How at the Institute of Information Systems and Marketing One Thing Leads to Another and Eventually Results in a Low-Trade Theorem

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Abstract This chapter portrays how research topics arise and develop in the creative environment of the research group Information & Market Engineering of the Institute of Information Systems and Marketing at the Karlsruhe Institute of Technology. It is somewhat long-winded in the beginning but identifies then a clear goal. In the following, it strays around several lines of research; touches on the question of why something like forecasting markets, the actual research topic, works at all – without answering it; and finally reaches a result that has little to do with the original objective. Along the way, the chapter provides some insights into the economic theory of double auctions.

1 Requirements

A good research paper features a clear question and a stringent methodological approach which leads to non-ambiguous results that answer the question. Particularly students and young scientists are reminded ad nauseam about the importance of a carefully formulated and precise research question. This question should be phrased *before* the details of the methodological approach are planned in order to, first, choose a method that is appropriate for the question and, second, avoid erroneous conclusions. Otherwise, there is the risk that random effects which are contained in the data may be interpreted as an own artefact or result. Likewise, a statistical hypothesis should be stated *before* the data of a study are analysed.

Under the direction of Christof Weinhardt, I conducted and supervised several experiments. At the research group Information & Market Engineering (IME), it was common that the experimenter had to specify the most relevant hypotheses relating to her or his research question. Moreover, she or he had to write down how relevant measures were to be operationalized and which tests used for assessing the

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hypotheses. The statistical necessity of this procedure becomes apparent in a simple thought experiment in which dice are thrown n times. Specific outcomes, such as particularly frequent observations of the number ‘6’, may suggest that the dice are not fair.

In order to actually assess whether the dice are fair or not, introductory statistics textbooks suggest, for example, Pearson’s chi-square test based on the relative frequencies of all numbers (e.g. Freedman et al. 2007, ch. 28). This test checks whether the frequencies of all numbers are equal. Dice that yield the numbers ‘1’, ‘2’, ‘3’, ‘4’, ‘5’, ‘6’, ‘1’, ‘2’, ... in this fix order are certainly not ideal dice. Pearson’s chi-square test, though, would not reveal any irregularity. Accordingly, there are many possibilities in which dice may deviate from perfectly independent and uniform randomness: A number of the pair ‘1’ and ‘2’ may be thrown more often than ‘3’ or ‘4’, odd numbers may be observed more often than even numbers or a low number may be followed more often by an even than an odd number and so forth. Given a respective data set, the hypothesis that all numbers occur independently and with equal probability could be rejected for some significance level α based on any of the above observations.

The significance level α describes the probability of error, that is, the possibility that a null hypothesis is rejected even though it is true. If one considers a sufficiently large number m of independent or orthogonal test statistics, then one will find up to $m \times \alpha$ reasons to reject the hypothesis of ideal dice even though it actually holds true. This means that if a researcher searches long enough, she or he may identify results that point in the wrong direction. A prior determination of hypotheses (in an appropriate number), a specification of their operationalization and the respective test procedure minimize the risk of this fatal error.

While in some projects it is at least possible to state the research question as well as the relevant hypotheses in advance, actual research often follows a different path. Some results are simply coincidental. An example is the famous apple that supposedly fell on Isaac Newton’s head, an event that allegedly led him to formulate the laws of gravitation (Stukeley 1752, p. 43). Whereas the historical content of Newton’s apple is at least contested,¹ it is considered certain that, for example, the discovery of penicillin is based on petri dishes of Alexander Fleming contaminated with fungal spores (Geo 2017). Similarly, the invention of Teflon or polytetrafluoroethylene traces back to unexpected polymerization in a pressure tank used by Roy Plunkett at DuPont for experiments while searching for a refrigerant (Deutschlandradio 2016), and cornflakes have their roots in dried wheat dough of the brothers John Harvey and Will Keith Kellogg (Geo 2019).

In the above examples, coincidences and mishaps led to new findings. While there is no need for adversities, it is important to seize accidental observations and develop them further. Innovation management investigates, among other things, how research environments can be designed and tailored to foster development. Tidd and

¹*Der Standard* (online issue of January 18, 2010, section “Zeit”) quotes Keith Moore, archivist of the Royal Society, as saying that Newton himself is likely to have polished this tale.

Bessant (2014, p. 62) mention, for example, *shared vision*, *key individuals*, *efficient working teams* and a *creative climate* as relevant components of an innovation organization.

