Chapter 2

Professional Competence for Teaching Mathematical Modelling

Professional competence is a widely discussed topic (see, e.g. Cochran-Smith & Fries, 2001; Darling-Hammond & Bransford, 2005) and was measured globally in various large-scale studies (see, e.g. Blömeke et al., 2014; Kunter et al., 2013). The dimensions for the subject of mathematics range from knowledge to mathematical content to pedagogical and didactic knowledge of teachers with the aim of bringing them together. In the context of the professionalisation of mathematics teaching education students, the question of the existence and structure of specific professional competence is also raised in order to verify skills gains in specific areas. Due to the numerous requirements in the care of cooperative modelling processes and “the strong implantation of real-world problem solving [...] into the curricula” (Schwarz et al., 2008, p. 788), it makes sense to differentiate professional competence in the field of mathematical modelling (Borromeo Ferri, 2018; Borromeo Ferri & Blum, 2010). A structural model describing and relating professional competence for teaching mathematical modelling was developed and empirically confirmed in cooperation between several German universities within the framework of the “Qualitätsoffensive Lehrerbildung” (Klock et al., 2019; Wess et al., 2021)—funded by the Federal Ministry of Education and Research (FKZ 01JA1605, FKZ 01JA1621). The conceptualisation of the model and empirical results for the structure are described in this section.

The first part of the section deals with the general concept of competence and the concept of professional competence of teachers. For this purpose, a clarification of the concept of profession and professional competence will first be made. Two conceptualised models of the professional competence of teachers are presented. With the help of a catalogue of didactic skills for teaching mathematical modelling (Borromeo Ferri & Blum, 2010), an interpretation of the COACTIV model (Baumert & Kunter, 2013) is made to conceptualise a structural model of professional competence for teaching mathematical modelling that can be verified empirically. Results for the empirical structure of the construct are reported in conclusion.

2.1 The Concept of Competence

The basic theoretical reference point of output orientation is the conceptualisation of the concept of *competence*. This is central to empirical studies that address the quality development and productivity of the education system (Klieme et al., 2008). While qualitative studies often use generative models of competence that distinguish between the actual competence and the performance, in the context of quantitative considerations, the functional pragmatic concept of competence, which “conceives of competencies as context-specific dispositions for achievement that can be acquired through learning. Furthermore, they functionally relate to situations and demands in specific domains” (Klieme et al., 2008, p. 8).

This concept is not explicitly interested in the generative, cognitive, situation-independent system, detached from normative educational goals, but focuses on a person’s ability to cope with challenges in certain situations. This work, therefore, relates—especially against the background of context dependence—to the extended concept of competence of Weinert (2001), which is frequently used in German-speaking countries, which defines competences as “intellectual abilities, content-specific knowledge, cognitive skills, domain-specific strategies, routines and subroutines, motivational tendencies, volitional control systems, personal value orientations, and social behaviors.” (Weinert 2001, p. 51).

In this sense, competence is perceived as a complex construct that addresses key aspects of the professional debate. Among other things, skills are assumed to be employable, as they are based on declarative and procedural knowledge. Weinert (2001) also names motivational, volitional, and social readiness; broadening the definition as context-specific cognitive performance management (Klieme et al., 2008). However, he points out that the motivational aspects must be considered as a separate construct in addition to the cognitive aspects, otherwise a lack of motivation is tantamount to a lack of competence. The concept of competence presented is used beyond the German-speaking countries. For example, Blomhøj and Højgaard (2003, p. 126) describe competence as “someone’s insightful readiness to act in a way that meets the challenges of a given situation.” This is in line with the Danish KOM project. A person is described as competent when he or she is able to master essential aspects of this field effectively, succinctly and accurately (Niss & Højgaard, 2011).

2.2 Professional Competence of Teachers

The professional competence of a teacher in his or her profession is to be understood as the aforementioned concept of competence based on different occupational requirements since motivational, volitional and social aspects play a role in addition to cognitive dispositions for achievement (Weinert, 2001). In order to clarify the

second part of the concept, the conditions under which competence can be described as “professional” are first specified.

