Chapter 3

Hydro-Mechanical Coupling

Abstract We summarize the basic principles that couple rock deformation and fluid flow. Topics covered include linear poroelasticity, consolidation, liquefaction, rock friction, and frictional instability. Together, these are the processes that serve as a starting point for understanding how water and earthquakes influence each other.

3.1 Introduction

The linkage between the hydrogeological and the mechanical processes in Earth's crust, i.e., hydro-mechanical coupling, underpins any quantitative analysis of the interactions between water and earthquakes. It embodies a spectrum of processes, from poroelastic deformation to the fluidization of an initially solid matrix. Here we discuss three major components of this broad field essential to the themes of this book: the linear theory of poroelasticity, nonlinear deformation (including consolidation and liquefaction), and friction and the rate-and-state model.

At small deformation, the response of rocks and sediments to an applied force is linearly elastic, and stress and strain are related by Hooke's law. Analysis of the elastic deformation of porous media and its effect on porous flow, and vice versa, is the domain of poroelasticity. In this chapter we first introduce poroelasticity and provide the equations essential for interpreting many important hydrogeological processes such as the response of groundwater to tidal forces and barometric loading (Chap. 5). At relatively large deformation, the relationship between stress and strain becomes nonlinear and deformations become irreversible. Common hydrogeological examples include consolidation and liquefaction (Chap. 8). The third topic of this chapter is rock friction that is closely related to the mechanisms of earthquakes, especially induced earthquakes (Chap. 4).

3.2 Linear Poroelasticity and Groundwater Flow

Deforming a rock changes the volume of pore space, which in turn causes the pore fluid pressure to change unless the fluid has enough time to escape. Subsurface fluid pressure and strain are thus generally ‘coupled’. For this reason, water level in confined aquifers can sometimes be used as a strain-meter. An example of this coupling is the periodic fluctuations of the water level in wells in response to the strain induced by Earth’s tides.

At the same time, changing fluid pressure in the subsurface will deform the rock. Thus, the cause-and-effect between the coseismic volumetric strain and the coseismic change of water level can become complex. The theory of linear poroelasticity is often invoked to explain the interactions between rock deformation and pore pressure change.

Linear poroelasticity addresses the coupling between small deformation of the rock matrix and fluid flow within the rock and is thus at the core of the discussions in this book. Several excellent texts (e.g., Wang 2000; Cheng 2016) provide more detailed expositions of the topic. The equations developed in this section summarize concepts from Chaps. 1–4 from Wang (2000). Roeloffs (1996) provides a more brief and applied presentation.

We begin with some examples and comments to illustrate the applications of poroelastic models and illustrate some of the interesting phenomena that can be understood. Figure 3.1 shows the response of water level in a well near the coast in southern Bangladesh to the loading of ocean tides (Burgess et al. 2017). Oscillations of the water level (colored curves in Fig. 3.1b) are not synchronous with, but lag behind, the ocean tides (grey curve in Fig. 3.1b) and their amplitudes at different depths are different. These amplitude differences reflect depth-varying poroelastic properties of the Bengal aquifer system (Burgess et al. 2017).

Figure 3.2 shows the responses of water level and volumetric strain to Earth tides measured in a Plate Boundary Observatory borehole in southern California. It also shows the response of the water level to the 2010 Mw7.2 El Mayor-Cucapah earthquake (Wang and Barbour 2017). Before the earthquake, water level and volumetric strain responded to Earth tides with the expected oscillations of opposite signs (expansion is positive). During the earthquake, volumetric strain suddenly decreased while pore pressure immediately increased. That is, the coseismic increase in pore pressure was caused by a coseismic volumetric contraction. Such changes are characteristic of the undrained response of aquifers to loading, i.e., a response that occurs without flow of pore water during loading. These responses are determined by the poroelastic Skempton’s coefficient $B \equiv -(\partial P / \partial \sigma)_f$, where P is pore pressure, σ is the applied volumetric stress, and the subscript f means that the process occurs at constant water content f . This relation may be re-expressed as $\Delta p = -B \Delta \sigma = -BK_u \Delta \epsilon$, where K_u is the undrained bulk modulus and $\Delta \epsilon$ the change of volumetric strain. Based on the BK_u estimated from the pre-seismic amplitudes of the tidal responses of water level and volumetric strain, Wang and Barbour (2017) converted the coseismic

Fig. 3.1 Water level in a well at Gabura (panel a), southern Bangladesh, showing hydraulic head in a coastal aquifer in response to the loading of local ocean tides (grey curve in b) recorded at Chittagong from June 20 to June 30, 2013 (panel b). Colored curves in b correspond to different screen depths in m below the ground surface. Notice the difference in phase and amplitude, reflecting different hydrogeological properties (modified from Burgess et al. 2017)

Fig. 3.2 Pore pressure and volumetric strain in Plate Boundary Observatory borehole B084 in southern California, before and after the 2010 Mw7.2 El Mayor-Cucapah earthquake in Mexico. Negative strain shows contraction (from Wang and Barbour 2017)

Fig. 3.3 Water level (blue, lower curve) and volumetric strain (black, upper curve) in the Fuxin well in northeastern China, before and after the 2011 Mw9.1 Tohoku earthquake, Japan (modified from Zhang et al. 2019)

change in water level to a coseismic change of volumetric strain of -61×10^{-9} , similar to the measured coseismic volumetric strain of -85×10^{-9} (Fig. 3.2).

If, on the other hand, the pressure response occurs together with flow of pore water, the process is no longer undrained. Figure 3.3 shows a case of coseismic increase of water level (lower curve) in the Fuxin well, NW China, which occurred together with a coseismic increase of volumetric strain during the 2011 Mw9.1 Tohoku earthquake, Japan (Zhang et al. 2019). The occurrence of a coseismic increase of water level together with a coseismic increase of volumetric strain suggests that the latter was caused by the coseismic increase of pore pressure, which can only be caused by the occurrence of coseismic flow. The two examples of water level responses to earthquakes illustrate how the cause-and-effect relationship between the coseismic change of volumetric strain and the coseismic change of water level may be complicated by the different boundary conditions near the well. Thus, the interpretation of coseismic changes of water level requires careful analysis and testing.

3.2.1 Constitutive Relations for Isotropic Stress: Biot (1941)

Consider a saturated and isothermal rock. Stress σ (extension positive) and pore pressure p are the independent variables; the strain ϵ and the increment of fluid content f of the rock may be expressed as

$$\epsilon = \epsilon(\sigma, p) \quad (3.1)$$

$$f = f(\sigma, p). \quad (3.2)$$

We consider first the case of an isotropic strain, for which $\epsilon = dV/V$ where V is volume. Changes in ϵ and f due to changes in σ and p are thus given by

$$d\epsilon = \left(\frac{\partial \epsilon}{\partial \sigma} \right)_p d\sigma + \left(\frac{\partial \epsilon}{\partial p} \right)_\sigma dp \quad (3.3)$$

$$df = \left(\frac{\partial f}{\partial \sigma} \right)_p d\sigma + \left(\frac{\partial f}{\partial p} \right)_\sigma dp. \quad (3.4)$$

Biot (1941) defined four parameters K , R , H and H_1 such that

$$d\epsilon = \frac{1}{K} d\sigma + \frac{1}{H} dp \quad (3.5)$$

$$df = \frac{1}{H_1} d\sigma + \frac{1}{R} dp \quad (3.6)$$

and thus

$$\frac{1}{K} = \left(\frac{\partial \epsilon}{\partial \sigma} \right)_p, \quad \frac{1}{H} = \left(\frac{\partial \epsilon}{\partial p} \right)_\sigma, \quad \frac{1}{H_1} = \left(\frac{\partial f}{\partial \sigma} \right)_p, \quad \frac{1}{R} = \left(\frac{\partial f}{\partial p} \right)_\sigma. \quad (3.7)$$

$1/K$ is the *compressibility* and K is the *bulk modulus*. Biot (1941) further assumed there is a potential density function

$$U = \frac{1}{2}(\sigma\epsilon + pf). \quad (3.8)$$

Because σ and p are independent variables, $\partial U/\partial \sigma = \epsilon/2$ and $\partial U/\partial p = f/2$; thus $\partial^2 U/\partial p \partial \sigma = \partial^2 U/\partial \sigma \partial p$ and

$$\left(\frac{\partial \epsilon}{\partial p} \right)_\sigma = \left(\frac{\partial f}{\partial \sigma} \right)_p \quad (3.9)$$

and hence

$$H = H_1. \quad (3.10)$$

3.2.2 Effective Stress

From (3.5) we have

$$d\epsilon = \frac{1}{K} \left(d\sigma + \frac{K}{H} dp \right) = \frac{1}{K} (d\sigma + \alpha dp) = \frac{1}{K} d(\sigma + \alpha p) = \frac{1}{K} d\sigma', \quad (3.11)$$

Fig. 3.4 Volumetric strain of Weber sandstone under confining pressure. **a** Strain versus confining pressure P_c . Solid circles represent data at elevated pore pressure. Open circles represent data for a dry sample in which $P = 0$. **b** Strain versus $P_c - P$, where P is pore pressure. **c** Strain versus the effective pressure $P_c - \alpha P$. Here all data overlap with the dry sample data, showing that the effective pressure produces the same change in strain as does confining pressure with $P = 0$ (from Nur and Byerlee 1971)

where α is the *Biot-Willis coefficient* (Sect. 3.2.3) and $\sigma' = \sigma + \alpha p$ is the effective stress, first proposed by Terzaghi (1925). The effective stress concept has been applied to many geological problems where pore pressure interacts with deformation. As an example, Fig. 3.4 shows laboratory measurements of the volumetric strain of Weber sandstone under controlled confining pressure and pore pressure (Nur and Byerlee 1971). It illustrates how the effective stress relation may be used to explain the volumetric deformation of rocks under confining pressure and pore pressure.

