Wavelet Filter Adjusting for Image Lossless Compression Using Pattern Recognition

Oleksiy Pogrebnyak, Ignacio Hernández-Bautista, Oscar Camacho Nieto,
and Pablo Manrique Ramírez

Instituto Politecnico Nacional, Centro de Investigacion en Computacion,
Ave. Miguel Othón De Mendizábal S/N, C.P. 07738, Mexico, D.F., Mexico
olek@pollux.cic.ipn.mx, ignaciohb@gmail.com,
{ocamacho, pmanriq}@cic.ipn.mx

Abstract. A method for image lossless compression using lifting scheme wavelet transform is presented. The proposed method adjusts wavelet filter coefficients analyzing signal spectral characteristics to obtain a higher compression ratio in comparison to the standard CDF(2,2) and CDF(4,4) filters. The proposal is based on spectral pattern recognition with 1-NN classifier. Spectral patterns of a small fixed length are formed for the entire image permitting thus the global optimization of the filter coefficients, equal for all decompositions. The proposed method was applied to a set of test images obtaining better results in entropy values in comparison to the standard wavelet lifting filters.

Keywords: image compression, lifting scheme, wavelets, pattern recognition.

1 Introduction

In the past two decades, the wavelet transform has become a popular, powerful tool for different image and signal processing applications such as noise cancellation, data compression, feature detection, etc. Nevertheless, a problem to choose or design the appropriate wavelet for a given application is still present. Different to many known orthogonal transforms, wavelet transform can use various basis functions and it is important that the chosen wavelet basis provides the best representation of the analyzed signal. For example, in signal detection applications, since the wavelet transform calculates the cross-correlation between the signal and wavelet, the wavelet matched to signal results in a greater peak in transformed domain. In signal compression applications the match results in better representation of the signal by wavelet, and therefore, in a better compression ratio.

The standard technique of finding orthonormal wavelet bases with compact support is Daubechies' classic technique [1], and in the case of biorthogonal wavelet bases the classical solution for univariate compactly supported wavelets of arbitrary high regularity is given by Cohen, Daubechies and Feauveau in [2]. Unfortunately, these techniques are not practical because they are complex and independent of the analyzed signal.

The first attempt at finding the optimal orthonormal wavelet basis for speech signals using parameterization was proposed by A. Tewfik *et al* in [3]. This method is limited to a finite number of scales and the optimization is performed in time domain in an iterative manner. Other existent design techniques mostly do not design the wavelet directly. Some techniques use a library of previously designed wavelets [4], some methods match the wavelet bases to the signal projecting the signal onto existing wavelet basis or transforming the wavelet basis [5].

Among the recent methods of wavelet matching those proposed by Chapa and Rao [6] and A. Gupta *et al.* [7] are the most notable. In [6], the authors adapt the generalized Meyer's wavelet minimizing the difference between wavelet and signal spectra, but their method is computationally expensive in phase matching and is designed for deterministic signal detection. In [7], the method of estimating of an analysis high pass wavelet filter is proposed for signals, which represent self-similar processes and their autocorrelation can be modeled analytically using the self-similarity index. Although authors state their method is simple, it requires in a self-similarity index a numerical search of a maximum likelihood estimator. Besides, the method yields filter banks without perfect restoration in general.

In this paper, we consider the application of wavelet transform for lossless image compression. For such uses, lifting scheme proposed by W. Sweldens [8] is a standard method for integer-to-integer transforms [9]. Lifting wavelet filters were obtained from known biorthogonal wavelet filters factoring the polyphase matrices [8], [10]. For lossless image compression, one step lifting filters CDF(2,2) and CDF(4,4) [9] are the most popular.

Various attempts of improving the performance of the lifting filters are known [11], [12], [13], [14], [15]. In common, the authors of these works try to optimize the lifting predictor minimizing the mean square prediction error at the output of the predictor in case of one step lifting, i.e., minimizing the energy of wavelet coefficients. With this, authors [11], [14], [15] obtain (sometimes, depending on the data) positive results if the combination with other improvements is used, but H. Thielemann [12] reported negative results for such least mean square optimization in comparison to CDF(2,2) performance. It is interesting to note, that the method of lifting predictor optimization presented in papers [14], [15] uses signal differences to compute the autocorrelation on them and not the standard calculation method.