A paradigm for such an organization is the IME, a research group of the Institute of Information Systems and Marketing (IISM) under the direction of Christof Weinhardt at the Karlsruhe Institute of Technology (KIT). Similar to a case study, this chapter depicts an example of a fruitful research project that continuously adapted to new stimuli it collected along its path. It first presents a rather clear question; strays then around several lines of research; touches on the question of why something like forecasting markets, the actual research topic, works at all (unfortunately without answering it); and finally reaches a result that has little to do with the original objective. Thus, the chapter provides some insights into the economic theory of double auctions. Apart from this, it does not fulfil the basic requirements of a good research report. The main reason is that it is incomplete. At the IME, the preliminary results would have been challenged and discussed in several research seminars. With much valuable and constructive feedback, they finally would have been sent back to the right path of academic virtue and the quality standards of the research group. Judged in retrospective, apparently, I left the IME too early to achieve this success.

2 Setting

Christof Weinhardt's research group IME at the IISM has been dealing with 'Market Engineering', the design of markets, for many years. Against this background, in the *STOCCER* project a prediction market was operated in the run-up to the 2006 Soccer World Cup in Germany, in the course of which, among other things, various market models and order types were examined (Luckner et al. 2005). The idea of such a prediction market is that virtual shares promise a payout that depends on future events. From a theoretical point of view, according to the so-called efficient market hypothesis (Fama 1970), the market price should reflect the aggregated assessment of the participants regarding the probability of occurrence of future events. Empirical evidence for the efficiency of prediction markets is given by Berg et al. (2001) referring to the *Iowa Electronic Market*.

When designing *STOCCER*, the IME discussed various incentive schemes, for example, the question of whether participants should be given monetary payments or non-monetary prizes for their participation and whether, in the case of monetary payments, they should be based on absolute performance (prediction accuracy), relative performance (ranking compared to other participants) or activity (e.g. based on transactions). In addition, the number of participants needed for such a market and the budget required for this market were also considered.

Unfortunately, these questions are not easy to answer. The auction theoretical literature provides comprehensive and far-reaching results on one-sided auctions of individual goods for which the bidders have private valuations. However, if

these criteria are relaxed, the results become much thinner. If the focus on one-sided auctions is abandoned and double auctions are addressed, a literature strand around Myerson and Satterthwaite (1983) deals with the question of whether and, if so under which conditions, efficient, incentive-compatible and budget-balanced mechanisms exist. For relatively general assumptions, the budget balance of a mechanism results in inefficiency. Thus, if the prices paid by the buyers equal in sum the revenues of the sellers, the mechanism does not reconcile all profitable transactions. Some possible gains from trade are left on the table. These results contrast with studies such as those of Wilson (1985) and Rustichini et al. (1994), which show that inefficiency falls quickly as the number of traders increases. If one takes into account the possibility that in a prediction market bidders can buy or sell not only one but several units of a good, the incentive to reduce quantity, a phenomenon called *strategic demand* or *supply reduction*, increases inefficiency (Ausubel et al. 2014). The analysis is further complicated by the fact that the shares traded on prediction markets are not goods for which the bidders have private valuations. After the end of the soccer world championship, the events to be forecasted are known and the shares have the same value for all players. Taking a common value component into account increases the complexity of the analysis, but, on the other hand, efficiency considerations become less important, since any allocation of common value goods is Pareto efficient.

With the exemplary questions ‘How does a performance-oriented payment scheme differ from rank- or transaction-based incentives with regard to the prediction accuracy of a forecast market?’ and ‘What influence does the size of the group have on the result of a forecasting market?’, the *STOCCER* project, with which the investigation of market models and order types actually pursued a completely different research agenda, generated further research challenges which were only waiting to be taken up. In the creative environment of the IME, they did not have to wait long. As a sort of *case study*, the present chapter reports how they were addressed and developed further.

3 Model

To assess the result of a market, a stock exchange is considered as a strategic game. The analysis aims to identify bidding equilibria and describe their characteristics. For simplification, a model with only two risk-neutral market participants, a buyer B and a seller S , is considered. The seller S owns a share that promises a payout depending on a future event. This payment is the same for both players (*common value* item, cf. Milgrom and Weber 1982). Other goods are not traded. With regard to the future event, both players possess a private signal or piece of information each, which is denoted by x (buyer) and y (seller). Both x and y are real numbers from an interval $[\underline{x}; \bar{x}]$ and $[\underline{y}; \bar{y}]$, respectively. The expected value of the share, given the two pieces of information x and y , is given by $v(x, y)$. The value function

$v(\cdot, \cdot)$ is monotonically increasing in both arguments. However, both the buyer and seller only know their own signal. They consider the signal of the other player as a realization of a random variable X and Y , respectively. The (conditional) densities $g(x|Y=y)$ and $h(y|X=x)$, in the following shorter $g(x|y)$ and $h(y|x)$, of these random variables X and Y are commonly known. The market mechanism is assumed to be a k -double auction with $k = 1/2$ (Satterthwaite and Williams 1989): The two players B and S each submit a bid $b = b(x)$ and $s = s(y)$, respectively. If $b(x) > s(y)$, B acquires the item from S and pays a price of $\frac{b(x)+s(y)}{2}$.² Otherwise, no trading occurs: S keeps the item and B makes no payment. The two functions $b(\cdot)$ and $s(\cdot)$ are assumed to be monotonically increasing and, thus, in particular to be invertible.