3.2.3 Related Poroelastic Constants

More commonly used poroelastic constants, such as the Skempton's coefficient B and the Biot-Willis coefficient α , may be expressed in terms of K , H and R . The *Skempton's coefficient* is defined as

$$B = -\left(\frac{\partial p}{\partial \sigma}\right)_f = \frac{R}{H} \quad (3.12)$$

using Eq. (3.7).

The *Biot-Willis coefficient* is defined as

$$\alpha = \frac{K}{H}. \quad (3.13)$$

We defined in the last chapter a quantity called the specific storage S_s . The specific storage at constant stress is

$$S_\sigma = \left(\frac{\partial f}{\partial p} \right)_\sigma = \frac{1}{R}. \quad (3.14)$$

The *specific storage at constant strain* $S_\epsilon = \left(\frac{\partial f}{\partial p} \right)_\epsilon$ can be derived by eliminating $d\sigma$ from Eqs. (3.5) and (3.6)

$$df = \frac{K}{H} d\epsilon + \left(\frac{1}{R} - \frac{K}{H^2} \right) dp \quad (3.15)$$

and hence

$$S_\epsilon = \left(\frac{\partial f}{\partial p} \right)_\epsilon = S_\sigma - \frac{K}{H^2} \quad (3.16)$$

and, from (3.13), (3.15) and (3.16)

$$df = \alpha d\epsilon + S_\epsilon p. \quad (3.17)$$

With the definitions of B and α , Eqs. (3.5) and (3.6) can be rewritten as

$$d\epsilon = \frac{1}{K} d\sigma + \frac{\alpha}{K} dp, \quad (3.18)$$

$$df = \frac{\alpha}{K} d\sigma + \frac{\alpha}{KB} dp. \quad (3.19)$$

Equation (3.18) is sometimes called the *Biot relation*. Equations (3.18) and (3.19) can be rearranged for stress and pressure

$$d\sigma = \left(\frac{K}{1 - \alpha B} \right) d\epsilon - \left(\frac{K}{1 - \alpha B} B \right) df \quad (3.20)$$

$$dp = - \left(\frac{K}{1 - \alpha B} B \right) d\epsilon + \left(\frac{K}{1 - \alpha B} \frac{B}{\alpha} \right) df. \quad (3.21)$$

We can define $K_u = d\sigma/d\epsilon$ for $f = 0$, and hence identify K_u as an *undrained bulk modulus* (the bulk modulus when there is no fluid gain or loss)

$$K_u = \frac{K}{1 - \alpha B}. \quad (3.22)$$

Furthermore, from (3.12), (3.13) and (3.14)

$$S_\sigma = \frac{\alpha}{KB}. \quad (3.23)$$

We can rewrite Eqs. (3.20) and (3.21) in terms of K_u

$$d\sigma = K_u d\epsilon - K_u B df \quad (3.24)$$

$$dp = -K_u B d\epsilon + \frac{K_u B}{\alpha} df. \quad (3.25)$$

This last relation (3.25) is the basis for using water wells as strain meters. If the pore pressure causes changes in the water height $dh = dp/\rho_w g$ in a well, and is an undrained response to a volumetric strain $d\epsilon$, the change of the volumetric strain and that of the water height are then related by

$$dh = \frac{1}{\rho_w g} dp|_{f=0} = -\frac{1}{\rho_w g} B d\sigma|_{f=0} = -\frac{K_u B}{\rho_w g} d\epsilon \quad (3.26)$$

As noted earlier, the validity of this undrained assumption depends on the rate of loading compared to the permeability of the formation.

From Eq. (3.25) we obtain

$$df = \alpha d\epsilon + \frac{\alpha}{K_u B} dp \quad (3.27)$$

which leads to a physical interpretation of α

$$\alpha = \frac{df}{d\epsilon}|_{dp=0}, \quad (3.28)$$

that is, α is the ratio of the increment in fluid content to volumetric strain at constant pressure. From (3.27) we can also obtain an expression for the *constrained specific storage coefficient*

$$S_\epsilon = \frac{df}{dp}|_{\epsilon=0} = \frac{\alpha}{K_u B} \quad (3.29)$$

and

$$\frac{S_\sigma}{S_\epsilon} = \frac{K_u}{K} = \frac{1}{1 - \alpha B}. \quad (3.30)$$

From Eq. (3.24) we obtain

$$d\epsilon = \frac{d\sigma}{K_u} + Bdf. \quad (3.31)$$

The volumetric strain ϵ is thus composed of two components: an elastic component under undrained conditions, and a second component from the transfer of fluid. Equation (3.31) provides another physical interpretation of Skempton's coefficient

$$B = \left. \frac{d\epsilon}{df} \right|_{d\sigma=0} \quad (3.32)$$

that is, B is the ratio of the increment in volumetric strain to fluid content at constant stress.

Equations (3.28) and (3.32) show that the Biot-Willis coefficient α and the Skempton coefficient B are closely related. These coefficients are often used in the study of different hydrogeological responses, as illustrated in the examples that follow. There are few direct comparisons between the volumetric strain and that estimated from the change of water level because most wells do not have the required measurements of both pore pressure and strain. At some special well-equipped wells, such as some Plate Boundary Observatory (PBO) boreholes (e.g., Barbour 2015), pore pressure measurement is collocated with the measurement of strain, allowing comprehensive characterization of the responses to tides, barometric pressure and teleseismic waves (Barbour et al. 2019).

3.2.4 Constitutive Relationship for Anisotropic Stress: Biot (1955)

Our variables are now pore pressure p , stress σ_{ij} , strain ϵ_{ij} and increment in fluid content df . The mean stress is $\sigma = \frac{1}{3}(\sigma_{xx} + \sigma_{yy} + \sigma_{zz}) = \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3)$ and the volumetric strain is $\epsilon = \epsilon_{11} + \epsilon_{22} + \epsilon_{33}$. In principal coordinates the constitutive relations are

$$d\epsilon_1 = \frac{1}{E}d\sigma_1 - \frac{\nu}{E}d\sigma_2 - \frac{\nu}{E}d\sigma_3 + \frac{dp}{3H} \quad (3.33)$$

$$d\epsilon_2 = -\frac{\nu}{E}d\sigma_1 + \frac{1}{E}d\sigma_2 - \frac{\nu}{E}d\sigma_3 + \frac{dp}{3H} \quad (3.34)$$

$$d\epsilon_3 = -\frac{\nu}{E}d\sigma_1 - \frac{\nu}{E}d\sigma_2 + \frac{1}{E}d\sigma_3 + \frac{dp}{3H} \quad (3.35)$$

$$df = \frac{1}{H}d\sigma + \frac{1}{R}dp \text{ with } \sigma = \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3) \quad (3.36)$$

The elastic constants E (Young's modulus) and ν (Poisson ratio) are defined for drained conditions ($dp = 0$).

In general coordinates, there are shear strains and stresses, thus seven equations since the stress and strain tensors are symmetric

$$d\epsilon_{xx} = \frac{1}{E}d\sigma_{xx} - \frac{\nu}{E}d\sigma_{yy} - \frac{\nu}{E}d\sigma_{zz} + \frac{dp}{3H} \quad (3.37)$$

$$d\epsilon_{yy} = -\frac{\nu}{E}d\sigma_{xx} + \frac{1}{E}d\sigma_{yy} - \frac{\nu}{E}d\sigma_{zz} + \frac{dp}{3H} \quad (3.38)$$

$$d\epsilon_{zz} = -\frac{\nu}{E}d\sigma_{xx} - \frac{\nu}{E}d\sigma_{yy} + \frac{1}{E}d\sigma_{zz} + \frac{dp}{3H} \quad (3.39)$$

$$d\epsilon_{xy} = \frac{1}{2G}d\sigma_{xy} \quad (3.40)$$

$$d\epsilon_{yz} = \frac{1}{2G}d\sigma_{yz} \quad (3.41)$$

$$d\epsilon_{xz} = \frac{1}{2G}d\sigma_{xz} \quad (3.42)$$

$$df = \frac{1}{H}d\sigma + \frac{1}{R}dp \quad (3.43)$$

where G is the shear modulus which can be related to E and ν via $G = E/2(1 + \nu)$.