An approach alternative to prediction error optimization was proposed in [16]. Analyzing the distribution of the wavelet coefficient distribution, the authors found that their distribution is well described by the generalized Gaussian distribution with the shape parameter close to 1 than to 2. As a result, they suggested the minimization of ℓ_1 -norm instead of ℓ_2 -norm, i.e., minimize the absolute value of error instead of error energy. Additionally, the update filter optimization by minimizing of ℓ_2 -norm of the difference between the approximation signal and the decimated version of the output of the ideal low-pass filter was proposed. Unfortunately, the minimization of ℓ_1 -norm is much more complex and requires sophisticated techniques, so convex analysis by proximity operators and an iterative algorithm were applied for the

problem of the weighted ℓ_1 -norm lifting prediction filter optimization. The case of lossy image compression was considered, and the obtained results show a slightly better performance than the standard 5/3 wavelet transform when the prediction filters were optimized separately. For other hand, the complexity of the proposed adaptive lifting algorithm is very high.

In this work, we introduce the method to improve the performance of the lifting predictor filters based on the use of the artificial intelligence techniques to design a model for wavelet filter automatic design for lossless image compression. To this end, we consider pattern classification algorithms, in particular, k -NN, to classify the image spectrum calculated in the discrete cosine transform domain. As a result, wavelet lifting filters coefficients are obtained not only for the lifting prediction but for the lifting update as well. The obtained wavelet lifting filters perform the wavelet transform more efficiently in terms of compression ratio comparing with the standard wavelet lifting filters CDF(2,2) and CDF(4,4).

The paper is organized as follows. In Section 2, the lifting scheme filters are described and generalized. Section 3 describes the proposed method to obtain the lifting filter coefficients using pattern recognition technique k -NN. Section 4 presents simulation results on lossless compression of different standard test grayscale images. Next, the conclusions are given in Section 5.

2 Lifting Scheme Generalization

The lifting scheme (or fast wavelet transform) is widely used in the wavelet based image analysis. Its main advantages are: the reduced number of calculations; less memory requirements; the possibility of the operation with integer numbers. The lifting scheme for integer-to-integer transform consists of the following basic operations: splitting, prediction and update [17].

Splitting is sometimes referred to as the lazy wavelet [8]. This operation splits the original signal $\{x\}$ into odd and even samples:

$$s_i = x_{2i}, \quad d_i = x_{2i+1}. \quad (1)$$

Prediction, or the dual lifting. This operation at the level k calculates the wavelet coefficients or the details $\{d^{(k)}\}$ as the error in predicting $\{d^{(k-1)}\}$ from $\{s^{(k-1)}\}$ [16]:

$$d_i^{(k)} = d_i^{(k-1)} + \left\lfloor \sum_{j=-\tilde{N}/2}^{\tilde{N}/2} p_j s_{i+j}^{(k-1)} \right\rfloor, \quad (2)$$

where $\{p_j\}$ are coefficients of the wavelet-based high-pass FIR filter and $\tilde{N}$ is the prediction filter order that corresponds to the number of vanishing moments. $\lfloor \theta \rfloor$ denotes a rounding operation; it truncates the real numbers without a bias: if $\theta \geq 0$, it maps $\theta + 0.5$ to the largest previous integer value; if $\theta < 0$, it maps $\theta - 0.5$ to the smallest following integer value.

Update, or the primal lifting. This operation combines $\{s^{(k-1)}\}$ and $\{d^{(k)}\}$, and consists of low-pass FIR filtering to obtain a coarse approximation of the original signal $\{x\}$ [17]:

$$s_i^{(k)} = s_i^{(k-1)} + \left\lfloor \sum_{j=-N/2}^{N/2} u_j d_{i+j}^{(k-1)} \right\rfloor, \quad (3)$$

where $\{u_j\}$ are coefficients of the wavelet-based low-pass FIR filter and N is the prediction filter order.