From applying the inverse $s^{-1}(\cdot)$ of the seller's bidding function on the inequality $b > s(y)$, it follows that the buyer acquires the good with a bid b , if for the signal y of the seller $y < s^{-1}(b)$ holds. In this case, the expected payment of B is

$$v(x, y) - \frac{b + s(y)}{2}.$$

The expected payment of the buyer is therefore calculated as

$$u_B(b, x) = \int_{\underline{y}}^{s^{-1}(b)} \left(v(x, y) - \frac{b + s(y)}{2} \right) h(y|x) dy.$$

The Leibniz rule for the derivative of parameter integrals provides the necessary condition for an optimal purchase bid:

$$\begin{aligned} \frac{du_B(b, x)}{db} &= \left(v(x, s^{-1}(b)) - \frac{b + s(s^{-1}(b))}{2} \right) h(s^{-1}(b)|x) s^{-1'}(b) \\ &\quad - \int_{\underline{y}}^{s^{-1}(b)} \frac{1}{2} h(y|x) dy \\ &= \left(v(x, s^{-1}(b)) - b \right) h(s^{-1}(b)|x) s^{-1'}(b) \\ &\quad - \frac{1}{2} H(s^{-1}(b)|x) \stackrel{!}{=} 0. \end{aligned} \tag{1}$$

²Whether or not trading takes place in case $b(x) = s(y)$ is not relevant for the following considerations. However, the result can be formulated more strikingly if no trading occurs in this case. Thus, strictly speaking, the result determines—somewhat unscientifically—the design of the model. For defense, the author argues that in the case of strictly monotonically increasing bidding functions and continuous signals, the probability that bids are equal (independent of the special properties of the result) is zero.

Regarding the seller, two cases are to be distinguished: If $s \geq b(x) \Leftrightarrow x \leq b^{-1}(s)$, the seller keeps the good with the value $v(x, y)$. If $s < b(x) \Leftrightarrow x > b^{-1}(s)$, S sells the item and receives a payment from buyer B of $\frac{b(x)+s}{2}$. The seller's expected payment is

$$u_S(s, y) = \int_{\underline{x}}^{b^{-1}(s)} v(x, y) g(x|y) dx + \int_{b^{-1}(s)}^{\bar{x}} \frac{b(x) + s}{2} g(x|y) dx$$

and we obtain the following first-order condition for equilibrium:

$$\begin{aligned} \frac{du_S(s, y)}{ds} &= v(b^{-1}(s), y) g(b^{-1}(s)|y) b^{-1'}(s) \\ &\quad - \frac{b(b^{-1}(s)) + s}{2} g(b^{-1}(s)|y) b^{-1'}(s) + \int_{b^{-1}(s)}^{\bar{x}} \frac{1}{2} g(x|y) dx \\ &= \left(v(b^{-1}(s), y) - s \right) g(b^{-1}(s)|y) b^{-1'}(s) + \frac{1}{2} \left(1 - G(b^{-1}(s)|y) \right) \stackrel{!}{=} 0. \end{aligned} \quad (2)$$

For illustration, consider an example where the signals of the players are independent of each other and uniformly distributed over the interval $[0; 1]$. The densities and distribution functions are then given by $g(x|y) \equiv h(y|x) \equiv 1$, $G(x|y) \equiv x$ and $H(y|x) \equiv y$. For the example, it is further assumed that the expected value of the traded share under the presence of the signals x and y be given by $v(x, y) = \frac{x+y}{2}$. Then the necessary equilibrium conditions (1) and (2) simplify to

$$\left(\frac{x + s^{-1}(b)}{2} - b \right) s^{-1'}(b) - \frac{1}{2} s^{-1}(b) \stackrel{!}{=} 0$$

and

$$\left(\frac{b^{-1}(s) + y}{2} - s \right) b^{-1'}(s) + \frac{1}{2} (1 - b^{-1}(s)) \stackrel{!}{=} 0.$$

A solution³ to the above functional equation system are the two bid functions

$$b(x) = \frac{1}{2}x \quad \text{and}$$

$$s(y) = y + \frac{1}{2}.$$

³There is a variety of solutions, including at least the family of linear solutions $b(x) = tx - t + \frac{1}{2}$; $s(y) = \frac{t}{3t-1}y + \frac{1}{2}$ with $t > \frac{1}{3}$.