2.2.1 Professional Competence

The concept of professional competence is used to describe the skills of teachers needed to meet their professional requirements. Shulman (1998) assigns six attributes to the concept of a profession:

- The obligations of service to others, as in a “calling”
- Understanding of a scholarly or theoretical kind
- A domain of skilled performance or practice
- The exercise of judgment under conditions of unavoidable uncertainty
- The need for learning from experience as theory and practice interact
- A professional community to monitor quality and aggregate knowledge

(Shulman, 1998, p. 516).

In a course, a pre-service teacher acquires basic scientific knowledge in his/her subjects. He/She serves society in the relevant field of education by carrying out his/her activities, and in doing so, through his/her evaluations, has a significant influence on the individuals to be formed. He/She sees himself/herself as a life-long learner and works professionally with colleagues to ensure the quality of school education. According to these characteristics, the profession of the teacher can be clearly described as a profession and professional competence can be regarded as a combination of the following factors with regard to the above concept of competence:

- “Specific declarative and procedural knowledge (competence in the narrow sense: knowledge and skills)
- Professional values, beliefs, and goals
- Motivational orientations
- Professional self-regulation skills”

(Baumert & Kunter, 2013, p. 28).

The specific competences of the above-mentioned aspects have been described differently in different conceptualisations. These conceptualisations are addressed in the following section.

2.2.2 *Conceptualisations of Professional Competence of Mathematics Teachers*

Shulman (1986, 1987) initiated an international discussion with his proposals to conceptualise the teacher knowledge. He developed and distinguished the following categories of professional knowledge as components of professional competence:

- *Content Knowledge*. This is pure expertise in the respective field. This includes knowledge about the systematic of the subject to organise the material according to the abilities of students. In this context, contents must be selected with regard to their importance for the subject.
- *Pedagogical Content Knowledge*. It contains content with regard to teaching. These include useful forms of representation of teaching content, analogies, examples and explanations. The teacher must have different approaches and forms of representation and be able to choose between them. The focus is on the knowledge about how students can learn content. For this purpose, the teacher must be aware of the difficulties of certain subjects and involve foresight, prior experience and misconceptions in the learning process.
- *Curricular Knowledge*. Knowledge about the educational plan that contains and arranges the topics in the different class levels is found in this knowledge dimension. It also includes the knowledge and understanding of various methods and materials for teaching instruction. A lateral curricular knowledge characterised by knowledge about current topics in other subjects and a vertical curriculum containing knowledge about topics and content that have been dealt with in the past and will be dealt with in the future can be distinguished.
- *Pedagogical Knowledge*. This is no subject-specific knowledge of a teacher, such as knowledge about effective class management and dealing with disciplinary problems. Shulman (1986, 1987) did not further differentiate this dimension since the focus is on content knowledge.

The teacher's perspectives, however, are not assigned to professional knowledge in the currently discussed conceptualisations, but to a specific construct, beliefs, attitudes or values (Baumert & Kunter, 2013).

There are different concepts of professional competence of mathematics teachers. In particular, the pedagogical content knowledge for mathematics teachers is conceived in a variety of ways, including in part content knowledge and pedagogical knowledge (Depaepe et al., 2013). Following the work of Shulman (1986, 1987), the professional competence of teachers was also examined in the German-speaking region using appropriate competence models. The studies "Mathematics Teaching in the 21st Century" (MT21; Schmidt et al., 2011; Tatto, Schwillen, Senk, Ingvarson, Peck and Rowley, 2008) and their follow-up studies "Teacher Education and Development Study—Mathematics" (TEDS-M; Blömeke et al., 2014) and TEDS—Follow Up (TEDS-FU; Kaiser et al., 2015). Other studies include "Cognitive Activation in the Mathematics Classroom and Professional Competence of Teachers" (COACTIV) and its follow-up studies "COACTIV Internship" (COACTIV-R) and "COACTIV