The inverse transform is straightforward: first, the signs of FIR filter coefficients $\{u_j\}$ and $\{p_j\}$ are switched. Next, the inverse update followed by inverse prediction is calculated. Finally, the odd and even data samples are merged [17].

A different look at the lifting scheme first was done in [18], where the FIR filters that participate in the prediction and update operation are described in the domain of Z-transform. According to this approach, the transfer function of the prediction FIR filter can be formulated as follows [19]:

$$H_p(z) = 1 + p_0(z + z^{-1}) + \dots + p_{\frac{\tilde{N}}{2}-1}(z^{\tilde{N}-1} + z^{-\tilde{N}+1}). \quad (4)$$

The $H_p(z)$ must have zero at $\omega = 0$, i.e., at $z = 1$. It can be easily found [18] that this condition is satisfied when

$$\sum_{i=0}^{\frac{\tilde{N}}{2}-1} p_i = -\frac{1}{2}. \quad (5)$$

When the admissibility condition (5) is satisfied, $H_p(-1) = 2$ and $H_p(0) = 1$ that means the prediction filter has gain 2 at $\omega = \pi$ and unit gain at $\omega = \frac{\pi}{2}$.

Following this approach, the transfer function for update filter can be obtained. We prefer to formulate this transfer function in the terms of $H_p(z)$ [19]:

$$H_u(z) = 1 + H_p(z) \left\{ u_0 \left[(z) + (z^{-1}) \right] + \dots + u_{\frac{N}{2}-1} \left[(z^{N-1}) + (z^{-N+1}) \right] \right\}. \quad (6)$$

Note that Eq. (6) does not take into account the sub-sampling (1) and therefore corresponds to the case of so-called redundant transform [20]. However, the update filter $H_u(z)$ frequency response is the same as in the case of “normal” lifting using sub-sampling (1).

$H_u(z)$ should have a zero at $\omega = \pi$, i.e., at $z = -1$. It can be easily found [18] that this condition is satisfied when

$$\sum_{i=0}^{\frac{N}{2}-1} u_i = \frac{1}{4}, \quad (7)$$

When the admissibility condition (7) is satisfied, $H_u(1)=1$ and the prediction filter has gain 1 at $\omega = 0$.

An elegant conversion of the formulas (5), (7) in the case of (4,4) lifting scheme was proposed in [18] to reduce the degree of freedom in the predictor and update coefficients. In our terms, the formulae for the wavelet filters coefficients are as follows:

$$p_0 = -\frac{128+a}{256}, \quad p_1 = \frac{a}{256}, \quad (8)$$

$$u_0 = \frac{64+b}{256}, \quad u_1 = -\frac{b}{256}, \quad (9)$$

where a and b are the parameters that control the DWT properties. It was shown, that for standard lifting filters CDF(2,2) with $p_0 = -0.5$, $p_1 = 0.0$, $u_0 = 0.25$, $u_1 = 0.0$, the values of a, b are $a = 0$, $b = 0$, and CDF(4,4) with $p_0 = -0.5625$, $p_1 = 0.0625$, $u_0 = 0.28125$, $u_1 = -0.03125$ has $a = 16$, $b = 8$. It can be concluded that changing the values of the coefficients a, b from 0 to 16, one can control the properties of the resulting wavelet lifting filters adjusting them to the signal spectral properties. This way, the filter characteristics are changed to achieve a higher image compression rate.

3 Proposed Method for Image Compression

The proposed method automatically obtains the wavelet filters coefficients from the image data, and can be described by the algorithm steps:

1. At the first step of the proposed algorithm the image power spectrum is calculated.
2. At the second step, the spectrum is analyzed using artificial intelligence methods to obtain the wavelet lifting filter coefficients.
3. Next, having the lifting filter coefficients, the fast wavelet transform is applied.
4. The transformed image has reduced entropy in wavelet coefficients and can be processed searching the non-zero coefficient trees [20, 21] and then coded by one of the existent entropy coders [22].

