the anamorphic skull at the bottom of the 1532 painting. This feature is shown in ■ Fig. 11.10. The way this appears to someone viewing it at a grazing angle is shown by linearly compressing it by $6\times$ in ■ Fig. 11.11 (Right), with a real skull for comparison in ■ Fig. 11.11 (Left).² Very obvious differences include that the jaw of Holbein's skull is much longer than the real skull, the slope of the top of the skull is steeper, and the eye sockets and nose are much more pronounced as well as aimed more in the direction of the viewer.

To see if optical projections may account for the appearance of this skull in the painting, we used a concave mirror of focal length 41 cm to project the image of a real skull onto a screen at a grazing angle in order to produce an anamorphic image. ■ Figure 11.11 (Left) is a photograph of the real skull taken from precisely the location of that concave mirror after the mirror had been removed from its holder. However, because of the limited depth of focus of the projected image on

2 The anamorphic skull is 106 cm long and 14.4 cm high. To visually compress its length to be the same as its height so that it appears approximately like ■ Fig. 11.11 (Right) requires viewing the painting at a grazing angle of $\sin^{-1}(14.4/106) \approx 8^\circ$. At this angle the far end of the anamorphic feature is over 100 cm further away from the viewer than is the near end, so that for reasonable viewing distances the magnification of the far end is significantly less than that of the near end. Also, since for any reasonable viewing distance the depth of the feature is greater than the depth of field of the eye, it requires the viewer to scan back and forth through the feature, with their eyes constantly refocusing when doing so, in order to "construct" a composite image in their mind that does indeed strongly resemble ■ Fig. 11.11 (Right). Although our analysis shows that this anamorphic feature was constructed with the aid of optical projections, the multiple positions of the lens needed to generate it, coupled with the multiple movements and refocusing of the eye needed to view it, along with the mental compositing need to construct the final image of it in the brain, results in an underlying complexity to ■ Fig. 11.11 (Right). For these reasons, because ■ Fig. 11.11 (Right) was generated by a linear transformation, it only approximately reproduces what the feature looks like to the viewer when examining the painting from a grazing angle.

■ **Fig. 11.10** *The French Ambassadors to the English Court* (detail). This detail shows the unusual feature at the bottom of Holbein's painting. Viewed from a grazing angle to visually compress it, this feature appears as shown in ■ Fig. 11.11 (Right). Possibly not apparent in this small B&W reproduction is that this anamorphic skull does not occupy the same visual space as the rest of the painting

■ **Fig. 11.11** (Left) Photograph of a skull taken from the position of the concave mirror used to project its image onto a tilted screen to form an anamorphic image. (Center) Composite of the individual in-focus segments of the projected anamorphic image of the skull after linearly compressing it horizontally. (Right) Anamorphic skull in *The French Ambassadors* after linearly compressing it horizontally

the tilted screen, it was necessary to refocus the concave mirror a number of times in order to generate the composite anamorphic image that we have compressed linearly to produce ■ Fig. 11.7 (Center).

The segments of each of the in-focus images are visible in this composite. What is striking about ■ Fig. 11.11 (Center) is how well it reproduces the very unusual visual appearance of the linearly compressed skull from Holbein's painting. Although mathematical and graphical methods can be used to construct anamorphic images, the optics-produced composite of ■ Fig. 11.11 (Center) is far more complex than is obtained from any such construction. The magnification of each segment in the anamorphic photographic composite is linear in the vertical direction, but is proportional to $1/\sin$ of the grazing angle in the horizontal. The overall composite of ■ Fig. 11.11 (Center) is thus the result of a nonlinear,

■ **Fig. 11.12** Anamorphic skull in *The French Ambassadors to the English Court*. For this figure we have rotated the feature in ■ Fig. 11.10 clockwise by 25° and then linearly compressed it by $6\times$. The height of the skull in this image compared to a real one gives a magnification $M = 0.71 \pm 0.5$. The lines indicate two regions where it can be seen Holbein duplicated features (notably, the two dark depressions just above the jaw, and the double-humped line midway up the skull). A discontinuity in the slope of the top of the skull is also visible at the left edge of the leftmost marked region

piecewise-segmented transformation. Although this complex transformation was naturally produced by the optical projection, it would be quite implausible to have resulted from any sort of a graphical or mathematical construction [9]. We conclude that the probability is extremely small that Holbein could have accidentally reproduced these complex features without having projected them with a lens.