Table 2.1 Conceptualisations of professional competence of (pre-service) teachers

MT21 (Tatto et al., 2008)	COACTIV (Kunter et al., 2013)
<i>Professional knowledge</i> Content knowledge Pedagogical content knowledge Teaching-related requirements (curricular and planning-related knowledge) Learning-related requirements (interaction-related knowledge) General pedagogical knowledge General didactic knowledge Pedagogical psychological knowledge Educational sociological knowledge	<i>Professional knowledge</i> Content knowledge Pedagogical content knowledge Explanatory knowledge Knowledge about the mathematical thinking of students Knowledge about mathematical tasks Pedagogical psychological knowledge Knowledge about effective class management Knowledge about teaching methods Knowledge about performance assessment Knowledge about individual learning processes Knowledge about individual peculiarities of students Advisory knowledge Organisational knowledge
<i>Other aspects of professional competence</i> Beliefs Epistemological beliefs Educational beliefs Professional beliefs	<i>Other aspects of professional competence</i> Beliefs, values, aims Epistemological beliefs Beliefs about teaching and learning mathematics Motivational orientations Career selection motivation Enthusiasm Self-efficacy Self-regulation

University Study” (Kunter et al., 2013). Table 2.1 compares the conceptualised components of professional competence in studies MT21 (Tatto et al., 2008) and COACTIV (Kunter et al., 2013).

The core of the MT21 study is a standardised test of the declarative and procedural knowledge as well as of the interdisciplinary, pedagogical knowledge of pre-service teachers, which were analysed in a multi-level model against the background of an effectiveness evaluation of mathematics education in international comparison. For this purpose, representative samples were drawn in 17 participating countries, taking into account two target populations of incoming mathematics teachers, namely those from primary (up to grade 4) and secondary (up to grade 8) (Tatto et al., 2008). It was shown that Germany belongs to a group of countries where both the content knowledge (CK) and the pedagogical content knowledge (PCK) of the secondary school pre-service teachers are significantly above the international average (Blömeke & Kaiser, 2014).

The COACTIV study was technically and conceptually linked to the second PISA test in Germany. Secondary school mathematics teachers whose students were part of the sample for the PISA 2003 survey of mathematical competence were interviewed. The study design used by COACTIV and PISA provided for the inclusion of complete

school classes. Beyond the survey date of 2003 which PISA set at the end of the 9th grade, teachers and students were surveyed again at the end of the 10th grade in order to generate a true longitudinal section of combined teacher-student data. Grammar school teachers performed much better in both their content knowledge (CK) and in their pedagogical content knowledge (PCK) than teachers in other forms of school. Teachers with a high level of pedagogical content knowledge use tasks with a high potential for cognitive activation and provide good support for individual learning of students (Krauss et al., 2008; Kunter & Baumert, 2013).

The concepts of professional competence developed in the COACTIV and MT21 and TEDS-M studies show great similarities. These include, among other things, the use of research findings on teacher expertise and the associated assumptions on the knowledge and skills of teachers (Draughtsman & Conklin, 2005) as well as the use of a concept of competence (Weinert, 2001) from empirical education research and an overarching model of professional competence of teachers (Depaepe & König, 2018). In particular, both concepts of professional competence look at professional knowledge, which is composed of different areas of knowledge. These are content knowledge (CK), pedagogical content knowledge (PCK) and pedagogical-psychological knowledge (PK). In addition, professional competence encompasses aspects of affective and value-oriented aspects in addition to the cognitively oriented knowledge dimensions mentioned above.

However, there are differences within the areas of knowledge. For example, the concept of Blömeke and Kaiser (2014) looks at teaching and learning process-related requirements within the field of pedagogical content knowledge, while Baumert and Kunter (2013) distinguish explanatory knowledge, knowledge about the mathematical thinking of students and knowledge about mathematical tasks. The concept of COACTIV (Baumert & Kunter, 2013) also uses the categories consulting knowledge, organizational knowledge, motivational orientations and self-regulation.