The first 6 of these equations can be written in standard index notation

$$\epsilon_{ij} = \frac{1}{2G} \left(\sigma_{ij} - \frac{\nu}{1 + \nu} \sigma_{kk} \delta_{ij} \right) + \frac{p}{3H} \delta_{ij} \quad (3.44)$$

with $\sigma_{kk} = \frac{1}{3}(\sigma_{xx} + \sigma_{yy} + \sigma_{zz})$ and the notation d has been dropped so that the variable now indicates a change in that variable. Using $\alpha = K/H$ and $K = \frac{2(1+\nu)}{3(1-2\nu)}G$ to include the last term into the parenthesis, (3.44) becomes

$$\epsilon_{ij} = \frac{1}{2G} \left(\sigma_{ij} - \frac{\nu}{1 + \nu} \sigma_{kk} \delta_{ij} + \frac{1 - 2\nu}{1 + \nu} \alpha p \delta_{ij} \right). \quad (3.45)$$

As defined in Eq. (3.11), the effective stress $\sigma'_{ij} = \sigma_{ij} + \alpha p \delta_{ij}$, then

$$\epsilon_{ij} = \frac{1}{2G} \left(\sigma'_{ij} - \frac{\nu}{1 + \nu} \sigma'_{kk} \delta_{ij} \right). \quad (3.46)$$

Equation (3.44) can be rearranged into equivalent but useful forms

$$\sigma_{ij} = 2G \left(\epsilon_{ij} + \frac{\nu}{1-2\nu} \epsilon_{kk} \delta_{ij} \right) - \alpha p \delta_{ij} \quad (3.47)$$

$$\epsilon_{ij} = \frac{1}{2G} \left(\sigma_{ij} - \frac{\nu_u}{1+\nu_u} \sigma_{kk} \delta_{ij} \right) + \frac{B}{3} f \delta_{ij} \quad (3.48)$$

where ν_u is the *undrained Poisson ratio* and is related to ν by

$$\nu_u = \frac{3\nu + \alpha B(1-2\nu)}{3 - \alpha B(1-2\nu)}. \quad (3.49)$$

3.2.5 Poroelastic Constants

There are a number of poroelastic constants that depend on different constraints on the Representative Elementary Volume (REV).

Compressibility There are several measures of the compressibility of porous rocks depending on the constraints on the REV. Following Wang (2000) we use a thought experiment to help define the different measures of compressibility. In the thought experiment, a rock is subjected to a confining pressure p_c and an independently controlled pore pressure p . We define a differential pressure $p_d = p_c - p$ and use p_d and p as the independent variables.

If the sample isunjacketed, $p_c = p$ and hence $p_d = 0$; we may define two bulk moduli

$$\frac{1}{K'_s} = -\frac{1}{V} \left(\frac{\partial V}{\partial p} \right)_{p_d=0} \quad \text{and} \quad \frac{1}{K_\phi} = -\frac{1}{V_p} \left(\frac{\partial V_p}{\partial p} \right)_{p_d=0} \quad (3.50)$$

where V is the sample volume, V_p is the pore volume, K'_s is the *unjacketed bulk modulus*, and K_ϕ is the *unjacketed pore modulus*.

If the sample is jacketed and drained, $p = 0$, $p_c = p_d$; we may define two more bulk moduli

$$\begin{aligned} \frac{1}{K} &= -\frac{1}{V} \left(\frac{\partial V}{\partial p_c} \right)_{p=0} = -\frac{1}{V} \left(\frac{\partial V}{\partial p_d} \right)_{p=0} \quad \text{and} \\ \frac{1}{K_p} &= -\frac{1}{V_p} \left(\frac{\partial V_p}{\partial p_c} \right)_{p=0} = -\frac{1}{V_p} \left(\frac{\partial V_p}{\partial p_d} \right)_{p=0} \end{aligned} \quad (3.51)$$

where K is the *drained bulk modulus* and K_p is the *drained pore modulus*. Then

$$\frac{dV}{V} = \frac{1}{V} \left[\left(\frac{\partial V}{\partial p} \right)_{p_d=0} dp + \left(\frac{\partial V}{\partial p_d} \right)_{p=0} dp_d \right] = -\frac{1}{K'_s} dp - \frac{1}{K} dp_d \quad (3.52)$$

$$\begin{aligned} \frac{dV_p}{V_p} &= \frac{1}{V_p} \left[\left(\frac{\partial V_p}{\partial p} \right)_{p_d=0} dp + \left(\frac{\partial V_p}{\partial p_d} \right)_{p=0} dp_d \right] \\ &= -\frac{1}{K_\phi} dp - \frac{1}{K_p} dp_d. \end{aligned} \quad (3.53)$$

Taking the confining pressure as the applied stress, we may replace $d\sigma$ by $-dp_c$ and rewrite Eq. (3.5) as

$$\epsilon = \frac{dV}{V} = -\frac{1}{K} dp_c + \frac{1}{H} dp = -\frac{1}{K} dp_d - \left(\frac{1}{K} - \frac{1}{H} \right) dp. \quad (3.54)$$

Comparing (3.54) with (3.52), we obtain

$$\frac{1}{K'_s} = \frac{1}{K} - \frac{1}{H} = \frac{1}{K} (1 - \alpha), \quad \text{or} \quad \alpha = 1 - \frac{K}{K'_s}. \quad (3.55)$$

The poroelastic constant K'_s can thus be obtained from the measureable constants K and α . Similarly, the other poroelastic constants K_p and K_ϕ can all be written in terms of the measurable constants K , K_f , α , B and ϕ .

For the drained pore modulus K_p , we start from its definition (3.51). For a fully saturated medium, $\phi = V_p/V = V_f/V$. The increment of water is

$$df = \frac{dV_p - dV_f}{V} = \phi \left(\frac{dV_p}{V_p} - \frac{dV_f}{V_f} \right) = \phi \left(\frac{dV_p}{V_p} + \frac{dp}{K_f} \right). \quad (3.56)$$

where K_f is the compressibility of the fluid

$$\frac{1}{K_f} = -\frac{1}{V_f} \frac{dV_f}{dp}. \quad (3.57)$$

From (3.6) we have

$$df = \frac{1}{H} d\sigma + \frac{1}{R} dp = -\frac{1}{H} dp_c + \frac{1}{R} dp. \quad (3.58)$$

Solving (3.56) for dV_p/V_p and using (3.58) we have

$$\frac{dV_p}{V_p} = -\frac{1}{\phi H} dp_c + \left(\frac{1}{\phi R} - \frac{1}{K_f} \right) dp$$

$$= -\frac{\alpha}{\phi K} dp_c + \left(\frac{\alpha}{\phi B K} - \frac{1}{K_f} \right) dp \quad (3.59)$$

and thus

$$\frac{1}{K_p} = -\frac{1}{V_p} \frac{dV_p}{dp_c} \Big|_{p=0} = \frac{\alpha}{\phi K}. \quad (3.60)$$

The unjacketed pore modulus K_ϕ can also be related to other poroelastic constants. Replacing p_c in (3.59) by $p_d + p$ we have

$$\frac{dV_p}{V_p} = -\frac{\alpha}{\phi K} dp_d + \left(\frac{\alpha}{\phi B K} - \frac{\alpha}{\phi K} - \frac{1}{K_f} \right) dp \quad (3.61)$$

and thus

$$\frac{1}{K_\phi} = -\frac{1}{V_p} \frac{dV_p}{dp} \Big|_{p_d=0} = \frac{\alpha}{\phi K} \left(1 - \frac{1}{B} \right) + \frac{1}{K_f}. \quad (3.62)$$

Storage capacity There are several measures of storage capacity of a rock depending on the constraints on the REV.

As defined earlier (3.14), the *unconstrained specific storage* S_σ is

$$S_\sigma = \frac{\partial f}{\partial p} \Big|_\sigma = \frac{1}{R} = \frac{\alpha}{K B}. \quad (3.63)$$

A micromechanical interpretation is provided by using (3.55) and (3.62) in (3.63)

$$S_\sigma = \left(\frac{1}{K} - \frac{1}{K'_s} \right) + \phi \left(\frac{1}{K_f} - \frac{1}{K_\phi} \right). \quad (3.64)$$

Also defined earlier (3.16) is the *constrained specific storage* S_ϵ

$$S_\epsilon = \frac{\partial f}{\partial p} \Big|_\epsilon = S_\sigma - \frac{K}{H^2} = S_\sigma - \frac{\alpha^2}{K} \quad (3.65)$$

S_ϵ can also be expressed in terms of G , ν and ν_u (see Detournay and Cheng 1993)

$$S_\epsilon = \frac{\alpha^2(1 - 2\nu_u)(1 - 2\nu)}{2G(\nu_u - \nu)}. \quad (3.66)$$

The *unjacketed specific storage* S_γ

$$S_\gamma = \left(\frac{\partial f}{\partial p} \right)_{p_d=0} = \phi \left(\frac{1}{K_f} - \frac{1}{K_\phi} \right). \quad (3.67)$$

The hydrogeological definition of the *uniaxial specific storage* S_s is

$$S_s = \rho_f g \left(\frac{\partial f}{\partial p} \right)_{\sigma_{zz}, \epsilon_{xx} = \epsilon_{yy} = 0} \quad (3.68)$$

where z is the vertical direction, and x and y are two horizontal directions. Hence it is the volume of water released per unit volume per unit decline in hydraulic head while maintaining zero lateral strain and constant vertical stress.

The constraints $\epsilon_{xx} = \epsilon_{yy} = 0$ and $\sigma_{zz} = 0$ can be used in Eq. (3.45) to obtain σ_{xx} and σ_{yy} which can be summed to obtain

$$\sigma_{kk} = -4\eta p \quad (3.69)$$

where

$$\eta = \alpha \frac{1 - 2\nu}{2(1 - \nu)}. \quad (3.70)$$

Thus the change in the mean stress is a scalar multiple of the change in pore pressure.

Substituting (3.69) and (3.63) into (3.19) and using $S = \frac{S_s}{\rho_f g}$, we have the storage coefficient

$$S = S_\sigma \left(1 - \frac{4\eta B}{3} \right). \quad (3.71)$$

It can also be shown that

$$S_\sigma \geq S \geq S_\epsilon. \quad (3.72)$$

The different storage coefficients illustrate the significance of boundary conditions on the REV for poroelastic behavior.