At the fourth step of the algorithm the proper image compression is performed, but in this paper we will concentrate on the steps 1-3.

The first step of the proposed algorithm is apply to the considered image $\mathbf{S}$ of size $M \times N$ the discrete cosine transform (DCT) [23] to obtain the mean power spectrum

$$S(i) = \frac{\alpha(i)}{MN} \left[\sum_{l=0}^{N-1} \sum_{q=0}^{M-1} s(l, q) \cos \left(\pi \frac{i(2q+1)}{2M} \right) \right]^2 \quad (10)$$

for $0 \leq i \leq M-1$.

where M is the number of rows and N is the number of columns of the processed image, $\alpha(i) = \begin{cases} 1, & 1 \leq i \leq M-1 \\ \frac{1}{\sqrt{2}}, & i = 0 \end{cases}$.

Subsequently, the resulting vector $\mathbf{X}$ is obtained interpolating the spectrum (10) to have a fixed length of 16 elements:

$$\mathbf{X} = F_{16}^{-1} \{F_M \{S(i)\}\}, \quad (11)$$

where $F_M \{\cdot\}$ denotes the direct Fourier transform of size M , and $F_{16}^{-1} \{\cdot\}$ is the inverse Fourier transform of size 16.

Thus, the image characteristic vector $\mathbf{X}$ of the reduced and fixed size is obtained. This vector can be considered as an input pattern. The set of coefficients a, b (8), (9) form the vector $\mathbf{y} = \{a, b\}$, which is the output pattern. Whilst the vector $\mathbf{X}$ is calculated using (10), (11), the vector $\mathbf{y}$ is found by exhaustive search varying the coefficients a, b to obtain those that minimize the transformed image entropy, or the data bitstream in the wavelet transform domain. The weighted entropy of the transformed image was proposed in [9] as a metrics to compare different image lossless compression algorithms. It can be formulated as:

$$H_q(\tilde{\mathbf{s}}) = - \frac{H(\tilde{\mathbf{s}}_a) + \sum_{q=1}^Q [H(\tilde{\mathbf{s}}_h) + H(\tilde{\mathbf{s}}_v) + H(\tilde{\mathbf{s}}_d)]}{MN}, \quad (12)$$

where $H(\tilde{\mathbf{s}}_a)$ is Shannon's entropy of the approximations $\tilde{\mathbf{s}}_a$, $H(\tilde{\mathbf{s}}_h), H(\tilde{\mathbf{s}}_v), H(\tilde{\mathbf{s}}_d)$ are entropies of the wavelet coefficients of each quadrant of q -th level of wavelet decomposition (horizontal $\tilde{\mathbf{s}}_h$, vertical $\tilde{\mathbf{s}}_v$ and diagonal $\tilde{\mathbf{s}}_d$ details), and Shannon's entropy in our terms can be calculated, as [22]

$$H(\tilde{\mathbf{s}}) = - \sum_{j=-n}^n p(\tilde{s}_j) \cdot \log_2 p(\tilde{s}_j), \quad (13)$$

where $\tilde{\mathbf{s}}$ is the transformed image, $\tilde{s}_j$ is the wavelet coefficient having value j ; $-n \leq j \leq n$ and n is the greatest value of the integer coefficients resulted from the wavelet lifting transform (1), (2), (3).

The patterns $\mathbf{X}, \mathbf{y}$ are associated with each image to form the fundamental set of patterns and perform the supervised learning and classification using 1-NN classifier.

We choose the k -NN technique because it is one of the most efficient and simple existed classification algorithms [24, 25].

With the characteristic vector $\mathbf{x}$ and the filter coefficients $\mathbf{y}$ the associative memory $\mathbf{M}$ is generated and learned. The learning stage in general form can be interpreted by the diagram:

Fig. 1. Block diagram of the learning stage

At this stage, the empirically obtained coefficients $\mathbf{y}$ are associated with the patterns $\mathbf{x}$ obtained with the proposed spectral model (10), (11). The diagram of the retrieval stage is shown below:

Fig. 2. Block diagram of the retrieval stage

At the retrieval stage, the patterns presented to the memory $\mathbf{M}$ are classified to obtain automatically the wavelet filter coefficients for their use in image compression.