Remarkably, given this solution, the item is never traded. The bids of the buyer are never larger than $\frac{1}{2}$, and the bids of the seller are never lower than $\frac{1}{2}$. Thus, the bid of the buyer is never sufficiently high to meet the seller's ask price.

Interestingly, this observation is not a peculiarity of the example. Consider the first-order condition from the buyer's point of view at the point $\hat{x}$ with $\underline{z} := b(\hat{x}) = s(\underline{y})$, that is, the signal at which the buyer's bid is just as high as the minimum seller's bid. From (1) follows

$$\left(v(\hat{x}, s^{-1}(\underline{z})) - \underline{z} \right) h(s^{-1}(\underline{z})|\hat{x}) s^{-1'}(\underline{z}) - \underbrace{\frac{1}{2} H \left(\overbrace{s^{-1}(\underline{z})}^{\underline{z}} | \hat{x} \right)}_{=0} \stackrel{!}{=} 0$$

$$\iff \left(v(\hat{x}, \underline{y}) - \underline{z} \right) h(\underline{y}|\hat{x}) s^{-1'}(\underline{z}) \stackrel{!}{=} 0.$$

Since $s(\cdot)$ is strictly monotonously increasing, $s^{-1'}(\cdot) > 0$; this also holds for the density $h(\cdot|\cdot) > 0$. Thus, the minimum amount that the seller asks for is

$$s(\underline{y}) = b(\hat{x}) = \underline{z} = v(\hat{x}, \underline{y}).$$

This means that the lowest bid of the seller is just $v(\hat{x}, \underline{y})$. Assume for the moment $\hat{x} < \bar{x}$. Then one obtains from (2) at the same point

$$\begin{aligned} & \left(v(b^{-1}(s(\underline{y})), \underline{y}) - s(\underline{y}) \right) \underbrace{g(b^{-1}(s(\underline{y}))|\underline{y})}_{>0} \underbrace{b^{-1'}(s(\underline{y}))}_{>0} \\ & + \underbrace{\frac{1}{2} \left(1 - G \left(\overbrace{b^{-1}(s(\underline{y}))}^{\hat{x} < \bar{x}} | \underline{y} \right) \right)}_{>0} \stackrel{!}{=} 0 \end{aligned}$$

the condition $s(\underline{y}) > v(b^{-1}(s(\underline{y})), \underline{y}) = v(\hat{x}, \underline{y})$. This means that the lowest bid of the seller is higher than $v(\hat{x}, \underline{y})$, which contradicts the above observation $s(\underline{y}) = v(\hat{x}, \underline{y})$. This contradiction can only be resolved if $\hat{x} = \bar{x}$ and, thus, $1 - G(\hat{x}|\underline{y}) = 0$. Consequently, the maximum bid that the buyer submits at signal $\bar{x}$ is just as high as the minimum bid of the seller that the latter submits at signal $\underline{y}$. Thus, the result already observed in the above example holds independently of the value function $v(\cdot, \cdot)$ and the distributions $g(x|y)$ and $h(y|x)$ of the signals x and y of B and S , respectively. And if the first-order conditions allow for several equilibrium candidates, all these candidates will share this property.

First of all, the result is surprising: It suggests that on a prediction market where shares can be traded that promise uncertain but equal payments for all participants, from a theoretical point of view, no trading should occur. This means that no market price would be generated and the market would be of no use as a forecasting tool. But not only that: Actual shares also have an unknown value that is identical for all participants. For these markets, too, the above model implies that – regardless of the information that individual traders might have regarding the value of the stock – there should be no trade.

The above result is quite radical, so it is reasonable to assume that it is possibly not correct or already known. Intuitively, however, it is easy to comprehend: An allocation of common value goods is always efficient. Consequently, transactions among participants do not promote welfare. Thus, there is no efficiency gain that can be shared. Rather, what one participant would gain, another participant would lose. There are no incentives that could tempt buyers and sellers alike to engage in a transaction.⁴ Thus, the possibility that the result is correct gains plausibility. Consequently, the question remains whether perhaps it is already known.