With so many different storage coefficients it may be useful to estimate their difference in practical applications. Here we examine the difference between $S = S_s/\rho_f g$ and S_σ . From (3.71) and the definition of η (3.70) we have $S/S_\sigma = 1 - \frac{2(1-2\nu)}{3(1-\nu)} \alpha B$. Assuming $\alpha B \sim 0.5$ and $\nu \sim 0.25$ for most rocks with experimental data, we have $S/S_\sigma \approx 0.8$. Hence, in practice, the differences among different specific storages may be small in comparison with the usually large uncertainties in this parameter.

Poroelastic expansion coefficient One of the Biot (1941) parameters, the *poroelastic expansion coefficient*, is, from (3.11) and (3.55),

$$\frac{1}{H} = \left(\frac{\partial \epsilon}{\partial p} \right)_\sigma = \frac{\alpha}{K} = \frac{1}{K} - \frac{1}{K'_s}. \quad (3.73)$$

The poroelastic expansion coefficient is thus the difference between the bulk compressibility and theunjacketed compressibility.

If there is no horizontal strain, i.e., $\epsilon_{xx} = \epsilon_{yy} = 0$, we may derive the vertical stress from (3.47) as

$$\sigma_{zz}|_{\epsilon_{xx}=\epsilon_{yy}=0} = \frac{2G(1-\nu)}{1-2\nu} \epsilon_{zz} - \alpha p = K_v \epsilon_{zz} - \alpha p, \quad (3.74)$$

where

$$K_v = \frac{2G(1-\nu)}{1-2\nu} \quad (3.75)$$

is the *vertical incompressibility*. Rearranging for the strain

$$\epsilon_{zz} = \frac{1}{K_v} \sigma_{zz}|_{\epsilon_{xx}=\epsilon_{yy}=0} + \frac{\alpha}{K_v} p. \quad (3.76)$$

Thus, for constant vertical stress, the volumetric strain is proportional to the pore pressure change, and the constant of proportionality

$$c_m = \alpha / K_v, \quad (3.77)$$

is known as the *Geertsma uniaxial expansion coefficient* (and can be shown to equal η/G).

Coefficients of undrained pore pressure buildup Skempton's coefficient $B = -\left(\frac{\partial p}{\partial \sigma}\right)_{f=0}$ can also be expressed in terms of compressibilities. Using $K_u = K/(1 - \alpha B)$ and $\alpha = 1 - K/K'_s$, we have

$$B = \frac{1 - K/K_u}{1 - K/K'_s}. \quad (3.78)$$

Using (3.62) and $\alpha = 1 - K/K'_s$, we also have

$$B = \frac{1/K - 1/K'_s}{1/K - 1/K'_s + \phi(1/K_f - 1/K_\phi)}. \quad (3.79)$$

Finally, B can also be written in terms of Poisson ratios

$$B = \frac{3(\nu_u - \nu)}{\alpha(1 + \nu_u)(1 - 2\nu)}. \quad (3.80)$$

Returning to the case with no horizontal strains, we can define a uniaxial undrained loading efficient as

$$\gamma = - \left(\frac{\partial p}{\partial \sigma_{zz}} \right)_{\epsilon_{xx}=\epsilon_{yy}=0, f=0}. \quad (3.81)$$

From the constitutive law (3.43) with $\epsilon_{xx} = \epsilon_{yy} = 0$ and $f = 0$, we have

$$\sigma_{xx}|_{\epsilon_{xx}=\epsilon_{yy}=f=0} = \sigma_{yy}|_{\epsilon_{xx}=\epsilon_{yy}=0, f=0} = \frac{v_u}{1-v_u} \sigma_{zz}. \quad (3.82)$$

Since $\sigma = \frac{1}{3}(\sigma_{xx} + \sigma_{yy} + \sigma_{zz}) = \frac{1}{3} \frac{1+v_u}{1-v_u} \sigma_{zz}$ and $B = - \left(\frac{\partial P}{\partial \sigma} \right)_f$

$$\gamma = - \left(\frac{\partial P}{\partial \sigma_{zz}} \right)_{\epsilon_{xx}=\epsilon_{yy}=f=0} = - \left(\frac{\partial P}{\partial \sigma} \right)_f \left(\frac{\partial \sigma}{\partial \sigma_{zz}} \right)_{\epsilon_{xx}=\epsilon_{yy}=f=0} = \frac{B}{3} \frac{1+v_u}{1-v_u} \quad (3.83)$$

The *tidal efficiency* TE can be defined as the water level change in a well divided by the water level change in the ocean. The uniaxial strain condition is often assumed for aquifers close to a shoreline; thus $TE = \gamma$.

γ can also be expressed in terms of other poroelastic constants, e.g.,

$$\gamma = \frac{\alpha}{K_v S}. \quad (3.84)$$

If we further assume $1/K'_s = 1/K_\phi = 0$, then

$$S = \frac{1}{K_v} + \phi \frac{1}{K_f}. \quad (3.85)$$

Combining (3.84) and (3.85) we have

$$TE = \gamma = \frac{1/K_v}{(1/K_v) + (\phi/K_f)} = \frac{K_f}{K_f + \phi K_v}. \quad (3.86)$$

Since ϕ and K_f can be measured, this equation may be used to estimate the vertical incompressibility (K_v) of aquifers beneath the ocean and near the shoreline from measurement of γ or TE .

Another surface source of loading is variation in atmospheric pressure. The *barometric efficiency* is defined as the ratio between the change in water level dh in a well to the change in atmospheric pressure dp_{atm} converted to an equivalent head $dp_{atm}/\rho_f g$, i.e.,

$$BE = -\rho_f g dh / dP_a. \quad (3.87)$$

where P_a is the barometric pressure, and the negative sign is included to make BE positive. The atmospheric pressure exerts a load both on the surface of the Earth and the water surface in the well. The former causes water level to rise by an amount $\gamma dP_a/\rho_f g$ and the latter for the level to drop by $dP_a/\rho_f g$. Thus $dh = (\gamma - 1)dP_a/\rho_f g$, and

$$BE = 1 - \gamma. \quad (3.88)$$

This relation is only for purely confined aquifers under undrained condition where the BE is a constant. Assume $1/K'_s = 1/K_\phi = 0$, we may have

$$BE = \frac{\phi K_v}{K_f + \phi K_v}. \quad (3.89)$$

Since K_f is known, independent measures of both TE and BE yield estimates of the aquifer porosity (ϕ) and the vertical incompressibility (K_v). For semi-confined aquifers (i.e., leaky aquifers), the barometric efficiency is no longer a constant but depends on frequency, i.e., $BE(\omega)$ and is sometimes denoted as the ‘barometric response function’ (e.g., Rojstaczer et al., 1988) to distinguish from the constant barometric efficiency for purely confined aquifers. Determination of $BE(\omega)$ and its hydrogeological applications are discussed in Chap. 5.

3.2.6 Governing Equations for Flow in Poroelastic Media

Coupling between elastic deformation and fluid flow occurs between the equation for fluid flow (assumed to be Darcy’s law) and conservation of mass. We begin with the continuity equation for fluid

$$\frac{\partial f}{\partial t} = -\nabla \cdot \mathbf{q} + Q, \quad (3.90)$$

where $\mathbf{q}$ is the specific discharge vector and Q is the fluid source per unit volume per unit time.

Substituting Darcy’s law into the conservation of mass equation yields

$$\frac{\partial f}{\partial t} = \frac{k}{\mu} \nabla^2 p + Q, \quad (3.91)$$

where k is permeability (assumed constant in space) and μ the fluid viscosity. Assuming uniaxial strain and constant vertical stress, then $f = Sp$ (Eq. 3.68), which when used in Eq. (3.90) leads to the standard flow equation in hydrogeology

$$S \frac{\partial p}{\partial t} = \frac{k}{\mu} \nabla^2 p + Q. \quad (3.92)$$

The assumption of uniaxial strain and constant vertical stress are not satisfied in 2D and 3D flows in general because the flow distorts the strain field. A more general flow equation can be obtained by replacing (3.27) in (3.91), leading to

$$\frac{\alpha}{KB} \left[\frac{B}{3} \frac{\partial \sigma_{kk}}{\partial t} + \frac{\partial p}{\partial t} \right] = \frac{k}{\mu} \nabla^2 p + Q, \quad (3.93)$$

where the first term on the left, the time derivative of the mean stress, is equivalent mathematically to a fluid source.

Since $S_\sigma = \alpha/KB$ (Eq. 3.63), then

$$S_\sigma \left[\frac{B}{3} \frac{\partial \sigma_{kk}}{\partial t} + \frac{\partial p}{\partial t} \right] = \frac{k}{\mu} \nabla^2 p + Q. \quad (3.94)$$

Using the relations $\sigma_{kk}/3 = K\epsilon - \alpha p$ and $S_\epsilon = (1 - \alpha B)S_\sigma$, Eq. (3.94) may be transformed to

$$\alpha \frac{\partial \epsilon_{kk}}{\partial t} + S_\epsilon \frac{\partial p}{\partial t} = \frac{k}{\mu} \nabla^2 p + Q, \quad (3.95)$$

in which fluid flow is coupled to the time variation of volumetric strain that acts mathematically as a fluid source.

These PDEs are inhomogeneous even when there are no explicit fluid sources Q . Simplification to a homogeneous diffusion equation needs to be justified throughout the region of interest.

