The Presence of Diagrams and Problems Requiring Diagram Construction: Comparing Mathematical Word Problems in Japanese and Canadian Textbooks

Mari Fukuda¹(✉), Emmanuel Manalo², and Hiroaki Ayabe³

¹ Graduate School of Education, The University of Tokyo, Tokyo, Japan
mari_fukuda@p.u-tokyo.ac.jp

² Graduate School of Education, Kyoto University, Kyoto, Japan
manalo.emmanuel.3z@kyoto-u.ac.jp

³ National Institute for Physiological Sciences, Okazaki, Aichi, Japan
ayabe@nips.ac.jp

Abstract. It is generally considered beneficial for learners to construct and use appropriate diagrams when solving mathematical word problems. However, previous research has indicated that learners tend not to use diagrams spontaneously. In the present study, we analyzed textbooks in Japan and Canada, focusing on the possibility that such inadequacy in diagram use may be affected by the presence (or absence) of diagrams in textbooks, the kinds of diagrams that are included, and whether problems requiring the construction of diagrams are provided in those textbooks. One set each of Japanese and Canadian elementary school textbooks were analyzed, focusing on the chapters dealing with division. Results revealed that the Japanese textbooks contain worked examples and exercise problems accompanied by diagrams more than the Canadian textbooks. Furthermore, the Japanese textbooks often use line diagrams and tables that abstractly represent quantitative relationships and they include more problems that require students to use diagrams. However, to encourage students to use diagrams spontaneously, it may be necessary to include problems that scaffold the use of diagrams in a step-by-step manner in both the Canadian and the Japanese textbooks.

Keywords: Mathematical word problem · Textbooks analysis · Utilizing diagrams

1 Introduction

Diagrams are one of the effective tools students can use in solving mathematical word problems [1]. For example, in the problem-solving process by Mayer [2], pictures representing the situation or scene of the problem may be effective in understanding the context of the problem, and diagrams such as arrays, blocks, and line diagrams representing quantities and their relationships may be effective in forming a representation of the whole problem and in facilitating the planning stage of problem-solving.

However, previous studies have revealed that students do not spontaneously use diagrams [3]. Since the use of learning strategies, such as utilizing diagrams in mathematics word problem solving, requires knowledge of strategies, it is necessary to provide scaffolding for developing the ability to use diagrams in the math classroom. One of the critical factors that influence teachers' teaching and students' learning are textbooks. Although textbooks and their systems vary from country to country, they are considered to be the physical tools most closely linked to teaching and learning [4], that provide opportunities for teacher professional development, and that are essential for both teachers and students to use [5]. If diagrams were not presented with word problems in textbooks, instruction that promotes the use of diagrams could be inhibited.

The present study comprised an initial exploratory analysis of textbook sets from two countries to determine the extent to which diagrams accompany mathematics word problems in those textbooks. The following questions were addressed:

RQ1. How many problems are accompanied by diagrams in the textbooks?

RQ2. What kinds of diagrams are presented with the problems?

RQ3. How many problems explicitly require the use of diagrams (e.g., construction or fill-in-the-blank) in the textbooks?

2 Method

We analyzed textbooks for elementary school students from one Japanese textbook company and one Canadian textbook company. Japanese textbooks are certified by the government and are required to be used in the classroom. The textbooks of Dainippon Tosho Publishing [6] were selected out of the six available companies. In Canada, the education system differs between provinces, and the contents of textbooks differ accordingly. Also, there is no requirement to use textbooks, and their selection and use depend on schools and teachers. In this study, "Math Makes Sense" by Pearson [7] was selected and analyzed. The units analyzed were about "Division," but the learning contents of the two countries' grade levels do not correspond perfectly. Based on these differences, fractional division was excluded from the analysis (due to mismatches), and only the units on whole numbers and decimal division were analyzed. Textbooks for grades 3–5 in Japan and grades 3–6 in Canada were used for the analysis.

The word problems were coded according to the problem type (i.e., worked example, exercise, review), the presence of diagrams, the number and kinds of diagrams, and the presence of problems requiring construction or filling in of parts of diagrams. Worked examples contain a problem sentence, formula, and answer, and exercises and reviews are problems for practicing and mastering skills. Exercises and reviews differ in that the exercises are included in each chapter for contents to be learned, and the reviews are aimed at applying the contents that have been learned (including multiple chapters).

