Measurement—Focusing Especially on Primary Education

JeongSuk Pang and Kees Buijs

Preparation

Measurement, as well as related topics of geometry, forms an important mathematics domain on the level of both primary school and pre-vocational secondary school in many countries. At this level it relates primarily to quantifying certain aspects of real world physical objects such as the length, area, capacity, weight/mass, temperature or volume of objects, and to the reconstruction and application of the current measuring systems in a country (metrical or non-metrical). It also includes the use of measuring instruments such as the folding ruler and measuring tape, the measuring jug and the kitchen scale. Related geometrical topics include understanding of and working with the concept of scale, and the reconstruction and application of formulas for the area of a rectangle, triangle and other geometrical figures.

TSG-8 addressed researchers, curriculum developers, and reflective practitioners (teachers) working in the field of measurement and related geometry on the level of primary school. It aimed at providing a forum for generating discussion, exchanging insights, and establishing a state of the art sketch of the domain, including indications for the status of measurement as a foundation for advanced mathematics domains.

The TSG-8 organizing team called for papers dealing with various aspects of measurement such as theoretical perspectives on mathematical growth of students' thinking related to measurement, the development of measurement sense in students, connections between measurement and related domains such as number

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Organizers Co-chairs: Jeong Suk Pang (Korea), Kees Buijs (Netherlands); Team members: Olimpia Figueras (Mexico), Silke Ruwisch (Germany), Andrea McDonough (Australia); Liaison IPC member: K. (Ravi) Subramaniam (India).

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sense and decimal numbers, curriculum development and implementation related to measurement, instructional approaches to foster students' development related to measurement, and culturally defined tools and practices for measurement and cultural supports for the learning and teaching of measurement.

Each of the 15 proposals which we had received was carefully and rigorously evaluated by three reviewers from the TSG-8 organizing team members with the support of K. Subramaniam. Having further discussed the initially accepted proposals amongst the TSG-8 team members, four papers were accepted for long oral presentation (30 min of presentation and 10 min of discussion) and eight papers for short presentation (15 min of presentation and 5 min of discussion). The remaining three papers were recommended for poster presentations during the general poster sessions of the ICME-12. Due to a cancellation, the final program of TSG-8 consisted of four long oral presentations and seven short ones.

We organized the accepted papers into four 90-minute sessions as follows:

- Session 1: Students' difficulties and teaching methods (July 10th),
- Session 2: Curricular materials and teaching methods (July 11th),
- Session 3: Delving into students' understanding (July 13th),
- Session 4: Measurement instrument and its use (July 14th)

Implementation

Session 1: Students' Difficulties and Teaching Methods

The first session was chaired by the co-chairs of TSG-8. At the beginning of the session, JeongSuk Pang from Korea welcomed all participants and introduced the organizing team members. Kees Buijs from Netherlands then delivered introductory remarks, showing a series of pictures taken in Seoul and related them to demonstrate measurement in a daily life.

Three papers were presented in this session (one long presentation and two short presentations) and vivid discussion was followed. First, Yah Hui Tan and Meng Hua Chua from Singapore investigated students' difficulties in learning the concepts of length and mass, and examined how teachers' use of an adapted version of the Kolb's *experiential learning cycle* was helpful to address their students' difficulties. They addressed the importance of using various measurement tools to assess students' understanding and misconceptions of measurement concepts.

Second, JeongSuk Pang, JeongWon Kim, and HyeJeong Kim from Korea identified key instructional elements in teaching measurement by comparing and contrasting two sets of measurement teaching practices which were recognized as good instruction in Korea. This presentation raised an issue on what counts as effective *measurement* instruction.

Third, Wayne Hawkins from Australia presented four primary teachers' pedagogical content knowledge in teaching measurement to students in Years 3 and 4. By exploring teachers' knowledge of mathematics along with knowledge of students and teaching, Wayne helped the audience understand the complex nature of pedagogical content knowledge and provoked a discussion on the dynamic nature of such knowledge.