4 Results

In this Section, the results obtained applying the proposed method are presented. The experiments were accomplished using the fundamental set of the standard gray scale test images [26, 27] of different size (2048x2560, 1524x1200, 1465x1999, 1024x1024, 512 x 512), some of them are shown in Fig. 3.

Fig. 3. Standard test images: Lenna, boats, F-16, man, baboon, sailboat, Elaine, couple, Tiffany, peppers, aerial, bike, cafe, tools, Zelda

For the classification, the associative memory $\mathbf{M}$ was generated using 1-NN classifier. The obtained classification results show that with this classifier all test images from the fundamental set were classified correctly.

The performance in image compression of the proposed method was compared to the performance of the standard wavelet lifting filters, CDF(2,2) and CDF(4,4). The

quantitative results were calculated in terms of the entropies (12) of the transformed images. For the validation of the proposed method, the Leave-One-Out (LOO) [25] technique was employed for each image from the fundamental image set.

The obtained results are presented in Table. From this Table, one can conclude that the proposed method outperforms the standard wavelet lifting filters in compression ratio for all tested standard natural images.

Table 1. Results of compression of test images, obtained by the standard wavelet lifting filters CDF(2,2), CDF(4,4) and the proposed method, bits/pixel. The best results are marked with bold.

Image	CDF (2,2)	CDF (4,4)	Entropy obtained with optimal filters	Entropy obtained with LOO
aerial	5.27382	5.22282	5.2218	5.22509
aerial2	5.33433	5.30469	5.30375	5.30783
baboon	6.11119	6.08569	6.08492	6.08622
baloon	3.02992	3.01551	3.01519	3.01751
barb2	5.13575	5.08559	4.96838	4.96945
bike	4.81578	4.81534	4.81405	4.81733
board	3.8856	3.87838	3.87676	3.87741
boats	4.22419	4.18232	4.18111	4.1866
cafe	5.62476	5.6231	5.61969	5.6231
couple	4.89369	4.91841	4.88488	4.88529
Elaine	4.88923	4.8613	4.86013	4.86058
F-16	4.16842	4.13125	4.13115	4.13545
finger	5.49482	5.354	5.3215	5.33779
girl	4.07426	4.13125	3.9699	3.97926
gold	4.67736	4.6738	4.67202	4.67175
hotel	4.70622	4.69675	4.69236	4.69881
Lenna	4.33814	4.28893	4.28734	4.30198
man	4.73744	4.71113	4.71089	4.71466
peppers	4.60908	4.59882	4.5944	4.60044
sailboat	5.1793	5.14887	5.14825	5.15003
Tiffany	4.28675	4.27145	4.27142	4.27186
tools	5.64493	5.62856	5.62782	5.63006
txtur2	5.51308	5.50565	5.50499	5.50706
woman	4.75623	4.72292	4.72139	4.72376
Zelda	3.84339	3.83507	3.80353	3.80497
Mean	4.76991	4.74766	4.73151	4.73537

5 Conclusions

In this paper, we have proposed the method for wavelet lifting filter optimization based on the image spectral analysis in the DCT domain and on the use of artificial intelligence model, in particular, 1-NN classifier that demonstrated to be competitive versus other known models.

The designed algorithm was tested on different standard test images. The obtained lossless compression results were compared to the results obtained with the standard wavelet lifting filters CDF(2,2) and CDF(4,4). With the proposed method, a higher compression ratio in terms of entropy was obtained for all considered test images.

Though the presented results not reflected the real bitstreams of the compressed data, the optimization of the entropy resulted in less bitstream data. Nevertheless, the future work will be concerned with the implementation of some wavelet sub-band non zero data trees forming technique and entropy codec. As a future work, we consider also the implementation of the proposed method of optimization at each wavelet decomposition level expecting the better results in term of compression ratio.

Acknowledgments. This work was supported by Instituto Politecnico Nacional as a part of the research project SIP#20141215.

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