3.2.7 Uncoupling Stress or Strain from Fluid Flow

Uncoupling in poroelasticity means that the mechanical term in the fluid flow equation is omitted. The uncoupling is one-way in that the pore-pressure field does affect stress and strain, but the changes in stress and strain do not affect fluid flow. In such cases, the transient flow equation can be solved independently and the resulting fluid pressure field then be inserted into the elastostatic equation and solved separately.

Uncoupling of the mechanical term in the fluid flow equation occurs in several classes of poroelastic problems. An important class is irrotational displacement in an infinite or semi-infinite domain without a body force, and includes deformation under barometric loading and deformation affected by groundwater flow between a well and aquifer. To illustrate how the uncoupling occurs in an irrotational displacement field, we insert the constitutive relation (3.47) into the force balance equation $\frac{\partial \sigma_{ij}}{\partial x_i} = 0$

to obtain the mechanical equilibrium equation (Wang 2000, Eq. 4.18)

$$G \nabla^2 u_i + \frac{G}{1-2\nu} \frac{\partial \epsilon}{\partial x_i} = \alpha \frac{\partial p}{\partial x_i}. \quad (3.96)$$

x_i For irrotational displacement fields, $\partial/\partial = \partial/\partial$, where is the displacement component in the direction; thus

$$G \nabla^2 \epsilon_{ii} + \frac{G}{1-2\nu} \frac{\partial^2 \epsilon}{\partial x_i^2} = \alpha \frac{\partial^2 p}{\partial x_i^2}, \quad (3.97)$$

Summing the three equations of (3.97) for $i = 1, 2$ and 3 , we arrive at, with (3.75)

$$\nabla^2 \left(\epsilon - \frac{\alpha}{K_v} P \right) = 0. \quad (3.98)$$

In the absence of a body force, equation (3.98) reduces to

$$\frac{\partial^2 w}{\partial z^2} = \frac{\alpha}{K_v} \frac{\partial P}{\partial z}. \quad (3.99)$$

For irrotational displacement fields, $\partial u_i / \partial x_j = \partial u_j / \partial x_i$, where u_i is the displacement component in the x_i direction; thus

$$\nabla^2 u_i = \frac{\partial^2 u_i}{\partial x_1^2} + \frac{\partial^2 u_i}{\partial x_2^2} + \frac{\partial^2 u_i}{\partial x_3^2} = \frac{\partial}{\partial x_i} \left(\frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2} + \frac{\partial u_3}{\partial x_3} \right) = \frac{\partial \epsilon}{\partial x_i}. \quad (3.100)$$

Equation (3.100) allows us to rewrite Eq. (3.99) as

$$\frac{\partial \epsilon_z}{\partial z} = \frac{\alpha}{K_v} \frac{\partial P}{\partial z}. \quad (3.101)$$

Integrating Eq. (3.101) yields

$$\epsilon_z = \frac{\alpha}{K_v} P + g(t), \quad (3.102)$$

where $g(t)$ is a ‘constant’ of integration that may be a function of t . Given the boundary conditions that ϵ_z and P must vanish at infinity, the function $g(t)$ must also vanish. Combining (3.102) with the constitutive relation in Eq. (3.18) leads to

$$\sigma_{kk} = -4\eta P. \quad (3.103)$$

Substituting (3.103) into Eq. (3.94) and using $S = S_\sigma(1 - 4\eta B/3)$ yields the decoupled flow equation

$$S \frac{\partial P}{\partial t} = \frac{k}{\mu} \nabla^2 P + Q. \quad (3.104)$$

Therefore, for an irrotational displacement field in an unbounded or semi-infinite space in the absence of body force, the flow equation is decoupled from stress or strain. These results are applied in the following section on the study of soil consolidation.

3.3 Consolidation

The relationships between stress and strain at sufficiently large deformation become nonlinear and the corresponding analysis may become complicated except in simplified cases. Thus we provide in this section mostly a qualitative description of a hydro-mechanical geologic process, i.e., the consolidation of sediments, which is ubiquitous on Earth's surface, especially in sedimentary basins, in which the hydrological responses to earthquakes have been abundantly documented. As illustrated in the cartoon of Fig. 3.5, earthquake shaking may be strong enough to break the grain-to-grain contact in unconsolidated sediments and cause the transfer of load from the solid matrix to the pore fluids; given time, the pressurized pore fluids may be expelled and the sediments may consolidate under gravity.

3.3.1 Consolidation of Sediments in Sedimentary Basin

Consolidation is the process of compaction of sediments owing to the effects of gravity under drained conditions, when enough time is available for pore fluids to

Fig. 3.5 Idealized schematic of pore-pressure change in soils during earthquakes. **a** Before an earthquake, individual soil grains are held in place by frictional or adhesive contact forces, creating a solid soil structure with water filling the spaces between the grains. Note the grain-to-grain contact. **b** After seismic shaking, particle rearrange with no change in volume (e.g., a lateral shift of a half diameter of every other row of particles in the figure), causing the particles to lose contact and go into suspension, and increased pore pressure as gravity load is transferred from the soil skeleton to the pore water. **c** As water flows out of the soil, pore pressure decreases, the soil particles settle into a denser configuration (National Academy of Sciences, Engineering and Medicine 2016)

move to the surface. Observations show that the porosity of sediments in sedimentary basins decreases with depth. Athy's law (Athy 1930), one of the several widely used empirical relations to describe the decrease of sediment porosity with depth (or increasing effective pressure), has the form

$$\varphi = \varphi_o e^{-bP'}, \quad (3.105)$$

where φ_o is the porosity at zero effective pressure, b is an empirical parameter, and P' is the effective pressure. Athy's law was based on field measurements, where additional processes such as chemical precipitation or pressure solution, in addition to consolidation, may have occurred to change the distribution of porosity with depth.

For porosities commonly encountered in sedimentary basins (0.2–0.5), b ranges from 10^{-7} to 10^{-8} Pa^{-1} for shale and from 10^{-8} to 10^{-9} Pa^{-1} for sandstone. It should be noted that in sedimentary basins, consolidation may not be the only process that changes sediment porosity. For example, the distribution of porosity may also be significantly affected by non-mechanical processes such as chemical precipitation and dissolution, and mineral transformations.

3.3.2 Terzaghi Theory of Consolidation

Terzaghi (1925) first analyzed the processes of soil consolidation with the effective stress relation. The discussion below follows the presentation in Wang (2000, Sect. 6.5). In Terzaghi's model, a constant stress $-\sigma_o$ is applied on the surface ($z = 0$) of a column of saturated soil of height L at time equal to 0. The piston applying the load is permeable such that the top boundary is drained. If the wall of the container is rigid, the deformation is uniaxial. As discussed in Sect. 3.2.7, the flow equation under uniaxial, poroelastic deformation is decoupled from stress or strain. We assume here that this decoupling may be extended to finite deformation. Thus the homogeneous diffusion equation (Eq. (3.104) with $D \equiv k/\mu S$ and $Q = 0$) applies

$$\frac{\partial P}{\partial t} - D \frac{\partial^2 P}{\partial z^2} = 0, \quad (3.106)$$

where D is the uniaxial hydraulic diffusivity.

The surface load produces an instantaneous undrained response

$$P_o = \gamma \sigma_o, \quad (3.107)$$

where γ is the loading efficiency (Eq. 3.81). Following the step deformation at $t = 0$, the sample consolidates gradually as water flows out of the top boundary. The boundary conditions are:

$$\sigma_{zz} = -\sigma_o, \quad (3.108)$$

$$w(L, t) = 0, \quad (3.109)$$

$$P(0, t) = 0, \quad (3.110)$$

$$\left. \frac{\partial P}{\partial z} \right|_{z=L} = 0. \quad (3.111)$$

The solution for the pore pressure in this boundary value problem is the same as that for the temperature in a classical heat conduction problem (Carslaw and Jaeger 1959, p. 96) and is

$$P(z, t) = \frac{4\gamma\sigma_o}{\pi} \sum_{m=0}^{\infty} \frac{1}{2m+1} \times \exp\left[\frac{-(2m+1)^2\pi^2 Dt}{4L^2}\right] \sin\left[\frac{(2m+1)\pi z}{2L}\right] \quad (3.112)$$

The calculated pore pressure, normalized by the initial pressure P_o , is plotted in Fig. 3.6 against z/L at several values of the dimensionless time $\tau = Dt/L^2$. Initially, the decline in pore pressure is confined to the top region near the drained boundary. At large τ , $P(z, \infty) = 0$.