Diagrams were categorized into one of five kinds: (1) Pictures (images representing/relating to the problem situation), (2) Concrete diagrams (illustrations/pictures, like counters and blocks, depicting quantitative relationships), (3) Schematic diagrams (arrows, lines, figures showing procedures and quantitative/functional relationships), (4) Line diagrams (line or tape diagrams, segments of which indicate quantities or show relationships between quantities), (5) Tables (arrays of numbers and words/letters).

3 Results and Discussion

3.1 Math Word Problems Accompanied by Diagrams in Textbooks

Table 1 shows the number of mathematical word problems accompanied by diagrams in Canadian and Japanese textbooks. Chi-square tests revealed that Japanese textbooks contained worked examples and exercises accompanied by diagrams more than Canadian textbooks (worked examples: $\chi^2(1) = 6.41$, $p = .011$, *Cramer's V* = 0.30; exercises: $\chi^2(1) = 4.46$, $p = .035$, *Cramer's V* = 0.14).

Next, we examined the number of diagrams included per question to determine whether the books differed. Table 2 shows the number of diagrams in word problems accompanied by diagrams. We found that the number of diagrams contained in each problem in the Japanese textbooks was higher, with an average of just over two diagrams per problem ($t(175) = 3.20$, $p = .002$, $d = 0.50$). Considering that Japanese textbooks are required to be used in classroom teaching, it is possible that diagrams are utilized more frequently in Japanese classes when teaching word problems.

Table 1. The number of mathematical word problems with/without diagrams

		With diagrams	Without diagrams	Total
Worked example	Canada	20 (87%)	3 (13%)	23
	Japan	47 (100%)	0 (0%)	47
Exercise	Canada	37 (41%)	54 (59%)	91
	Japan	69 (55%)	56 (45%)	125
Review	Canada	5 (28%)	13 (72%)	18
	Japan	8 (15%)	45 (85%)	53
Total	Canada	62 (47%)	70 (53%)	132
	Japan	124 (55%)	101 (45%)	225

Table 2. The number of diagrams in word problems containing diagrams

	Problems with diagrams	Total diagrams	Per problem	
			<i>Mean</i>	<i>SD</i>
Canada	62	88	1.42	1.28
Japan	124	273	2.20	2.07

3.2 Types of Diagrams Presented with Math Word Problems

Table 3 shows the number of each diagram type included in the Canadian and Japanese textbooks. The first and second coders independently coded 20% of the total diagrams.

Interrater reliability was satisfactory ($\kappa = .88$), discrepancies were resolved after discussion, and all remaining coding was done by the first coder. Chi-square test showed that the Canadian textbooks contain more picture and concrete diagrams, while the Japanese textbooks contain more line diagrams and tables ($\chi^2(4) = 39.81, p < .001$, *Cramer's V* = 0.33). The results suggest that the Canadian textbooks use more concrete diagrams to clarify the problem context, while the Japanese textbooks use more abstract diagrams to clarify the quantity relationships.

Table 3. The number of each diagram type included in word problems

	Picture	Concrete	Schematic	Line	Table	Total
Canada	51 (58%)	34 (39%)	0 (0%)	3 (3%)	0 (0%)	88
Japan	106 (39%)	62 (23%)	17 (6%)	80 (29%)	9 (3%)	274
Total	157 (43%)	96 (27%)	17 (5%)	83 (23%)	9 (2%)	362

3.3 Problems Requiring the Use of Diagrams

The number of problems that require students to construct a diagram or fill in parts of provided diagrams were counted. Using the same procedure as before, two coders coded 20% of the problems independently, and high interrater reliability was confirmed ($\kappa = .94$); discrepancies were discussed and resolved. The Japanese textbook contained 19 of such problems (8.4% of total problems), while the Canadian textbook contained five of such problems in total (3.8% of total problems). The questions in the Canadian textbook required students to construct a picture from scratch (e.g., “Draw a picture. Use grid paper”), whereas many of the problems in the Japanese textbook required students to write only numbers that can be read from the problem text in the blanks of a given figure (e.g., “Let’s complete the number line”).

Such scaffolding of diagram use is possibly effective considering the common difficulties students manifest in constructing diagrams, but the construction of diagrams from scratch may be difficult for many students. Therefore, it may be necessary to divide the construction of diagrams into steps, starting with filling in the blanks and gradually fading the scaffolding by having students construct more of the diagrams by themselves. Further investigation is needed to see if such scaffolding and fading support of diagram construction is effective. As this study is preliminary and focused only on division, it is necessary to expand to other topics and problem types to examine whether there are differences there, and to investigate the relationship between teachers’ use of textbook diagrams in class and the students’ use of diagrams and textbooks at home.

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