■ Figure 11.12 shows ■ Fig. 11.11 (Right) at a larger scale. Marked on this figure are two regions where we observed that Holbein has duplicated features of the skull. Because the lens and canvas (or, less likely, the skull) has to be moved a number of times when piecing together an anamorphic image from segments projected at such a high magnification, it is very easy to accidentally duplicate a region, so its presence provides additional evidence that Holbein had to refocus a

lens. The duplicated segment corresponds to a region 3.0 ± 0.5 cm wide on a real skull. That same region corresponds to a width of 8.2 cm on the actual painting which gives us an approximate lower limit measure for the depth of focus. From the results of our experiments shown in ■ Fig. 11.11 (Left) and (Center), that region of the skull is at an angle of $25^\circ \pm 5^\circ$ with respect to the perpendicular to the axis of the lens, so its depth into the scene is 1.3 ± 0.5 cm. Although a more accurate value for the depth of focus can be obtained by convoluting this measured DOF into the calculation, for our purposes here the approximate value 8.2 cm will suffice. Using this value, along with a circle of confusion of 2 mm and the measured $M = 0.71$, we calculate as a lower limit

$$\begin{aligned} f\# &\geq \text{Depth of Focus} / [2C \times (M + 1)] \\ &= 12.0 \end{aligned}$$

Because we have neglected the DOF in the calculation shown here, this value for the $f\#$ of Holbein's lens is somewhat smaller than the actual value, as well as represents a lower limit. However, this calculation is sufficient to show that the $f\#$ of Holbein's lens is consistent with the values we obtained for Lotto's and Campin's lenses (22 and 25.2, respectively).

11.2.4 Robert Campin, *The Annunciation Triptych (Merode Altarpiece)*, c1425–c1430

Robert Campin was a contemporary of Jan van Eyck and they are documented to have known each other. The center and right panels of Robert Campin's *Merode Triptych* of c1425_B28 contain the earliest evidence we have found to date of the use of direct optical projections. A detail of the right panel is shown at the lower left of ■ Fig. 11.13. As we previously showed, this portion of the painting exhibits the same complex changes in perspective seen in Lorenzo Lotto's *Husband and Wife*, resulting from Campin also having refocused his lens twice [4, 6].

The upper right in ■ Fig. 11.13 shows one of the two sets of slats (the set that is numbered on the lower inset), with each slat individually rotated to be vertical and expanded horizontally by a factor of $3.5\times$ to accentuate any deviations from being straight. Marked on the slats are the locations of "kinks" exhibited by each of them, with those kinks connected by lines. The positions of the lines connecting the kinks are shown on the inset at the lower left. Comparing with ■ Fig. 11.2 of Reference 5 it can be seen that the slats are kinked at the same two depths into the painting where we previously showed, with a different type of analysis using different data, that Campin had to refocus due to the DOF of his lens. Geometrical constructions can be devised which exhibit kinks, but not in the overall configuration of this painting. The complex perspective exhibited by the latticework in this portion of the painting is a direct and inevitable outcome from the DOF of a lens, but would be extremely unlikely to have resulted from any geometrical construction, or from the use of a straightedge.

Using the height of the head in the full painting as a scale, the magnification of this portion of the painting is $M \approx 0.27$. If we assume a circle of confusion of 1 mm Eq. (11.7) yields $f\# = 25.2$. We can obtain an estimate for the focal length with the assumption the lens or concave mirror had a diameter of 3 cm, in which case the focal length $FL = f\# \times 3 \text{ cm} = 76 \text{ cm}$, which is quite reasonable.

■ **Fig. 11.13** (Lower Left) Detail of the *Merode Altarpiece* with one set of slats numbered. (Upper Right) Slats rotated to be vertical and expanded horizontally by $3.5\times$. We have connected the “kinks” that are apparent in the slats by lines, the locations of which are shown in the detail at the lower left

11.3 Conclusions

These discoveries demonstrate that highly influential European artists used optical projections as aids for producing some of their paintings early in the fifteenth century, at the dawn of the Renaissance, at least 150 years earlier than previously thought possible. In addition to van Eyck and Lotto we have also found optical evidence within works by well-known later artists including Bermejo (c1475), Holbein (c1500), Caravaggio (c1600), de la Tour (c1650), Chardin (c1750), and Ingres (c1825), demonstrating a continuum in the use of optics by artists, along with an evolution in the sophistication of that use. However, even for paintings where we have been able to extract unambiguous, quantitative evidence of the direct use of optical projections for producing certain of the features, it does not mean that these paintings are effectively photographs. Because the hand and mind of the artist are intimately involved in the creation process, understanding these images requires more than can be obtained from only applying the equations of geometrical optics. As to how information on optical projections came to these artists, evidence points to it having come via the Cairo-based scholar Ibn al Haytham [12].

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