The instantaneous mechanical response is the undrained poroelastic response and may be obtained from the uniaxial constitutive relation (3.75)

Fig. 3.6 Evolution of normalized pore pressure in a column of sediments of height L under a vertical load on the surface, suddenly applied at $t = 0$ (from Wang, 2000). The successive contours are for dimensionless time $\tau = ct/L^2$

$$K_v \equiv \frac{\delta\sigma_{zz}}{\delta\epsilon_{zz}}|_{\epsilon_{xx}=\epsilon_{yy}=f=0} = \frac{2G(1-\nu)}{1-2\nu}. \quad (3.113)$$

Integrating (3.113) we have

$$\sigma_{zz}|_{\epsilon_{xx}=\epsilon_{yy}=0} = \frac{2G(1-\nu_u)}{1-2\nu_u}\epsilon_{zz}. \quad (3.114)$$

Thus,

$$\frac{dw}{dz} \equiv \epsilon_{zz}|_{f=\epsilon_{xx}=\epsilon_{yy}=0} = \frac{1-2\nu_u}{2G(1-\nu_u)}\sigma_o = \frac{1}{K_v^{(u)}}\sigma_o. \quad (3.115)$$

Integrating Eq. (3.115) from L to z with the boundary condition $w = 0$ at $z = L$, we have

$$w(z, 0^+) = \frac{\sigma_o(L-z)}{K_v^{(u)}} \quad (3.116)$$

Thus, the initial displacement at the surface ($z = 0$) is

$$w_o \equiv w(0, 0^+) = \frac{\sigma_o L}{K_v^{(u)}}. \quad (3.117)$$

The time-dependent displacement $\Delta w(z, t)$ during the drainage phase may be calculated from the pore pressure,

$$\Delta P(z, t) = P(z, t) - P_o. \quad (3.118)$$

where $P(z, t)$ is given in Eq. (3.112) and P_o is given in Eq. (3.107). Utilizing (3.102) by replacing P in this equation by ΔP in Eq. (3.118) and integrating $\partial w / \partial z = \epsilon_z$ with the resulting series solution for w from $z = L$ (where $\Delta w = 0$ hence $g = 0$) to $z = 0$, we obtain (Wang 2000, Eq. 6.41)

$$\Delta w(0, t) = \frac{\alpha\gamma\sigma_o L}{K_v} \left\{ 1 - \frac{8}{\pi^2} \sum_{m=0}^{\infty} \frac{1}{(2m+1)^2} \exp\left[-\frac{(2m+1)^2\pi^2 Dt}{4L^2}\right] \right\} \quad (3.119)$$

where, as defined earlier, α and K_v are the Biot-Willis coefficient and the vertical compressibility, respectively.

The series solution (3.119) varies from 0 at $t = 0$ to $\frac{\alpha\gamma\sigma_o L}{K_v}$ at $t = \infty$, and is plotted against $\log(Dt/L^2)$ in Fig. 3.7. The total surface displacement is

$$w(0, t) = \frac{\sigma_o L}{K_v^{(u)}} + \frac{\alpha\gamma\sigma_o L}{K_v}. \quad (3.120)$$

Fig. 3.7 Surface displacement change of a column of saturated soil of height L during the drainage phase, following a step loading at the surface (3.119), normalized with $\Delta w(0, \infty) = \alpha \gamma \sigma_o L / K_v$ and plotted against $\log(Dt/L^2)$ (from the authors)

The poroelastic parameters in the above model are assumed constant to simplify integration. In reality, these parameters are not constant at finite deformation. On the other hand, the solutions so obtained do provide some insight into the consequence of sediment consolidation and an order-of-magnitude estimate of the settlement of a sedimentary layer under consolidation.

3.4 Liquefaction

In the previous section we showed that deformation of sediments in sedimentary basins due to consolidation is a function of time and that, in response to instantaneous loading, pore pressure in the sediments increases. Here we review a particular time-dependent deformation of sediments, which occurs when saturated sediments are subjected to cyclic loading such as that generated by earthquakes. This phenomenon became widely studied in the earthquake engineering communities after the great 1960 M9.2 Alaska earthquake that caused widespread liquefaction and property damage. Many laboratory and field measurements have been performed on saturated sediments to simulate the response of sedimentary basins and soils to seismic shaking. These experiments demonstrate that, as seismic waves propagate through saturated sediments, pore pressure increases. Pore pressure increases because sediments consolidate under cyclic deformation, during which part of the load on the

solid matrix is transferred to the pore fluids (e.g., Figure 3.5). If there is little time for pore pressure to dissipate, pore pressure may continue to rise under ongoing shaking. If the rising pore pressure stays below the overburden pressure, the effective pressure will remain positive, and the sediments will remain solid. Pore pressure will eventually dissipate and sediments will consolidate. On the other hand, if the rising pore pressure reaches the magnitude of the overburden pressure during seismic shaking, the overburden is supported entirely by the pore pressure and the sediments will lose strength and become fluid-like—a phenomenon known as liquefaction.

Figure 3.8 shows the deformation of porous sediments under cyclic loading in a drained condition (Luong 1980). If the shear stress is below a characteristic threshold, cyclic shearing causes the volume of the sheared sample to decrease (Fig. 3.8a). On the other hand, at shear stress above the threshold, cyclic shearing causes the volume of the sheared sample to increase (Fig. 3.8b). The transition between the two responses, where no contraction or dilatancy occurs, corresponds to a ‘critical state’ in soil mechanics (Wang et al. 2001; National Academies

Fig. 3.8 Deviatoric stress versus volumetric strain for two sand samples of the same constitution. **a** Shearing at a maximum deviatoric stress ‘ q ’ of 0.2 MPa. Note that the volumetric strain decreases with increasing number of stress cycles and the sample contracts under cyclic shearing. **b** Shearing at a maximum deviatoric stress of 0.25 MPa. Note that the volumetric strain increases with increasing number of stress cycles and the sample dilates under cyclic shearing. (from Luong 1980)

Fig. 3.9 Experimental results for pore-pressure generation in eight different sands with dry density from 20 to 80% of the mineral density, under initial confining pressure from 0.25 to 1.9 kPa, and 10 cycles of uniform loading. Note that, for sands with such diverse densities and confining pressures, pore-pressure buildup begins at a threshold strain of 10^{-4} (From Vucetic 1994)

of Science, Engineering and Medicine 2016) where volume stays unchanged during deformation.

Note that the case in Fig. 3.8a is normally reported because experiments are normally performed at shear stresses below that required to cause shear-induced dilatation. Using a wide variety of saturated sediments under a wide range of confining pressures, Dobry et al. (1982), Vucetic (1994) and Hsu and Vucetic (2004) showed that pore pressure begins to increase when sediments are sheared above a strain amplitude of 10^{-4} (Fig. 3.9), defining a threshold strain amplitude for the initiation of sediment consolidation.

Seed and Lee (1966) showed how deformation and pore pressure changed in two sand specimens under cyclic shearing at constant stress amplitude of ± 50 kPa (Fig. 3.10). Note that the loose sand failed at the 9th stress cycle, while the dense sand withstood hundreds of stress cycles without complete failure. In the loose sand experiment, the axial strain was small during the first eight cycles, but increased significantly afterwards. The large strains during 9th stress cycle suggest that the sample was failing. The loss in rigidity, and thus the occurrence of liquefaction, is marked by the drastic increase in shear strain at the 10th cycle of the experiment.

Experiments performed at constant strain amplitude are complimentary to those performed at constant stress amplitude (Fig. 3.11). Note that pore pressure starts to increase from the first strain cycle and continues to increase while the stress amplitude continues to decline, suggesting that the sample is weakening. Stress amplitude is reduced to zero (i.e., sample liquefies) when pore pressure becomes equal to the confining pressure.

Fig. 3.10 Experimental stress-strain relations for **a** loose sand and **b** dense sand under cyclic shear stress of constant stress amplitude of 2 kg per cm² (0.2 MPa). The loose sand, at a dry density of 38% of the mineral density, showed relatively small deformation during the first eight cycles but failed at the 9th stress cycle. The dense sand at a dry density of 78%, on the other hand, did not fail for hundreds of stress cycles (from Seed and Lee 1966)

3.5 Rock Friction and Instability

Earth's shallow crust is permeated by fractures; frictional sliding and instability on these fractures are important processes controlling crustal deformation and the occurrence of shallow earthquakes. Because intact rocks usually have poor permeability, fractures may also control crustal permeability. Some fractures are 'opened' for fluid flow, while others may be clogged by precipitates and/or colloidal particles. Thus rock friction and instability can involve multiple interacting hydro-mechanical processes.

Fig. 3.11 Experimental results showing pore-pressure generation and mechanical weakening of a saturated sand specimen subjected to cyclic shearing at constant strain amplitude of $\pm 0.2\%$. The sediment specimen had an initial void ratio of 87% and was subjected to a confining pressure of 0.15 MPa and an initial pore pressure of 0.1 MPa. Note that the axial stress is reduced to zero when pore pressure becomes equal to the confining pressure (from Seed and Lee 1966)

3.5.1 Friction and Frictional Instability

Rock masses containing joints or fractures may be mechanically stable if the shear stresses on these surfaces are smaller than the frictional resistance. If the shear stress on one of the surfaces reaches the frictional resistance, the rocks on the two sides of the surface may slip past each other. If sliding occurs abruptly, the accumulated shear stress and strain are suddenly released. The sudden change in the state of stress and strain may give rise to an earthquake that is characterized by crustal vibrations and the emission of seismic waves. The time-dependent evolution of stress accumulation, development of instability, release of the crustal stress and strain, and the eventual recovery of the state of stress, is known as the elastic rebound theory for earthquakes (Reid 1911). The shear strength of the fault is linearly related to the normal stress and the friction coefficient by the well-known Coulomb effective friction law, which is discussed later in this section. Complications arise from the dependence of the friction coefficient on the slip velocity, which leads to slip instability and fault rupture, and is discussed in Sect. 3.5.2.

Brace and Byerlee (1966) proposed that instability in frictional sliding is the mechanism for earthquakes. Since then, a large amount of work has been done to

Fig. 3.12 Schematic diagram of shear force versus shear displacement in a frictional sliding experiment. At shear force below point C, the relation is linear and deformation is elastic. Stable sliding occurs between points C and D (stick). At point D, instability occurs and shear force suddenly releases (slip). Equilibrium re-establishes at point E. The stick-slip process repeats between points E and F (modified from Byerlee 1978)

understand friction in rocks and its instability. The schematic diagram in Fig. 3.12 plots the shear force on a sliding surface against shear displacement at a constant normal force. At small shear force (below point C on the curve), shear force is linearly proportional to displacement and deformation is elastic. Stable sliding occurs between points C and D, and displacement increases nonlinearly with shear force. At point D, instability occurs with a sudden release of shear force and little displacement. The system reaches a new equilibrium at point E at a lower shear force. Stable sliding commences again when shear force is once more increased until the second unstable release of shear force occurs at point F. The sequence of stable sliding followed by instability may repeat with continued deformation, and is commonly referred to as the stick-slip process.