Solitary thinking about the critical but constructive culture of scientific discussions at the IME led the author to take another look at the literature before presenting the above result in the research group's colloquium. And indeed, with the knowledge of what to look for, namely, the lack of trade in markets, that which was searched for was quickly found: As early as 1982, Milgrom and Stokey established a general *No-trade* theorem. They show that based on a Pareto-efficient initial allocation of goods, additional private information about the value of the goods does not induce trade. Similarly, in the present model the initial endowment according to which *S* owns a good is efficient regardless of the signals of the traders. According to the *No-trade* theorem, private information (an individual signal) does not induce trade. In comparison with the above calculations, Milgrom and Stokey relax the assumption of risk-neutral to (weakly) risk-averse market participants and thus arrive at more general insights than those presented in this chapter.

Today, Milgrom and Stokey's *No-trade* theorem is applied in a somewhat more far-reaching interpretation primarily in the context of prohibiting insider trading. Regarding the existence of trading in stock markets, there are various approaches to mitigate the *No-trade* theorem. Grossman and Stiglitz (1980) consider a model in which traders can acquire information and thereby gain an information advantage. Uninformed and informed traders then meet in the market. In contrast to the present chapter (or the model by Milgrom and Stokey), in which traders receive their signals for free, Grossman and Stiglitz assume that the acquisition of information is costly. Moreover, Grossman and Stiglitz' traders do not act strategically with respect to price formation, but rather accept as price-taker the price that results from mapping aggregated supply with aggregated demand. Grossman and Stiglitz also emphasize the importance of *noise* as a fuzziness of the pricing system with regard to the

⁴It is no secret at the IME that the author became aware of this simple, intuitive explanation only after he had finally come to the presented result after long calculations.

disclosure of information. Kyle (1985), Black (1986) and De Long et al. (1990) as well as the literature based on these articles also refer to *noise traders* to explain trading.

Another strand of literature is based on excessive reliance on one's own information. According to Odean (1998, 1999), *overconfidence*, that is, too much emphasis on one's own information, is an essential reason for motivating an actor to trade. Odean claims that '[t]rading volume increases when price takers, insiders, or marketmakers are overconfident' (1998, p. 1888). In a broader interpretation, actual private motives for trading may exist as well. The latter could be based on the fact that an institutional investor must invest liquid funds according to certain parameters, or a private investor sells securities not because the assets are considered overvalued but because other financial needs must be covered. The core idea of these considerations is to abandon the assumption of a common value. The next section considers a corresponding variation of the model, in which private information – in game theory modeling the type of an actor – is weighted more heavily.

4 Excursus

If we replace the assumption of a common value in favour of private valuations for the traded good, the value function depends only on a market participant's own respective signal. For the buyer it simplifies to $v_B(x)$ and for the seller to $v_S(y)$. Without loss of generality, we equate the private signal of a player with his valuation and substitute $v_B(x) = x$ and $v_S(y) = y$. Apart from this, nothing else changes in the model. The equilibrium conditions (1) and (2) are now given by

$$(x - b) h(s^{-1}(b)|x) s^{-1'}(b) - \frac{1}{2} H(s^{-1}(b)|x) \stackrel{!}{=} 0$$

and

$$(y - s) g(b^{-1}(s)|y) b^{-1'}(s) + \frac{1}{2} \left(1 - G(b^{-1}(s)|y)\right) \stackrel{!}{=} 0.$$

For simplification it shall be assumed that the signals x and y are independently uniformly distributed over the interval $[0; 1]$ as in the example above. Thus, the equilibrium conditions shorten to

$$(x - b) s^{-1'}(b) - \frac{1}{2} s^{-1}(b) \stackrel{!}{=} 0$$

and

$$(y - s) b^{-1'}(s) + \frac{1}{2} \left(1 - b^{-1}(s)\right) \stackrel{!}{=} 0.$$

Harmonizing the running variable to z instead of b and s , substituting $z = b(x) = s(y)$ and accordingly $x = b^{-1}(z)$ and $y = s^{-1}(z)$ and using the abbreviations $u(z) = b^{-1}(z)$ and $v(z) = s^{-1}(z)$ yields the following system of ordinary differential equations (ODE) with respect to the inverse functions $u(\cdot)$ and $v(\cdot)$ of $b(\cdot)$ and $s(\cdot)$, respectively:

$$v'(z) = \frac{v(z)}{2(u(z) - z)}$$

$$u'(z) = \frac{1 - u(z)}{2(z - v(z))}.$$

The above ODE has, among other solutions, the (linear) solution

$$v(z) = \frac{3}{2}z - \frac{3}{8} \text{ and}$$

$$u(z) = \frac{3}{2}z - \frac{1}{8},$$

the reversal of which yields the searched bidding functions in equilibrium

$$b(x) = \frac{2}{3}x + \frac{1}{12} \text{ and}$$

$$s(y) = \frac{2}{3}y + \frac{1}{4}.$$

The experience with Milgrom and Stokey's *No-trade* theorem reminds us to re-examine the literature before claiming a new finding. In fact, this result is not new either. Already Chatterjee and Samuelson (1983) have derived these bidding functions for the considered double auction with two bidders. Based on Chatterjee and Samuelson's result, Satterthwaite and Williams (1989) examine the double auction with private values that are uniformly distributed in the interval $[0; 1]$ and find that the necessary equilibrium conditions describe in the interior of the tetrahedron $0 \leq y \leq z \leq x \leq 1$ a vector field, so that *any* arbitrary point $0 < y < z < x < 1$ represents an initial value condition for a trace $(s^{-1}(z), z, b^{-1}(z))$ that relates to a pair of functions $b(z)$ and $s(z)$ which constitute equilibrium bidding strategies of the double auction. All these traces or the respective bidding functions leave the tetrahedron in one direction through the edge $(0, z, z)$ and in the other direction through the edge $(z, z, 1)$. This means that any possible bid z with $0 < z < 1$ and any x and y with $x > z > y$ may represent an equilibrium, so that $x = b^{-1}(z)$ and $y = s^{-1}(z)$.

Let's try not to completely lose the central train of thought: With the assumption of a *common value* good, we obtain the result that in equilibrium no trade occurs. On the other hand, anything can happen if a *private value* good is traded, in the sense that every point $0 < y < z < x < 1$ relates to some bidding equilibrium. It also follows that in the case of a double auction with a private value good, there is a positive probability of trading in each equilibrium. However, Satterthwaite and Williams also show that not only the linear equilibrium mentioned above but all equilibria are inefficient, that is, there are always cases in which $y < x$, but trading does not occur. In fact, the linear solution given above and known from Chatterjee and Samuelson maximizes among all solutions the expected trading volume and efficiency.⁵ In this equilibrium, trading takes place if $b(x) > s(y) \iff \frac{2}{3}x + \frac{1}{12} > \frac{2}{3}y + \frac{1}{4} \iff x > y + \frac{1}{4}$. The probability of trade is

$$\int_0^{\frac{3}{4}} \int_{y+\frac{1}{4}}^1 1 \, dx \, dy = \frac{9}{32} \approx 28, 1\%.$$

If the equilibrium were efficient, trade, however, would occur in 50% of all cases.

5 Digression

The excursus above shows one way in which the model can be changed so that trade occurs in a market even among purely rational actors. However, the approach is not very helpful with regard to trading in securities whose payoffs are the same for all actors. In this respect, this section examines a hybrid model that combines *common* and *private value* aspects. For this purpose, the value function is adjusted again without changing the other elements of the model. In order to allow a numerical comparison with the example of the private values, the assumption of the independently and uniformly distributed signals over the interval $[0; 1]$ is maintained.

To model the value of the traded good, the pair of functions

$$v_B(x, y) = ax + (1 - a)y \text{ for the buyer and}$$

$$v_S(y, x) = ay + (1 - a)x \text{ for the seller}$$

with $a \in [1/2; 1]$ are being used. These functions describe, on the one hand, a common value component in the amount of $(1 - a)(x + y)$; on the other hand, they also feature a private value component with a value of $(2a - 1)x$ (buyer) and $(2a - 1)y$ (seller).

⁵For a proof, see Satterthwaite and Williams (1989).

For $a = 1/2$, the model is identical to the above common value approach; the larger a is, the stronger the influence of the private value component, and for $a = 1$, the private value case is modeled.

The equilibrium conditions of the hybrid approach stem again from (1) and (2). We obtain:

$$\left(ax + (1 - a) s^{-1}(b) - b \right) s^{-1'}(b) - \frac{1}{2} s^{-1}(b) \stackrel{!}{=} 0$$

$$\left(ay + (1 - a) b^{-1}(s) - s \right) b^{-1'}(s) + \frac{1}{2} \left(1 - b^{-1}(s) \right) \stackrel{!}{=} 0.$$

Again, there is a continuum of solutions for the inverse $b^{-1}(s)$ and $s^{-1}(b)$ of the bidding functions of B and S , including for each a a pair of linear functions. From their reversal, we get the linear equilibrium

$$b(x) = \frac{2}{3}x + \frac{3a - 2}{6(3a - 1)}$$

$$s(y) = \frac{2}{3}y + \frac{a}{2(3a - 1)}.$$

The correctness of the above solution can be easily checked by inserting it into the equilibrium conditions (1) and (2).