2.3 Competence Dimensions for Teaching Mathematical Modelling

In order to be able to interpret the COACTIV model for the field of mathematical modelling, it is necessary to identify requirements for teachers that arise in the preparation and implementation of mathematical modelling processes. Borromeo Ferri and Blum (2010) describe skills they consider necessary for teaching mathematical modelling (see Table 2.2). Each of these dimensions is concretely specified by three facets of knowledge and/or ability. These include, in addition to declarative and procedural knowledge (e.g. recognition of phases in the modelling process) also action skills (e.g. conduct reality-related mathematics lessons) by (pre-service) teachers.

The *theoretical dimension* provides a background necessary and important for practical work, which is based on theoretical conceptualisations and empirical studies

Table 2.2 Competence dimensions for teaching mathematical modelling (cf. Borromeo Ferri, 2018)

Dimension	Facets
Theoretical dimension	Modelling cycles Aims/Perspectives of mathematical modelling Criteria/Types of modelling tasks
Task-related dimension	Processing of modelling tasks Cognitive analysis of modelling tasks Development of modelling tasks
Teaching-related dimension	Planning of reality-related mathematics lessons Conduct reality-related mathematics lessons Interventions during modelling processes
Diagnostic dimension	Identifying phases in the processing process Identifying difficulties in the processing process Evaluation of modelling tasks

of the current modelling discussion (Borromeo Ferri, 2018). It includes knowledge about modelling cycles and their suitability for various purposes (see Sect. 1.1.2). The educational aims associated with mathematical modelling (see Sect. 1.2) as well as knowledge about the different criteria of modelling tasks (see Sect. 1.3) form the basis of any teaching.

In particular, the last facet has strong constrictions on the *task-related dimension*. This includes the knowledge and ability to handle modelling tasks in various ways. This allows the teacher to identify different approaches for the task and thus gain an idea of the variety of solutions. Besides the cognitive analysis of the modelling task difficulties are anticipated (see Sect. 1.4.2). If modelling tasks are to be used in the classroom in a targeted manner, the development of tasks is necessary. This allows the development of individual partial competencies or modelling competence in a broad sense (see Sect. 1.1.3) depending on the requirement.

The *teaching dimension* includes knowledge and ability aspects for theory-guided planning and subsequent implementation of reality-related mathematics lessons (Borromeo Ferri & Blum, 2010). As described in Sect. 1.4, this is characterised by the choice of an appropriate learning environment. While teaching, attention should be paid to the observance of the characteristics of self-directed and cooperative learning. In the case of difficulties in the modelling process, the teacher supports by adaptive intervention (see Sect. 1.4.1).

The *diagnostic dimension* focuses on the classification of students' modelling activities in the different phases of the modelling cycle and on the identification of any cognitive hurdles in the processing process (see Sect. 1.4.2). For this purpose,

teachers need knowledge and skill aspects from the field of pedagogical diagnostics as well as concrete access to the recognition and documentation of progress, difficulties and errors in the modelling process of students (Borromeo Ferri, 2018). Therefore, knowledge about the different phases of the modelling cycle is essential for the effective and results-oriented execution of these activities. Finally, developing and evaluating a performance test with modelling tasks is another facet of the diagnostic dimension.

2.4 A Competence Model for Teaching Mathematical Modelling

The competence dimensions shown in Table 2.2 are used in the interpretation of the COACTIV model (see Table 2.1), in particular the professional knowledge, to derive a structural model of professional competence for teaching mathematical modelling. In order to distinguish between interdisciplinary professional competences of mathematical teachers, the structural model must be as specific as possible to the field of mathematical modelling. Certain aspects and areas of the professional competence of teachers, therefore, seem more important in the context of mathematical modelling (see Fig. 2.1).