Experimental observation shows that the shear stress at which unstable sliding occurs is related to the normal stress by the Coulomb failure criterion

$$\tau = c - \mu\sigma = c + \mu|\sigma|, \quad (3.121)$$

where τ is the applied shear stress at which unstable sliding occurs, σ is the normal stress (extension positive), μ the friction coefficient, and c the cohesion strength across the surface. Here we use the absolute magnitude of the normal stress $|\sigma|$ in Eq. (3.121), rather than $-\sigma$, in order to retain the sign convention in rock mechanics experiments where data are presented with compression taken to be positive.

Figure 3.13 shows the experimental data for τ versus σ on rock surfaces (Byerlee 1978) measured at elevated normal stress. The slope of the curve given by the experimental data defines the friction coefficient μ . At normal stress above ~ 100 MPa,

Fig. 3.13 Shear stress plotted as a function of normal stress at the maximum friction for a variety of rock types (from Byerlee 1978)

μ is ~ 0.6 for most rocks, although it may be much lower if the sliding surface is covered with clay minerals such as montmorillonite and vermiculite. At high normal stresses, such as those on faults at seismogenic depths, c is relatively small and is often neglected.

When the sliding surface is subjected to pore pressure, its frictional properties change significantly. Byerlee and Brace (1972) showed that, when fluids are present, it is not the normal stress that is important but the effective normal stress, as defined in Eq. (3.11). In other words, the failure criterion (3.121) should be replaced by

$$\tau = c + \mu(|\sigma| - \alpha P). \quad (3.122)$$

The relation between the state of stress on a surface and the failure criterion may be illustrated by the schematic diagram in Fig. 3.14. The state of stress on any surface in a rock may be represented by a point (τ, σ) on the Mohr circle (Fig. 3.14) where the angle θ is the angle between the surface and the direction of the maximum principle stress, and the failure criterion is represented by the straight line, i.e., equation (3.121). The surface is stable if this point falls below the failure criterion, but is unstable if

Fig. 3.14 Left: Definition of normal and shear stresses on a sliding surface inclined at angle θ with respect to σ_1 . Right: Graphic representation of the magnitude of normal and shear stresses as a function of the angle θ compared with the magnitude of shear resistance ($\tau + \tau_c$) (modified from Wang and Manga 2010)

it touches the failure criterion. It is convenient to define the difference between the shear stress on a surface and the frictional resistance on this surface as τ_c (Fig. 3.14), i.e.,

$$\tau_c = \tau - [c + \mu(|\sigma| - \alpha P)]. \quad (3.123)$$

When the shear stress τ acting on a surface (fault) equals the shear resistance (sum of cohesion and the product of the absolute magnitude of the effective normal stress and the friction coefficient), τ_c becomes zero and failure occurs. Failure may be brought about in different ways; Fig. 3.15 shows that failure may be induced by increasing the absolute magnitude of the maximum principle stress (Fig. 3.15a), by

Fig. 3.15 Three ways to induce failure on a fault by causing the Mohr circle to move to the failure criterion: **a** increasing the absolute magnitude of the maximum principle stress, **b** decreasing the absolute magnitude of the minimum principle stress, and **c** increasing the pore pressure on the fault (modified from Wang and Manga 2010)

Fig. 3.16 Average shear stress and the absolute magnitude of the normal stress on a laboratory fault (sawcut) in granite. Symbols of different shapes represent results of different experiments. Open symbols: stress condition on locked area before injection of water or just after the preceding stick-slip event. Solid symbols: stress condition just prior to moment of unstable stick-slip. Each arrow connecting two states of stress (open symbol and solid symbol) shows the change of stress conditions that led to instability, or rupture of the sliding surface. Dashed line: failure criterion for frictional sliding (from Shi and Wang 1985)

decreasing the absolute magnitude of the minimum principle stress (Fig. 3.15b), or by increasing the pore pressure on the fault (Fig. 3.15c).

These concepts were successfully tested in laboratory experiments. Figure 3.16 shows the results of an experiment of frictional sliding on a laboratory fault in granite. Many sets of experiments were carried out. Each open symbol shows the initial average shear stress and the absolute magnitude of the normal stress on the fault in an experiment when the fault was stable. Pressurized water was then injected into the fault to raise pore pressure. As a result, the absolute magnitude of the effective normal stress on the fault was reduced, as indicated by an arrow. Unstable sliding occurred when pore pressure was raised so high that the absolute magnitude of the average effective normal stress was reduced to the position of the solid symbols. The dashed line shows the failure criterion for frictional sliding. The assemblage of solid symbols falls along the failure criterion, confirming the concept illustrated in Fig. 3.15c that instability on faults is controlled by the effective normal stress, rather than by the normal stress itself.

3.5.2 The Rate-and-State Equation

While Brace and Byerlee (1966) proposed instability in frictional sliding as a mechanism for earthquakes, they did not address the transition from stable sliding to instability with the sudden release of shear stress. Dieterich (1994) bridged this gap with the rate-and-state equation, which is based on the experimental finding (Dieterich 1979) that the friction between two surfaces is not a constant but rather depends on the velocity of their relative displacement (Fig. 3.17). Dieterich (1994) assumed a spring-slider model to represent the nucleation of a single source and reformulated the frictional criterion as

$$\tau(t) - k\delta(t) = \sigma \left[\mu_o + A \log\left(\frac{v}{v_o}\right) + B \log\left(\theta(t) \frac{v_o}{d_c}\right) \right] \quad (3.124)$$

where $\tau(t)$ is the applied shear stress, $\delta(t)$ the frictional slip, σ the effective normal stress, v the slip velocity ($d\delta/dt$), v_o is a reference velocity, μ_o is a reference frictional coefficient corresponding to v_o , k the effective stiffness of the source patch, assumed constant, $-k\delta(t)$ the shear stress relaxed by slip, A and B are constitutive parameters relating friction to changes in slip speed and state, respectively, $\theta(t)$ is a state variable in the fault constitutive formulation, and is a characteristic slip distance over which fault state evolves.

Ruina's (1983) formulated the so-called aging law for the evolution of the state variable θ

Fig. 3.17 Shear stress on the sliding surface versus displacement in which the loading velocity is stepped by one decade. The normal stress is held constant at 5 MPa. The top curve is data. The lower two curves are simulations with different rate-and-state equations (from Linker and Dieterich 1992)

$$\dot{\theta} = 1 - \frac{\theta}{d_c} \dot{\delta} - \frac{\alpha \theta}{B \sigma} \dot{\sigma}, \quad (3.125)$$

where α is a parameter to account for the effect of the normal stress. Using this relation, Dieterich (1994) derived his rate-state equation

$$R = \frac{r}{\gamma \dot{\tau}_r}, \quad (3.126)$$

and

$$\dot{\gamma} = \frac{1}{A \sigma} \left[1 - \gamma \left(\dot{\tau} + \left(\frac{\tau}{\sigma} - \alpha \right) \dot{\sigma} \right) \right], \quad (3.127)$$

where R is the rate of seismicity, r is the background rate of seismicity, $\dot{\tau}_r$ is the background shear stressing rate and γ is a state variable.

The above relation may be tested against field seismicity data, but the state variable γ is difficult to estimate. Segall and Lu (2015) eliminated the state variable γ between the two equations to obtain a single equation for R

$$\frac{dR}{dt} = \frac{R}{t_a} \left(\frac{\dot{\tau}}{\dot{\tau}_r} - \frac{R}{r} \right), \quad (3.128)$$

where $t_a = \frac{A \sigma_o}{\dot{\tau}_r}$ is a characteristic decay time. This equation was used in recent studies of the seismicity in the mid-continental USA induced by the injection into deep aquifers large amounts of wastewater coproduced from the extraction of hydrocarbons (Segall and Lu 2015; Zhai et al. 2019), a subject discussed in Chap. 4. The slight difference between the above equation and Segall and Lu's Eq. (12) is to keep the definition of $R = dN/dt$, i.e., the rate of seismicity, instead of the rate of seismicity relative to the background rate.

Heimisson and Segall (2018) revisited and re-derived the rate-state equation. They omitted the 1 in Eq. (3.125) by assuming that the seismogenic source is 'well above the steady state' and then integrated the equation to obtain

$$\theta(t) = \theta_o e^{F(t)}, \quad (3.129)$$

where

$$F(t) = - \left[\frac{\delta(t)}{d_c} + \frac{\alpha}{B} \log \left(\frac{\sigma(t)}{\sigma_o} \right) \right]. \quad (3.130)$$

Inserting Eq. (3.129) and (3.130) into (3.124) they obtained

$$\frac{\tau(t) - k\delta(t)}{\sigma} = \mu_o + A \log\left(\frac{v}{v_o}\right) - B \left[\frac{\delta}{d_c} + \frac{\alpha}{B} \log\left(\frac{\sigma(t)}{\sigma_o}\right) + \log\left(\frac{v_o}{d_c} \theta_o\right) \right] \quad (3.131)$$

which may be solved for δ and v (Heimisson and Segall 2018)

$$\delta = -\frac{A}{H} \log\left(1 - \frac{H v_o}{A} \int_o^t K(t') dt'\right), \quad (3.132)$$

$$v = \frac{v_o K(t)}{1 - \frac{H v_o}{A} \int_o^t K(t') dt'}, \quad (3.133)$$

where

$$H = \frac{B}{d_c} - \frac{k}{\sigma_o}, \quad (3.134)$$

$$K(t) = \exp\left(\frac{\tau(t)}{A\sigma(t)} - \frac{\tau_o}{A\sigma_o}\right) \left(\frac{\sigma(t)}{\sigma_o}\right)^{\alpha/A}, \quad (3.135)$$

v_o is the sliding velocity at time $t = 0$ (taken to be the same as the background velocity) and τ_o and σ_o are the background shear stress and normal stress.

