How Can We Statistically Analyze the Achievement of Diagrammatic Competency from High School Regular Tests?

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Abstract. Owing to the recent global changes in education goals, students nowadays need to achieve ‘key competencies’ in school. ‘Diagrammatic competency’ is an essential part of such competencies. To cultivate diagrammatic competency, it is necessary to evaluate teachers and students and provide feedback on the students’ degree of achieving diagrammatic competency. Regular school tests can provide useful opportunities for assessing such achievement. However, in such tests, Japanese high schools mainly focus on evaluating the understanding of learning contents rather than the development of competencies (such as diagrammatic competency). The current study was a collaboration between educational psychologists and a high school mathematics teacher. Together they modified a regular school test to incorporate tasks that require diagrammatic competency to solve them, thus enabling the assessment of such achievement. The study was conducted in an actual high school. The students’ performance was analyzed using cognitive diagnostic models [1], which statistically estimate how well students have mastered the elements of cognitive abilities and skills required to solve problems, generating ‘attribute mastery probabilities’. The attribute mastery probabilities obtained demonstrated that students’ achievement of diagrammatic competency was insufficient, indicating a need for cultivating such competency in subject learning instruction provided in schools.

1 Necessity of Assessing Diagrammatic Competency in School

Although traditional school instruction emphasizes the acquisition of the contents of various subjects, due to the recent global changes in educational goals, developing students’ learning competencies are now increasingly considered an essential objective of school education. This perspective is embodied in the idea of ‘key competencies’ proposed by the Organization for Economic Cooperation and Development (OECD). The ability to use external resources, such as diagrams, has been considered one of the most critical among those key competencies.

An essential aspect that needs to be cultivated in school is using diagrams effectively, which can be referred to as ‘diagrammatic competency’. For example, Kragten et al. [2] emphasized that “diagrammatic literacy” is vital in secondary science education. Furthermore, based on findings about the efficacy of self-constructed diagrams

(e.g., [3]), the ability to construct diagrams has also been integrated into the concept of diagrammatic competency.

To cultivate diagrammatic competency, it is necessary to evaluate and provide feedback to students and teachers on the students' extent of achieving it. Regular school tests can provide valuable opportunities for assessing such achievement. However, such tests in Japanese high schools mainly focus on evaluating the understanding of learning contents rather than the development of competencies like diagrammatic competency. Thus, this study proposes a framework that can evaluate diagrammatic competency in school students statistically, with the use of regular school tests.

In this study, the students' performance was analyzed using cognitive diagnostic models (CDMs) [1]. As described later in more detail, this analysis assumes that students can solve problems if they master the elements of cognitive abilities and the skills required for problem-solving (i.e., 'attributes'), and it estimates how well students have mastered attributes (i.e., 'attribute mastery probabilities'). The current study incorporated diagrammatic competency into the analysis as one of the attributes and attempted to statistically calculate how students master diagrammatic competency using CDMs.

2 Development of Tasks to Assess Diagrammatic Competency

Educational psychologists and a high-school mathematics teacher collaborated for this study; they worked together to modify a regular school test to incorporate tasks requiring diagrammatic competency to solve problems.

The problems assessing diagrammatic competency essentially were of two types: the first ones were tasks that were also included in the traditional regular test; however, it could also assess the diagrammatic competency (e.g., item 1 in Fig. 1). The second type required diagrammatic competency for problem-solving, which was newly incorporated in this regular test (e.g., item 2 in Fig. 1). As reflected by these problems, the current study defined diagrammatic competency operationally as the ability to imagine a necessary diagrammatic representation when solving a given problem. The regular test was conducted in an actual high school, and 40 students took the test.

In this study, the attributes representing diagrammatic competency along with four other attributes were identified. In addition to 'diagrammatic competency (A1)', 'comprehension of math terminology (A2)', 'application of a mathematical formula (A3)', 'understanding relations between numerical expressions (A4)', and 'computational skills (A5)' were incorporated as attributes in this study.