In this way, the aspects of *beliefs* can be interpreted as part of beliefs/values/aims and *self-efficacy expectations* as part of the motivational orientations through modelling-specific aspects. On the other hand, the aspect of *self-regulation* refers to the personality characteristics of the teacher and is therefore independent of any specific concretisation. In terms of *professional knowledge*, the area-specific interpretation of *pedagogical content knowledge* can be focused, since it is assumed that the pedagogical content knowledge is the central factor in determining the cognitive activation potential of teaching (Baumert & Kunter, 2013).

The conceptualisations of the three competence aspects and areas, the beliefs and self-efficacy expectations for mathematical modelling and the modelling-specific pedagogical content knowledge are presented in the following sub-sections. Based on the COACTIV study, the pedagogical content knowledge is understood as declarative and procedural knowledge (knowledge and skills) (Baumert & Kunter, 2013), which is measurable in its competence facets about competencies in the narrow sense. The constructs were examined for empirical testing using a structural equation model (Klock et al., 2019; Wess et al., 2021).

Fig. 2.1 Structure model of professional competence for teaching mathematical modelling (Wess et al., 2021)

2.4.1 *Modelling-Specific Pedagogical Content Knowledge*

The COACTIV model breaks down the pedagogical content knowledge into explanatory knowledge, knowledge about the mathematical thinking of students and knowledge about mathematical tasks (see Table 2.1). These three competence facets were based on the competence dimensions for teaching mathematical modelling (Borromeo Ferri, 2018; Borromeo Ferri & Blum, 2010) and further broadens the *knowledge about concepts, aims and perspectives* (see Fig. 2.1).

The explanatory knowledge facet is interpreted as knowledge about interventions using the skills “Performing *reality-related mathematics education*” and “Interventions during modelling processes” in Table 2.2. This facet of competence includes knowledge about the characteristics of adaptive interventions and the effect of different interventions on the learner’s solution process. Capabilities to assess interventions in terms of adaptability and to perform interventions adequately are typical requirements for teachers in the management of mathematical modelling processes. In the field of mathematical modelling, it is specific that the interventions are characterized by a high degree of independence orientation (Smit et al., 2013; Van de Pol et al., 2010) and minimal intervention in the solution process, so that only in a few cases a direct explanation of the teacher is appropriate in the sense of explanatory knowledge. The competence facet “planning reality-related mathematics teaching” includes, among other things, the selection of appropriate social forms and methods, which is part of the scope of pedagogical-psychological knowledge. Therefore, this facet is not included in the conceptualisation.

The facet involving the knowledge about students’ mathematical thinking is interpreted as a *knowledge about modelling processes by means of the competencies* “Identification of modelling phases” and “Identification of difficulties and errors” in Table 2.2. This competence facet includes the skills to diagnose modelling phases and difficulties in the modelling process and to set support goals for interventions based on these. This requires specific knowledge about the modelling process and influencing factors as well as of typical difficulties. This diagnostic component is central to the modelling-specific pedagogical content knowledge. On the one hand, diagnostic skills in the modelling process are a prerequisite for intervention-related competencies as described in the Knowledge about interventions facet. On the other hand, they are necessary for the diagnosis of demands and thus for the selection and development of cognitively activating tasks, which also have links to task-related knowledge.

The Knowledge about mathematical tasks facet is interpreted as *knowledge about modelling tasks* based on the aspects “Characteristics of modelling tasks,” “Processing of modelling tasks,” “Cognitive analysis of modelling tasks” and “Development of modelling tasks” in Table 2.2. This competence facet includes knowledge about different types and criteria of modelling tasks. It also includes skills for the criteria-driven development of modelling tasks, as well as their analysis and processing with regard to multiple solutions. These become even more solid when the (pre-service) teachers are given the opportunity to develop their own modelling tasks

(Borromeo Ferri & Blum, 2010). The development of mathematical tasks for certain topics and content fields is a demanding, complex and time-consuming activity. Furthermore, due to the many requirements that teachers must fulfil every day in school, there is little room for students to do modelling tasks. The comprehensive classification scheme for the categorisation and analysis of modelling tasks in accordance with Maaß (2010) in conjunction with the explanations for the task design in Czocher (2017) provide a theoretical basis for the facets mentioned here. Therefore, there is a great need for good and high-quality teaching materials, including mathematics, and in particular modelling problems (Borromeo Ferri, 2018).