Besides the given linear equilibrium, there is usually a continuum of further solutions for all a . For the pure common value case with $a = 1/2$, an alternative equilibrium has already been presented above (see also footnote 3). Numerical calculations, which will not be discussed in detail here, show that for all a the given linear equilibrium maximizes the trading probability and efficiency among all solution candidates. This observation is consistent with the results of Satterthwaite and Williams (1989), who had already established this for the private value case. Using the efficiency argument as a dominance criterion, we focus on linear equilibria. In these equilibria, trade occurs if

$$b(x) > s(y)$$

$$\iff \frac{2}{3}x + \frac{3a - 2}{6(3a - 1)} > \frac{2}{3}y + \frac{a}{2(3a - 1)}$$

$$\iff x > y + \frac{1}{2(3a - 1)}.$$

The probability of trading thus calculates to

$$\int_0^{1-\frac{1}{2(3a-1)}} \int_{y+\frac{1}{2(3a-1)}}^1 1 dx dy$$

$$= \frac{9(2a-1)^2}{8(3a-1)^2}$$

For $a=1$ and $a=1/2$, we obtain the already known values $\frac{9}{32}$ and 0, respectively.

The above expression also shows a beautiful regularity: The larger a is, that is, the higher the private value component, the higher the probability of trade. An efficient market result, however, is never reached. For all $a > 1/2$, efficiency would require that in exactly 50% of the cases trade occurs. In contrast, the inefficiency of the double auction, which is well known for the case of private values, increases with an increasing common value component of the traded good. This means that with an increasing common value component, not only the size of the cake, which buyers and sellers may share, but also the portion of the cake that is actually realized decreases. The author is inclined to assume a more general ‘low-trade’ theorem behind this phenomenon, which is shown in the above example of the model under consideration.

6 Assessment

The research question which was raised in this chapter and which targeted the prediction accuracy of a forecasting market under different incentive schemes and different numbers of traders could not be answered. The foregoing only examined the case of a utility function, which depends linearly on the payment of the traded good. The characteristics of rank- or transaction-based payments were not addressed. Neither was the impact of the group size explored, as only a game with two players was considered. Furthermore, the model did not take into account that in a typical prediction market several goods are traded in larger numbers.

In the considered model of a market with two participants, one potential buyer and one potential seller, who can trade a single good, it was found, though, that trade among rational actors occurs only with a relatively low probability. Furthermore, this probability of trade depends on the weight of the private component with respect to the total value of the good. Thus, only a small portion of the possible welfare gain is realized.

With regard to the actual object of investigation, prediction markets on which common value goods are traded, the obtained result is somewhat irrelevant. On the contrary, the design of a prediction market should ensure that private value components do not play a role. Rather, market prices of the traded shares should

reflect some objective and unbiased value relating to the forecasted future event, which is the same for all traders.

If, however, one frees oneself from the compulsion to answer a previously posed research question, the chapter, which somewhat cheekily has also been dubbed a ‘case study’, shows how in a cooperative environment, such as the IME, different research projects can cross-fertilize each other and how questions posed in one project can provide food for thought for further projects. Good research management features breeding ground for developing these questions and leveraging synergies.

The connection between the share of private and common value components of a good in relation to the probability that this good will be traded has – in contrast to other results presented in this chapter – not been known to me from the literature. Nevertheless, I consider this an interesting observation and have enjoyed the freedom at the IME to devote myself to this but also to other questions that, so to speak, suddenly fell from the sky, and to be able to discuss them in a great team with great colleagues and an outstanding supervisor. The openness to take up questions, develop them further and thereby open up new areas of research and activity has shaped and helped me greatly in my subsequent work. A special feature of the IME, however, is to actually answer these questions and complete projects without getting bogged down. Unfortunately, 10 years after I left the IME, I have not been able to do this with this chapter.

I learned a lot from Christof Weinhardt during my time at the IME. But in many ways he remains unrivaled. This chapter, which has also been described as a case study, does not go far enough to learn from it. A case study should actually identify and work out relevant aspects. So, what is the secret of the atmosphere that Christof has created at his institute? Certainly, the joy of work, the good relationship among colleagues and the inspiration and stimulation of a creative and inspiring director. But how one can replicate such a productive environment remains a mystery (at least to me). I am thankful for the experience I was able to gain and for the many things I learned at the IME and from Christof. And concerning the remainder that I did not learn, my respect remains. Chapeau!