Session 2: Curricular Materials and Teaching Methods

The second session was chaired by the TSG-8 organizing team member, Olimpia Figueras from Mexico. Three papers were presented in this session (one long presentation and two short presentations) and insightful issues were discussed afterwards. First, JeongSuk Pang, SuKyoung Kim, and InYoung Choi from Korea reported a comparative analysis of the statements in two Korean elementary mathematics textbook series in terms of two coding criteria: degree of guidance and key learning elements of the measurement domain. This presentation suggested the need of re-conceptualizing key learning elements of measurement as well as the possibility of developing a new coding system for textbook analysis. Several participants showed their interest in using this coding system in analyzing their textbooks.

Second, Silke Ruwisch from Germany presented third grade students' understanding of capacity and proposed the need for explicit comparison and measurement actions with many different containers before building up mental representation.

Third, Jeenath Rahaman from India presented different ways in which multiplicative thinking was involved in the measurement of area. She shared some tasks that had prompted students to use multiplicative thinking in finding the area of given figures. This also gave the participants an opportunity to reflect on the importance of designing tasks to explore the connection between multiplicative thinking and measurement of area.

Session 3: Delving into Students' Understanding

The third session was chaired by the TSG-8 organizing team member, Silke Ruwisch from Germany. Three papers were presented in this session (one long presentation and two short presentations) and thought-provoking issues were raised. First, Kees Buijs from Netherlands reported gaps between the informal and formal knowledge of 13–14 years old pre-vocational students, and suggested some ways to bridge such gaps. This presentation provided unique information mainly because of the characteristics of the students who had participated in this study. Despite their reasonable knowledge of measurement units and basic measurement sense, the difficulties that

students had in solving more theoretical measurement problems were striking. As such, this presentation addressed a core issue in designing a measurement curriculum.

Second, Oyunaa Purevdorj from Mongolia presented second grade students' difficulties in understanding the given word problem, drawing a rectangle, and finding out the perimeter of a rectangle, and attributed the causes of such difficulties to the ways curriculum and textbook were designed, and the ways that teachers taught them in the country. This presentation helped participants understand the close relationship among curricular documents, teaching methods, and students' learning outcomes.

Third, Andrea McDonough from Australia reported on a design experiment to teach lower primary students about the measurement of mass. By illustrating multiple tasks and hands-on lessons in which students were expected to focus on the key measurement understandings of comparison and unit, Andrea prompted the audiences to grasp how to maximize the opportunity to learn the measurement of mass.

Session 4: Measurement Instrument and Its Use

The final session was chaired by the TSG-8 organizing team member, Andrea McDonough from Australia. Two papers were presented in this session (one long presentation and one short presentation) and general discussion was followed. First, K. Subramaniam from India presented measurement units and modes in the Indian context. He illustrated unique informal measurement units and multiple modes of quantification that are still being used in the Indian context. The presentation raised issues of how to design the school mathematics curriculum to incorporate students' practical knowledge of measurement and measurement sense.

Second, Bona Kang from USA reported four emerging sociomathematical norms regarding linear measurement and then the students' meaningful shift to use rigid tools. As such, she suggested the positive impact of social processes on the students' use of informal tools in measurement. This presentation raised an issue of a reflexive relationship between social and cognitive processes in measurement activity.

The final session was closed by two co-chairs. They appreciated all the participants who presented their studies, engaged in a rich discussion, and provided comments throughout the four sessions.

Reflection

The adequate number of papers presented in each session enabled TSG-8 to have an opportunity for participants to present their results, share ideas, and discuss issues within an affordable time frame. On the one hand, such an opportunity was effective

in comparison to other TSGs because they had to run parallel sessions at the same time to provide more opportunities to present papers but had difficulties in sharing participants' ideas as a whole group. On the other hand, it was surprising that not many papers were submitted to TSG-8, even though measurement and related geometry are considered as an essential part of the mathematics curriculum especially at primary level in many countries. One reason might be a lack of attention to this domain. Another reason might be that a number of proposals were submitted to other TSGs by drawing more attention to the genre of research rather than the content domain of mathematics.

Generally speaking, TSG-8 had regular attendants who were ready to bring up rich discussion within a permissive atmosphere throughout the four sessions. Despite the relatively small number of papers presented in this group, a number of important issues came up and participants agreed the necessity of further international comparative studies in the domain of measurement. We hope that the topic study group dealing with measurement continues to serve a well-recognized group of the congress.

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