The remaining competencies of the theoretical dimension “Modelling cycles” and “Goals/perspectives of modelling” in Table 2.2 are interpreted as *knowledge about concepts, aims and perspectives* in another competence facet. It consists of selected aspects of theoretical background knowledge. On the one hand, knowledge about modelling cycles and their suitability for various purposes is described, for example as a metacognitive strategy for learners or as a diagnostic tool for teachers. On the other hand, various perspectives of mathematical modelling research are presented (Kaiser & Sriraman, 2006), for example modelling as a means of learning mathematics and fulfilling other curriculum needs (Julie & Mudaly, 2007). In addition, teachers should be aware of the relevant aims of mathematical modelling in the classroom and of the different relevance of reality references for students.

In addition to specialised knowledge specific to modelling, beliefs and self-efficacy in mathematical modelling processes are also part of the professional competence to teach mathematical modelling.

2.4.2 Beliefs Regarding Mathematical Modelling

In German literature, the concepts of convictions, beliefs, ideas, notions, subjective theories, world-views and attitudes are often used in parallel without any clear distinction being made (Voss et al., 2013). In English literature, a similar blur is found, but the term “beliefs” is used predominantly (Leather et al., 2002): “The term “belief” is often used loosely and synonymously with terms such as attitude, disposition, opinion, perception, philosophy, and value” (Leder & Forgasz, 2002, p. 96).

Since the conceptual definitions overlap, uniform concrete specification is difficult (Leder & Forgasz, 2002). In this work, the concept of beliefs is therefore preferred, following the COACTIV study. These include “psychologically held understandings and assumptions about phenomena or objects of the world that are felt to be true, have both implicit and explicit aspects, and influence people’s interactions with the world” (Voss et al., 2013, p. 249).

They are relatively stable cognitive structures (Voss et al., 2013) since their filtering function strengthens the perception of content that corresponds to one’s own beliefs and reduces the perception of inconsistent content (Törner, 2002). According to Patrick and Pintrich (2001), change requires an intensive examination of one’s

beliefs and other perspectives. It is also possible to specify beliefs for specific content areas (Voss et al., 2013). Törner (2002) structures beliefs in the following three hierarchical aspects:

- *Global Beliefs.* General beliefs include beliefs about teaching and learning mathematics, the nature of mathematics, and the development of mathematical knowledge.
- *Domain-specific Beliefs.* Domain-specific beliefs include beliefs about specific mathematical sub-fields, such as analysis, stochastics or geometry. These may differ, for example, in their beliefs about the accuracy of mathematics in each field.
- *Subject-matter Beliefs.* Subject-matter beliefs are beliefs that refer to a concrete mathematical term (e.g. derivative), a mathematical object (e.g. function) or a mathematical procedure (e.g. bisection).

The first aspect is particularly attractive if beliefs for mathematical modelling are to be conceptualised, since it has a relatively high degree of generality. Woolfolk Hoy et al. (2006) distinguish *epistemological beliefs* and *beliefs about teaching and learning* mathematics with respect to the teaching–learning processes. Epistemological beliefs refer to the structure and genesis of knowledge (Buehl & Alexander, 2001). Rösken and Törner (2010) capture them via the mathematical world views that represent beliefs with respect to components of mathematics. They distinguish between the formalism aspect, the application aspect, the process aspect and the schema aspect. In terms of mathematical modelling, the application aspect is of particular interest, which relates to the meaning and utility of mathematics in the real world. Rösken and Törner (2010) summarise under this mathematical world view beliefs about the benefits of mathematics and its everyday and social significance. Due to the application orientation and the realism of modelling tasks, this aspect is suitable for the conceptualisation of *epistemological beliefs for mathematical modelling*. Thus, statements that convey a practical benefit to mathematical modelling in the world are understood as epistemological beliefs about mathematical modelling (Klock et al., 2019; Wess et al., 2021).

