Bio-inspired Aging Model-Particle Swarm Optimization and Geometric Algebra for Structure from Motion

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Abstract. On computer vision field Structure from Motion (SfM) algorithms offer good advantages for numerous applications (augmented reality, autonomous navigation, motion capture, remote sensing, object recognition, image-base 3D modeling, among others), nevertheless, these algorithms show some weakness; in the present paper we propose the use of Bio-inspired Aging Model-PSO (BAM-PSO) to improve the accuracy of SfM algorithms. The BAM-PSO algorithm is used over a Geometric Algebra (GA) framework in order to compute the rigid movement on images and this allows us to obtain a numerically stable algorithm.

Keywords: SfM, Conformal Geometric Algebra, PSO.

1 Introduction

Computer vision has been a very active research field in the last decades, but the problems to be solved have proven to be difficult. Nevertheless, new paradigms improve the robot capabilities in the field. The present paper proposes the use of evolutionary computation over a geometric mathematical framework to improve some aspects of Structure from Motion (SfM) Algorithms.

Structure from Motion represents a family of computer vision algorithms whose paradigm is based on extracting structures when movement is detected or extracting movement when structures are detected using 2D correspondences from multiple images, [7,3]. There are several approaches for SfM algorithms, for example, the use of monocular camera systems for 3D reconstructions [10] or segmentation from multi-body motion [10].

Most of two view SfM algorithms are based on the epipolar restriction (Eq. 1) between at least, two images of the same scenario: image- i and image- j

$$x'Ex = 0 \quad (1)$$

where $E = \hat{T}R$ is called the *essential matrix* (i.e. the matrix which relates corresponding points in stereo images assuming that the cameras satisfy the pinhole camera model), therefore E represents the rigid motion between image- i

and image- j , $\hat{T}$ is the skew-symmetric matrix that contains the translation and R is the rotation matrix, x stands for a point on image- i , while x' represents the corresponding point for x on image- j . The computation of this matrix can be hard to achieve, and this is the main difference between SfM algorithms.

Some SfM algorithms compute the *essential matrix* directly using for example an 8-point algorithm, the result is limited by the calibration of the camera, [7]. Most of SfM compute the *fundamental matrix* through 8-point, RANSAC or others algorithms [7], [13], and they get a weak perspective bifocal tensor, after that, we can compute the *essential matrix* with the intrinsic parameters, or use a self-calibration technique. Other SfM algorithms use trifocal ([2]) or n-focal tensors ([12], [3]) to get a stronger perspective which is very useful, although, they are computationally expensive and they also depend on the calibration process.

In this work, we propose a novel algorithm to compute the movement which defines the *essential matrix* by using a variation of the evolutionary computation algorithm called Particle Swarm Optimization (PSO) [5] combined with the mathematical framework known as Geometric Algebra (GA) in order to find the motor (rotation and translation versor) that represents the motion between views. Evolutionary computation algorithms like PSO offer a fast convergence in solving optimization problems, and using them on SfM, can provide small reprojection errors respect to others reconstruction approaches.

The main task on PSO algorithms is to find a suitable objective function; using a mathematical framework such Geometric Algebra, we can address this problem with the advantage of using explicitly different geometric entities as 2D image correspondences (besides points, we can define planes, spheres, circles, etc. and their operators using the same algebra). By using geometric entities which are more complex than points we can access to richer information of the structure of the environments and therefore we can made more accurate 3D reconstructions as it is shown in Sec. 6. The use of GA framework also allows us to obtain a numerically stable algorithm: the computation of the rigid movement is done with a versor representation called motor (M), this includes the rotation and translation movement in its definition, and it provides an uniquely representation of the (scaling) rotation transformation in 3D, which does not happen when Euler angles or matrix algebra are used, because the three Euler angles from which a rotation matrix can be constructed do not suffice by themselves [11].

This paper is organized as follows: In Sec. 2 is presented the evolutionary computation algorithm used in order to obtain the motion transformation between two views. In Sec. 3 it is given a brief introduction to Conformal Geometric Algebra (CGA). Sec. 4 shows a SfM algorithm. Sec. 5 presents our algorithm, proposed to solve SfM problems. In Sec. 6 we show the results obtained in reconstruction 3D environments using our approach and some comparatives with other SfM algorithms, while Sec. 7 is devoted to conclusions and future work.

2 Bio-inspired Aging Model-PSO

Particle Swarm Optimization [5] is an evolutionary algorithm based on observations of bird flocks and insect swarm behavior that provides high convergence speed and shows excellent results solving many of modern engineering problems [8]. The PSO algorithm consists of an iterative adaptation of set of multidimensional vectors called particles that communicate information among them as a swarm which provides a set of candidate solutions for an objective function. Recently, there has been an increasing interest on developing improvements to the PSO algorithm. These improvements intent to alleviate the premature convergence problem of the original PSO algorithm. In this work, it is used one of these new proposed algorithms called BAM-PSO [1] due to it has proved to solve the above mentioned problem and improve the accuracy of the results obtained with other PSO variant algorithms.

The BAM-PSO algorithm is an evolutionary computation algorithm based on the Aging Leader and Challengers (ALC-PSO) algorithm [9]. The basic idea of both algorithms is to include the aging process to each particle of the swarm. Meanwhile, ALC-PSO uses a linear aging model, BAM-PSO proposes to use a bio-inspired aging model.

Each particle in the swarm has a velocity v_{ij} Eq. 2 and a position x_{ij} Eq. 3 and these quantities are adjusted in each iteration of the algorithm as follows:

$$v_{ij}(t+1) = w * v_{ij}(t) + c_1 R_1(p_{ij}(t) - x_{ij}(t)) + c_2 R_2(leader + x_{ij}(t)) \quad (2)$$

$$x_{ij}(t+1) = x_{ij}(t) + v_{ij} \quad (3)$$

where i is the i th particle of the swarm, j is the j th element of dimension problem, t is the iteration counter, w represents the inertial weight factor (constant), R_1 and R_2 are random, normalized and uniformly distributed values, c_1 and c_2 represent the social and cognitive parameter, x_{ij} is the particle ij position for the iteration t ; $x_{ij}(t+1)$ is the particle ij position for $t+1$ iteration, $v_{ij}(t)$ is the particle ij velocity for t iteration. $p_{ij}(t)$ represents the local best position for particle ij in iteration t and *leader* is the particle holding the best solution found by the swarm at iteration t .

3 Conformal Geometric Algebra

Geometric Algebra also known as Clifford Algebra, in fact represents a family of algebras that depend on a chosen vector space and on a special kind of product (the geometric product). The GA is a mathematical framework where we can find embedded concepts from linear algebra, tensor algebra, quaternion algebra, complex numbers and others. We represent a geometric algebra with $G_{p,q,r}$ ¹.

The Conformal Geometric Algebra (CGA) is a 5-D Geometric Algebra that uses a conformal transformation to span euclidean geometric algebra represented

¹ For a more extensive introduction to Geometric Algebra please see [4], [11].

by G_3 , to $G_{4,1}$. One of the advantages of use more dimensions is that problems are intuitively and easily formulated.

In CGA we use the Euclidean basis vectors e_1, e_2, e_3 that we can span with a conformal transformation over the Minkowski's plane with the basis e_-, e_+ that holds the Eq. 4. We can define another basis called null with Eq. 5

$$e_+^2 = -1, \quad e_-^2 = 1, \quad e_+ \cdot e_- = 0. \quad (4)$$

$$e_0 = \frac{1}{2}(e_- - e_+), \quad e_\infty = e_- + e_+. \quad (5)$$

Where e_0 represents the 3D origin and e_∞ represents the point to the infinity, they are null vectors and hold the Eq. 6

$$e_0^2 = e_\infty^2 = 0, \quad e_\infty \cdot e_0 = -1. \quad (6)$$

With this basis, geometric entities are represented as multivectors, in fact, there is a dual representation for every entity; the inner product null space representation (IPNS) and the outer product null space representation (OPNS).

The basic geometric entities and their representations are listed in the Table 1; where x_E stand for an euclidean point, x_C for a conformal point, r for the radius of the sphere, n a unitary vector and d a distance from the origin (Hesse representation of the plane) and finally $\mathbf{n}$ and $\mathbf{m}$ stand for the Plucker coordinates of the line.

Table 1. Geometric entities on CGA

Entity	IPNS	OPNS
Point	$x_C = x_E + \frac{1}{2}x_E^2 e_\infty + e_0$	$x_C^* = s_1 \wedge s_2 \wedge s_3 \wedge s_4$
Sphere	$s = x_E - \frac{1}{2}(x_E - r)^2 e_\infty + e_0$	$s^* = x_1 \wedge x_2 \wedge x_3 \wedge x_4$
Plane	$P = n + d e_\infty$	$P^* = x_1 \wedge x_2 \wedge x_3 \wedge e_\infty$
Line	$L = \mathbf{n}I_E - e_\infty \mathbf{m}I_E$	$L^* = x_1 \wedge x_2 \wedge e_\infty$
Circle	$C = s_1 \wedge s_2$	$C^* = x_1 \wedge x_2 \wedge x_3$

We can use the same basis to represent rigid transformations, in general any conformal transformation can be expressed as Eq. 7.

$$\sigma X' = H X \tilde{H} \quad (7)$$

where $X, X' \in G_{4,1}$, H is a versor and σ a scalar, σ is apply to ensure that the null cone is invariant under H .

Rotations are represented by *Rotors* and they are defined in form of Eq. 8.

$$R = e^{-\frac{\theta}{2}\mathbf{n}} \quad (8)$$

where $\mathbf{n}$ represents the dual entity of the rotation axis and θ represents the rotation angle. We also define the *translator* in the form of Eq. 9.

$$T = 1 + \frac{1}{2}te_{\infty} \quad (9)$$

t is the euclidean translation. The null cone is invariant under R and T so $\sigma = 1$ then the rigid transformations are represented by Eq. 10.

$$X' = RX\tilde{R}, \quad X' = TX\tilde{T}. \quad (10)$$

We can use the last equations to express a rigid body motion, we denote such operator and the conformal transformation with Eq. 11.

$$M = TR, \quad X' = MX\tilde{M}. \quad (11)$$

This equation of rigid movement can be applied on every entity of the algebra.

4 Structure from Motion

In this section, a SfM algorithm is shown as reference framework for our proposed algorithm ([3]).

One important step in SfM is to solve the correspondence problem, but this is out scope of this work, therefore, the correspondences will be defined by a supervised method. The SfM algorithm is shown in Algorithm 1.

Algorithm 1. Structure from motion

Require: Correspondences of images, Calibration matrix (K).

Ensure: Rotation and Translation between images.

1: Use the correspondences to calculate the *fundamental matrix* F {using RANSAC}

2: Calculate the *essential matrix* with $E = K^T F K$

3: Decompose $E = \hat{T}R$ on its singular values $E = U\Sigma V^T$ {to extract the movement}

4: Use the $W = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ or W^T defined on [3] to extract the movement with

$$\hat{T} = VW\Sigma V^T \text{ and } R = UW^{-1}V^T.$$

5: Select supervisely the correct movement from the fourth possible answers {There are possibles solutions due use of W or W^T , see [3]}

5 Bio-inspired Aging Model-PSO with Geometric Algebra for SfM

Object mapping approach allows a compact representation of the environment, based in the idea that we have a map with geometric entities (that could be obtained by the Hough Transform as shown in [6]); it is possible to use BAM-PSO over a CGA to estimate the motion of the entities.

To establish a search space for the BAM-PSO, we need to define the parameters on the rigid transform with the versor M expressed with the Eq. 12 where

T stands for the translation (see Eq. 9) and R_x , R_y and R_z for the rotation on x , y , and z axes respectively (see Eq. 8).

$$M = TR_zR_yR_x \quad (12)$$

In order to operate the M versor we define α_x , α_y and α_z as the rotation angle of R_x , R_y and R_z respectively and t_x , t_y and t_z as the euclidean translation on T . Therefore, the parameters of the search space can be define like Eq. 13.

$$\chi = \{\alpha_x, \alpha_y, \alpha_z, t_x, t_y, t_z\} \quad (13)$$

This search space will be bounded for the first three parameters by $[-\pi, \pi]$. Using the RMSE as error the fitness function for BAM-PSO is defined in Eq. 14.

$$f_O = \min_{\chi} \sum_{k=1}^q \sqrt{\frac{\sum_{l=1}^s (P_l(E'_k) - P_l(ME_k\tilde{M}))^2}{n}} \quad (14)$$

Where the function $P(\cdot) : G_{4,1} \rightarrow \mathbb{R}^n$ extract the parameters of the geometric entities, q and s represent the number of entities in the object and the number of parameters in the entity respectively. For a conformal point, sphere, plane or line $P(\cdot) = \{x_E\}$, $\{p\}$, $\{n, d\}$ or $\{\mathbf{n}, \mathbf{m}\}$ respectively, (see Table 1). It is important to note that the objective function not only works with basic geometric entities, we can also define multi-entities as rigid objects.

6 Results

Consider the couple of images in Fig. 1 as the input for a SfM algorithm. For comparative purposes lets define 5 interest objects extracted from the human figure on the images: 1) Head and truck, 2) right arm, 3) left arm, 4) right leg, 5) left leg. For SfM we defined supervised correspondences of each object.

For the BAM-PSO for SfM it is needed to extract objects from Fig: 1 in a supervised way to get geometric entities and their correspondences (we use stereo vision to create a 3D cloud point) then, we can form the map of geometric entities shown in Fig: 2.a) using the GA framework and definitions shown in Tab. 1.

The results of BAM-PSO for the calculate of the motion is shown on Fig: 2.b) that manifest the error minimization over 200 generations, it's easy to see the fast convergence to a small error. Finally, on Tab: 2 is presented a comparative between BAM-PSO with CGA for SfM and classic SfM; measuring time, PSO error (error reached for BAM-PSO algorithm) and an error that stands for the accuracy of the method described in the Eq. 15 where x_i is a new 3d point in the object on the initial frame, x_f the same point on the final frame and H stand for the homogeneous transform.

$$E = ||x_f - Hx_i|| \quad (15)$$

Fig. 1. Input images

Fig. 2. BAM-PSO. (a) Map of geometric entities, (b) Error function over generations.

Table 2. Comparative between BAM-PSO with CGA for SfM and classic SfM

Object	BAM-PSO with CGA for SfM			Classic SfM	
	Time (s)	PSO error	E (Eq: 15)	Time (s)	E (Eq: 15)
Head and Trunk	3.8165	2.2087e-04	0.0002	2.1451	1.4072
Right arm	2.0896	0.0421	0.0094	2.0855	1.08575
Left arm	5.8751	0.0372	0.0103	2.0468	1.9581
Right leg	4.5828	0.0164	0.5854	1.9643	3.5366
Left leg	6.2066	0.0266	0.1984	1.9201	3.8789

7 Conclusions and Future Work

As we already proved, the use of BAM-PSO over geometric entities represented on CGA improve the accuracy of SfM algorithms, the use of entities that are richer of structure information provides an error decrease and numerical stability for SfM. The future work based in this paper will include the parameterization of the circle (As the only entity missing on Sec. 5 and the process automation with hough transform as is shown on [6]. Also a parallelization of BAM-PSO algorithm will be implemented in order to reduce the processing time of SfM.

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