To conduct the analysis of CDMs, it was necessary to specify the item-attribute relationship in the form of a matrix, called a 'Q-matrix'. The Q-matrix for this study was specified by the math teacher and two educational psychologists. Inter-rater agreement between the math teacher and one of the educational psychologists was confirmed as substantially equivalent (Cohen's $\kappa = .66$, 95% confidence interval, $.54-.78$). Inconsistencies in the Q-matrix were discussed between the two educational psychologists.

An example of the Q-matrix for the tasks in Fig. 1 is presented in Table 1. It indicates, for instance, that to solve item 1 in Fig. 1, other attributes such as 'A4', 'A5' are necessary, together with diagrammatic competency (A1). A total of 40 students took the regular test that was administered.

	Item 1	Item 2
Task	Let D be the domain represented by the three inequalities $y - x \leq 0, y + 2x \geq 0, 2x - y - 4 \leq 0$. Find the maximum and minimum values of $x + y$ when point (x, y) moves within this domain.	Let C be a parabola $y = f(x)$ and take a point $P(a, 2a^2)$ on C , where $a > 0$. Let the tangent at the P on C be l . Let point Q be the intersection of line l and the x -axis, line m be perpendicular to l through Q , and point A be the intersection of m and the y -axis. Choose one of the following (1) to (4) as the most appropriate representation of $T = \int_0^a \{2x^2 - (4ax - 2a^2)\} dx$ (1) Area of the square $OQPA$ (2) Area of the figure bounded by curve C and line l (3) Area of the figure enclosed by curve C , line l , and y -axis (4) Area of the figure enclosed by curve C , line l , and the x -axis
Necessary Diagram

Fig. 1. Example tasks assessing diagrammatic competency

Table 1. Example of Q-matrix

Attribute Task	A1:Diagrammatic Competency	A2:Comprehension of Math Terminology	A3:Application of a Mathematical Formula	A4:Understanding Relations between Numerical Expressions	A5:Computational Skills
Item1	1	0	0	1	1
Item2	1	0	1	1	1

3 Results and Discussion

In the current study, DINA (the deterministic inputs, noisy “and” gate) model was applied for analysis. The R2jags package in the R program was used with five chains, 30,000

iterations, and 10,000 burn-in. All the model parameters were confirmed to converge. Attribute mastery probabilities for each student were calculated. Table 2 shows examples of outputs. It also illustrates that the analysis with CDMs makes it possible to empirically demonstrate unique patterns of students’ acquisition of each attribute and their acquisition of diagrammatic competency.

Following previous studies, in the current study, 0.5 was used to decide whether students were judged as mastering the attribute. The result of the analysis revealed the percentages of students who mastered each attribute: acquisition of ‘diagrammatic competency’ was 52.5%, ‘comprehension of math terminology’ was 52.5%, ‘application of a mathematical formula’ was 67.5%, ‘understanding relations between numerical expressions’ was 45.0%, and ‘computational skills’ was 60.0%. This suggests that almost half of the students in a class did not sufficiently master diagrammatic competency.

Table 2. Example of output: attribute mastery probabilities (excerpt)

Student ID	A1:Diagrammatic Competency	A2:Comprehension of Math Terminology	A3:Application of a Mathematical Formula	A4:Understanding Relations between Numerical Expressions	A5:Computational Skills
1	0.19	0.55	0.90	0.54	0.80
17	0.97	0.90	1.00	0.05	0.99
23	0.09	0.89	0.93	0.31	0.92
37	0.44	0.83	0.71	0.59	0.60

In this study, cluster analysis was conducted to categorise the students into several groups. Four clusters were obtained, as listed in Table 3. The students belonging to the two larger clusters did not achieve diagrammatic competency. The results indicate the necessity of cultivating diagrammatic competency in school practices.

Table 3. Results of cluster analysis (ward’s method) of students

Cluster ID	<i>N</i>	A1:Diagrammatic Competency	A2:Comprehension of Math Terminology	A3:Application of a Mathematical Formula	A4:Understanding Relation between Numerical Expressions	A5:Computational Skills
1	12	0.30	0.62	0.87	0.59	0.79
2	17	0.46	0.41	0.31	0.46	0.41
3	8	0.92	0.64	1.00	0.85	0.99
4	3	0.98	0.93	1.00	0.03	1.00

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