Beliefs about teaching and learning mathematics include beliefs about educational goals, teaching methodological preferences and classroom management. According to Kuhs and Ball (1986), three approaches can be distinguished: a learner-focused approach, a content-focused approach with a core focus on conceptual understanding, and a content-focused approach with a core focus on performance. Teachers with learner-focused beliefs see mathematical learning as an active process of knowledge construction (Voss et al., 2013). The content-oriented beliefs differ depending on whether the focus of the teacher is on promoting a conceptual understanding of the content treated or on developing the ability of the students to apply mathematical rules and procedures. This also involves differentiating the cognitive learning aims in terms of routine construction, problem solving and modelling, arguing and reasoning, and evidence. *Beliefs about modelling in mathematics education* and its goal can thus be assigned to beliefs about teaching and learning.

Both areas of beliefs about the teaching–learning processes can be viewed at a meta-level from the perspective of behaviourist and constructivist learning theories. This means that both epistemological beliefs and beliefs about teaching and learning mathematics can be *understood from a more transmissive or constructivist perspective* (Voss et al., 2013). Such correlations between beliefs about learning and beliefs for modelling could also be demonstrated empirically (Kuntze & Zöttl, 2008; Schwarz et al., 2008). Therefore, both positively correlated constructivist beliefs and negatively correlated transmissive beliefs contribute to the description of beliefs in mathematical modelling.

2.4.3 *Self-Efficacy Expectations for Mathematical Modelling*

Self-efficacy expectations are an empirically founded feature of professional competence (Kunter, 2013). The notion of the self-efficacy expectation is understood as an assessment of one's own effectiveness in certain situations. "A teacher's efficacy was a judgement of his or her capabilities to bring about desired outcomes of student engagement and learning, even among those students who may be difficult or unmotivated." (Tschannen-Moran & Woolfolk-Hoy, 2001, p. 783).

They are important for teaching and influence the performance, beliefs and motivation of students (Philippou & Pantziara, 2015). They go hand in hand with a higher quality of teaching, the use of more innovative and effective methods in teaching and a higher level of commitment of the teacher (Kunter, 2013).

Self-efficacy can be specified and concretised in relation to teacher's ideas about their own effectiveness in mathematical modelling processes. The content of the activities is determined by the facets of the modelling-specific pedagogical content knowledge. One of the main activities of the teacher during cooperative modelling processes is the diagnostics of the processing procedure. Since the diagnostic component is related to both the intervention and task-related knowledge facets (see Sect. 2.3), the self-efficacy expectations regarding the assessment of one's ability to diagnose the performance potential of students in the modelling process are operationalised.

The modelling process of the students is characterised by different activities and cognitive processes in different phases. Different diagnostic processes are necessary for the different modelling phases in which the students are currently working. This justifies the assumption that the self-efficacy of the teacher also differs according to the modelling phase. Regarding the activities of the students and the associated diagnostics, it is particularly possible to identify phases that are not specific to the modelling process and in which the activities can be traced by written materials (mathematical work), phases that are specific to the modelling process (simplification/structuring; mathematisation; interpreting; validation). The self-efficacy expectations for mathematical modelling are therefore conceptualised for the diagnosis of performance potentials for the *activities of mathematical working and modelling*.

2.4.4 *Empirical Validation of the Structural Model*

The preceding sections present the underlying structural model of professional competence for teaching mathematical modelling, which results from a specific design of the COACTIV model (see Table 2.1), taking into account theoretical and empirical insights of the current research on mathematical modelling. This naturally exploits the different research traditions that form the basis of the COACTIV model: thus, the emphasis on knowledge and skills—here in the form of modelling-specific pedagogical content knowledge—as a core of professionalism finds its connection in the highlighted work on expert research and regarding structural elements in the dimensions of Borromeo Ferri and Blum (2010). The beliefs, values and aims are also based on the statements of Rösken and Törner (2010), which were supplemented by perspectives on teaching and learning mathematics (Kuhs & Ball, 1986). Likewise, this specific interpretation shows the clear alignment of cognitive characteristics of teachers, which justify a conceptual summary of both aspects as “modelling-specific expertise.” In this context too, the notion of competences as cognitive abilities and skills that can be learnt in principle is emphasised (Cochran-Smith & Draughtsman, 2005; Darling-Hammond & Bransford, 2005). Extending it to include motivational orientations—in the form of self-efficacy expectations—the present structural model follows the example of the COACTIV study and goes beyond the understanding of expertise presented by reinterpreting these aspects with regard to selected facets of teaching mathematical modelling (see Baumert & Kunter, 2013).

For the empirical review of the conceptualised structure (see Fig. 2.1), a structural equation analysis was carried out based on a data set of 156 pre-service teachers from several German universities (for a deeper consideration, see Klock et al., 2019). The fit indices CFI, RMSEA and SRMR as well as the non-significant χ^2 test point to a global fit of the model, relying on the Hu and Bentler guide values (1998). The local fit is impaired by a minor and insignificant charge of the transmissive beliefs ($\lambda = -0.09$) and a low variance elucidation ($R^2 = 0.01$). A negative load is fully in line with expectations, as constructivist and transmissive beliefs correlate negatively due to the different theoretical perspectives of learning (Voss et al., 2013). All other scales load significantly with a medium to high significance on the respective constructs. The structure of the model can therefore be confirmed by the analysis. In addition, a significant but only weak latent connection between the beliefs about mathematical modelling and the modelling-specific pedagogical content knowledge can be demonstrated empirically ($r = 0.38$). Both these two constructs also do not correlate significantly and with little significance with the self-efficacy expectations for mathematical modelling.

Based on the inclusion of beliefs, self-efficacy and cognitive dispositions for achievement, competences are measured in a broad sense (see Sect. 2.1). The results must be compared before a relatively small and non-representative sample for the evaluation methodology. Therefore, a review of the structural model on the basis of further data was necessary in order to demonstrate any significant interaction with the expectations of self-efficacy. A sample of 349 pre-service teachers from several

Fig. 2.2 Structural equation analysis ($N = 349$) with two belief scales (Wess et al., 2021)

German universities was used. On the one hand, the associated results demonstrate the above-mentioned interactions with self-efficacy expectations and on the other, eliminate the deficiencies in the local fit. The χ^2 test, on the other hand, becomes significant, which can be considered problematic for the global fit (Hu & Bentler, 1998; in detail in Chap. 3). However, in view of the explanations of the beliefs on mathematical modelling (see Sect. 2.4.2), another theoretically sound model structure can be examined (see Fig. 2.2; for a deeper consideration, see Wess et al., 2021), which, instead of four belief scales, only looks at two scales located at a meta-level: a constructivist-oriented scale and a transmissive-oriented scale.

In view of the fit indices, the model specified in this manner shows a very good global fit to the current data set (Hu & Bentler, 1998). In addition, correlations of medium practical relevance between self-efficacy expectations and beliefs about mathematical modelling ($r = 0.57$) and between these and high scores can be demonstrated in the modelling-specific pedagogical content knowledge ($r = 0.53$). A significant correlation of medium to high practical significance between beliefs and specific pedagogical content knowledge can also be identified ($r = 0.78$). In addition, all scales have significant loads of high significance.

Overall, the above results show that the professional competence to teach mathematical modelling can be structurally validated in the three areas of competence. The significant latent correlations between beliefs and self-efficacy expectations for mathematical modelling, as well as between these and the modelling-specific pedagogical content knowledge, point to interdependencies between constructs, which points to an affiliation with an overarching construct—the professional competence to teach mathematical modelling.

In the context of quality development in teacher education, the theoretical fundamentals outlined serve to operationalise professional competence for teaching mathematical modelling. The development of the associated test, which also served as a survey tool for the above analyses, forms the centre of this book and is discussed in detail in the following chapter.

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