References

- Ausubel, L. M.; Cramton, P.; Pycia, M.; Rostek, M.; Wernetka, M. (2014): “Demand Reduction and Inefficiency in Multi-unit Auctions”, *Review of Economic Studies* 81, 1366–1400.
- Berg, J. E.; Forsythe, R.; Nelson, F.; Rietz, T.A. (2001): “Results from a Dozen Years of Election Futures Markets Research”, in: Plott, C.; Smith, V. L. (eds.) *Handbook of Experimental Economic Results*, 742–751.
- Black, F. (1986): “Noise”, *Journal of Finance* 41, 529–543.
- Chatterjee, K.; Samuelson, W. (1983): “Bargaining under Incomplete Information”, *Operations Research* 31, 835–851.
- De Long, J. B.; Shleifer, A.; Summers, L. H.; Waldmann, R. J. (1990): “Noise Trader Risk in Financial Markets”, *Journal of Political Economy* 98, 703–738.

- Deutschlandradio (2016): “75 Jahre Teflon – Siegeszug einer zufälligen Entdeckung“, *Deutschlandfunk Kultur* 04.02.2016, www.deutschlandfunkkultur.de/75-jahre-teflon-siegeszug-einer-zufaellig-entdeckung.932.de.html?dram:article_id=344524 (accessed Nov 04, 2020).
- Fama, E. F. (1970): “Efficient Capital Markets, A Review of Theory and Empirical Work”, *Journal of Finance* 25, 383–417.
- Freedman, D.; Pisani, R.; Purves, R. (2007): *Statistics*, W. W. Norton & Company, New York.
- Geo (2017): “Wie Alexander Fleming durch eine Schlampererei das Penicillin entdeckte”, *Geo Chronik* 1, www.geo.de/magazine/geo-chronik/19648-rtkl-antibiotika-wie-alexander-fleming-durch-eine-schlampererei-das (accessed Nov 04, 2020).
- Geo (2019): “Wie die Kelloggs-Brüder durch Zufall die Cornflakes erfanden”, *Geolino* Nr. 5, www.geo.de/geolino/mensch/21389-rtkl-geschichte-der-cornflakes-wie-die-kelloggs-brueder-durch-zufall-die (accessed Nov 04, 2020).
- Grossman, S. J.; Stiglitz, J. E. (1980): “On the Impossibility of Informationally Efficient Markets”, *American Economic Review* 70, 393–408.
- Kyle, A. S. (1985): “Continuous Auctions and Insider Trading”, *Econometrica* 53, 1315–1335.
- Luckner, S.; Kratzer, F.; Weinhardt, C. (2005): “STOCER – A Forecasting Market for the FIFA World Cup 2006”, *Proceedings of the 4th Workshop on e-Business (WEB 2005)*, Las Vegas, 399–405.
- Milgrom, P. R.; Stokey, N. (1982): “Information, Trade and Common Knowledge”, *Journal of Economic Theory* 26, 17–27.
- Milgrom, P. R.; Weber, R. J. (1982): “A Theory of Auctions and Competitive Bidding”, *Econometrica* 50, 1089–1122.
- Myerson, R. B.; Satterthwaite, M. A. (1983): “Efficient Mechanisms for Bilateral Trading”, *Journal of Economic Theory* 29, 265–281.
- Odean, T. (1998): “Volume, Volatility, Price, and Profit When All Traders Are above Average”, *Journal of Finance* 53, 1887–1934.
- Odean, T. (1999): “Do Investors Trade Too Much?”, *American Economic Review* 89, 1279–1298.
- Rustichini, A.; Satterthwaite, M. A.; Williams, S. R. (1994): “Convergence to Efficiency in a Simple Market with Incomplete Information”, *Econometrica* 62, 1041–1063.
- Satterthwaite, M. A.; Williams, S. R. (1989): “Bilateral Trade with the Sealed Bid k -Double Auction: Existence and Efficiency”, *Journal of Economic Theory* 48, 107–133.
- Standard (2010): “Die ganze Wahrheit über Newtons Apfel“, Standard, 18. Januar, Wien, www.derstandard.at/story/1263705369432/die-ganze-wahrheit-ueber-newtons-apfel (accessed Nov 04, 2020).
- Stukeley, W. (1752): *Memoirs of Sir Isaac Newton's Life*, MS/142, Royal Society Library, London, ftp.royalsociety.org/accessible/SpreadDetails.aspx?BookID=1807da00-909a-4abf-b9c1-0279a08e4bf2 (accessed Nov 04, 2020).
- Tidd, J.; Bessant, J. (2014): *Strategic Innovation Management*, Wiley, Chichester.
- Wilson, R. (1985): “Incentive Efficiency of Double Auctions”, *Econometrica* 53, 1101–1115.

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