Instability occurs when the slip velocity v becomes singular; i.e., when the denominator on the right side of Eq. (3.133) vanishes,

$$\int_o^{t_{inst}} K(t') dt' = \frac{A}{H v_o}, \quad (3.136)$$

where t_{inst} is the time to instability. Solving this equation at constant effective normal stress σ_o and shear stressing rate $\dot{\tau}_r$, i.e., $K(t) = \exp(\dot{\tau}_r t / A\sigma_o)$, Heimisson and Segall (2018) obtained for the single spring-slider system

$$t_{inst} = \frac{A\sigma_o}{\dot{\tau}_r} \log\left(1 + \frac{\dot{\tau}_r}{H v_o \sigma_o}\right). \quad (3.137)$$

For a population of background seismic sources that fail at constant rate r , the time to instability of the N th source is N/r , and Eq. (3.136) may be rewritten as, assuming that N can take non-integer values,

Fig. 3.18 **a** Comparison of the predicted seismicity rate by Dieterich's (1994) rate-and-state equation (dashed lines) with the Coulomb stress approximation (dotted lines), and with the prediction by the formulation by Heimissson and Segall (2018) (solid lines). Black line indicates the shape of the Gaussian normal stress perturbation and $\Delta\sigma_p$ is the peak stress of the Gaussian perturbation. **b** Same as for (a), but showing the cumulative number of events (from Heimissson and Segall 2018)

$$\int_o^{N/r} \exp\left(\frac{\dot{\tau}_r t'}{A\sigma_o}\right) dt' = \left(\frac{A}{Hv_o}\right)_N, \quad (3.138)$$

which gives the value A/Hv_o for the N th source in a population of sources that fail at constant rate r under background conditions. Inserting (3.138) in Eq. (3.136) we have

$$\int_o^{t_{inst}} K(t') dt' = \int_o^{N/r} \exp\left(\frac{\dot{\tau}_r t'}{A\sigma_o}\right) dt'. \quad (3.139)$$

From this equation Heimissson and Segall (2018) derived the cumulative number of events N

$$\frac{N}{r} = t_a \log\left(1 + \frac{1}{t_a} \int_o^t K(t') dt'\right). \quad (3.140)$$

From $R = dN/dt$, the seismicity rate R is

$$\frac{R}{r} = \frac{K(t)}{1 + \frac{1}{t_a} \int_o^t K(t') dt'}. \quad (3.141)$$

Figure 3.18 compares the seismicity rate and the cumulative number of events predicted by Heimisson and Segall (2018) and those by Dieterich (1994). The two sets of predictions are similar except at large changes in the normal stress.

The rate-and-state equation has been central in the recent discussions of induced seismicity (e.g., Segall and Lu 2015; Zhai et al. 2019) as discussed in Chap. 4.

References

- Athy LF (1930) Density, porosity, and compaction of sedimentary rocks. *Am Asso Petrol Geol Bull* 14:1–24
- Barbour AJ (2015) Pore pressure sensitivities to dynamic strains: observations in active tectonic regions. *J Geophys Res* 120:5863–5883
- Barbour AJ, Xue L, Roeloffs E, Rubinstein JL (2019) Leakage and increasing fluid pressure detected in Oklahoma's wastewater disposal reservoir. *J Geophys Res* 124:2896–2919
- Biot MA (1941) General theory of three dimensional consolidation. *J Appl Phys* 12:155–164
- Biot MA (1955) Theory of elasticity and consolidation for a porous anisotropic solid. *J Appl Phys* 26:182–185. <http://dx.doi.org/10.1063/1.1721956>
- Brace WF, Byerlee JD (1966) Stick-slip as a mechanism for earthquakes. *Science* 153:990–992
- Burgess WG, Shamsudduha M, Taylor RG et al (2017) Terrestrial water load and groundwater fluctuation in the Bengal Basin. *Sci Rep* 7:3872. <https://doi.org/10.1038/s41598-017-04159-w>
- Byerlee JD (1978) Friction of rocks. *Pure Appl Geophys* 116:615–626
- Byerlee JD, Brace WF (1972) Fault stability and pore pressure. *Bull Seismol Soc Am* 62:657–660
- Carslaw HS, Jaeger JC (1959) *Conduction of heat in solids*. Clarendon Press, Oxford
- Cheng AHD (2016) *Poroelasticity*. Springer
- Detournay E, Cheng AHD (1993) Fundamentals of poroelasticity. In: Hudson JA (ed) *Comprehensive rock engineering: principles, practice and projects*, vol 2. Pergamon Press
- Dieterich JH (1979) Modeling of rock friction, 1, experimental results and constitutive equations. *J Geophys Res* 84:2161–2168
- Dieterich JH (1994) A constitutive law for rate of earthquake production and its application to earthquake clustering. *J Geophys Res* 99:2601–2618. <https://doi.org/10.1029/93JB02581>
- Dobry R, Ladd RS, Yokel FY, Chung RM, Powell D (1982) Prediction of pore water pressure buildup during earthquakes and liquefaction of sands during earthquakes by the cyclic strain method, vol 138. National Bureau of Standards
- Heimisson ER, Segall P (2018) Constitutive law for earthquake production based on rate-and-state friction: Dieterich 1994 revisited. *J Geophys Res Solid Earth* 123:4141–4156
- Hsu CC, Vucetic M (2004) Volumetric threshold shear strain for cyclic settlement. *J Geotech Geoenviron Eng* 130:58–70
- Linker MF, Dieterich JH (1992) Effects of variable normal stress on rock friction: observations and constitutive equations. *J Geophys Res* 97:4923–4940
- Luong MP (1980) Stress-strain aspects of cohesionless soils under cyclic and transient loading. In: Pande GN, Zienkiewicz OC (eds) *Proceedings of the international symposium on soils under cyclic and transient loading*. AA Balkema, Rotterdam, Netherlands, pp 315–324
- National Academy of Sciences, Engineering and Medicine (2016) *State of the art and practice in the assessment of earthquake-induced soil liquefaction and its consequences*. The National Academies Press, Washington, DC. <https://doi.org/10.17226/23474>
- Nur A, Byerlee JD (1971) An exact effective stress law for elastic deformation of rock with fluids. *J Geophys Res* 76:6414–6419
- Reid HF (1911) *The mechanics of the earthquake*. Volume II of *The California Earthquake of April 18, 1906*. Carnegie Institution of Washington, DC
- Roeloffs E (1996) Poroelastic techniques in the study of earthquake-related hydrologic phenomena. *Adv Geophys* 37:135–195
- Ruina A (1983) Slip instability and state variable friction laws. *J Geophys Res* 88:10359–10370

- Seed HB, Lee KL (1966) Liquefaction of saturated sands during cyclic loading. *J Soil Mech Found Div* 92:105–134
- Segall P, Lu S (2015) Injection-induced seismicity: Poroelastic and earthquake nucleation effects. *J Geophys Res Solid Earth* 120:5082–5103. <https://doi.org/10.1002/2015JB012060>
- Shi XJ, Wang CY (1985) Instability on a weakening fault. *Pure Appl Geophys* 122:478–491
- Terzaghi K (1925) *Erdbaumechanik*. Franz Deuticke, Vienna
- Vucetic M (1994) Cyclic threshold of shear strains in soils. *J Geotech Eng* 120:2208–2228
- Wang HF (2000) *Theory of linear poroelasticity*. Princeton University Press, Princeton, NJ
- Wang CY, Cheng LH, Chin CV et al (2001) Coseismic hydrologic response of an alluvial fan to the 1999 Chi-Chi earthquake, Taiwan. *Geology* 29:831–834
- Wang CY, Barbour AJ (2017) Influence of pore pressure change on coseismic volumetric strain. *Earth Planet Sci Lett* 47:152–159. <https://doi.org/10.1016/j.epsl.2017.07.034>
- Wang CY, Manga M (2010) *Earthquakes and water*. Springer
- Zhai G, Shirzaei M, Manga M et al (2019) Pore pressure diffusion, enhanced by poroelastic stresses, controls induced seismicity in Oklahoma. *Proc Natl Acad Sci* 116:16228–16233. <https://doi.org/10.1073/pnas.1819225116>
- Zhang Y, Wang CY, Fu LY et al (2019) Unexpected far-field hydrological response to a great earthquake. *Earth Planet Sci Lett* 519:202–212. <https://doi.org/10.1016/j.epsl.2019.05.007>

Open Access This chapter is licensed under the terms of the Creative Commons Attribution 4.0 International License (<http://creativecommons.org/licenses/by/4.0/>), which permits use, sharing, adaptation, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons license and indicate if changes were made.

The images or other third party material in this chapter are included in the chapter's Creative Commons license, unless indicated otherwise in a credit line to the material. If material is not included in the chapter's Creative Commons license and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder.