Chapter 12

The Precision Limits in a Single-Event Quantum Measurement of Electron Momentum and Position

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Abstract A modern state-of-the-art “*quantum measurement*” [The term “quantum measurement” as used here implies that parameters of atomic particles are measured that emerge from a single scattering process of quantum particles.] of momentum and position of a **single** electron *at a given time* [“at a given time” means directly after the scattering process. (It should be noticed that the duration of the reaction process is typically extremely short => attoseconds).] and the precision limits for their experimental determination are discussed from an **experimentalists point of view**. We show—by giving examples of actually performed experiments—that in a single reaction between quantum particles *at a given time* only the momenta of the emitted particles but not their positions can be measured with sub-atomic resolution. This fundamental disparity between the conjugate variables of momentum and position is due to the fact that during a single-event measurement only the total momentum but not position is conserved as function of time. We highlight, that (other than prevalently perceived) Heisenberg’s “Uncertainty Relation” UR [1] does not limit the achievable resolution of momentum in a **single-event measurement**. Thus, Heisenberg’s statement that in a single-event measurement only either the position or the momentum (velocity) of a quantum particle can be measured with high precision contradicts a real experiment. The UR states only a correlation between the mean statistical fluctuations of a large number of repeated single-event measurements of two conjugate variables. A detailed discussion of the real measurement process and its precision with respect to momentum and position is presented.

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1 Introduction

Otto Stern was the pioneer in high-resolution momentum spectroscopy of atoms and molecules moving in vacuum. Gerlach and Stern performed between 1920 to 1922 in Frankfurt their famous Stern-Gerlach experiment (SGE). They obtained for Ag atoms a sub-atomic momentum resolution in the transverse direction of 0.1 a.u. [2]. Today, modern state-of-the-art spectrometer devices such as the Scienta electron spectrometers [3] or the COLTRIMS Reaction Microscope C-REMI [4] can provide even a much better resolution. The imaging system C-REMI can even measure several particles in coincidence by detecting the momenta of all charged fragments emitted in a quantum process. Thus the complete entangled dynamics of such a single quantum process can be visualized. However, in such high resolution experiments the experimenter cannot obtain any direct information on the relative positions of particles. For a single event the absolute and also relative positions inside the quantum reaction are not measurable. The purpose of this paper is to illustrate for a single-event scattering measurement, as discussed by Heisenberg [1], the precision limits of electron momentum and position by presenting experimental examples.

The goal of a quantum measurement, e.g. scattering of a quantum projectile on a target atom in vacuum, is to obtain information on the quantum mechanical collision process. How can such a measurement be performed? The experimenter must prepare projectiles and target objects in a well-defined momentum and as far as possible also in a well-defined position state. This is typically achieved by classical methods. As shown below the momentum state of projectile and target object can be prepared with sub-atomic precision, but positions at a given time can never be controlled with atomic size accuracy. The reason is, no particle in vacuum can be brought completely at rest in the system of measurement and thus positions are not conserved with time. In other words the experimenter cannot predict with sub-atomic precision the impact parameter of the collision and the impact parameters are statistically distributed. Thus numerous single event measurements one after the other have to be summed up to obtain a statistical distribution.

The statistical distribution contains two sources of errors: First, the systematical error of each single-event measurement. This is de facto the horizontal error bar which is given by the quality of preparation and of the classical detection device only. This error bar depends very little on Heisenberg's uncertainty relation. Second, there is a statistical error in the ordinate values, which depends on the number of detected events and which does not depend on the precision of the single-event measurement. The sum of all single-event measurements, i.e. the statistical distribution, is relevant for comparison with theory.

What are the precision limits for parameters in the quantum experiment? The detection apparatus delivers only auxiliary values, from which then information on the quantum process can be deduced. Such auxiliary values are: The time, when the collision occurs. It can be determined by classical methods (see below) with about 50 pico-second precision. During the collision electrons and ionic fragments can be emitted each with a so-called final momentum. Immediately after emission they

move in a spectrometer device in which the charged quantum particles exchange momentum due to the electro-magnetic force with the macroscopic detection device. Finally the particles impact on a detector, which can be placed at any distance from the collision zone. The auxiliary values, that are measured by the detector, are: detector impact position and time. Typically they are measured with a precision of $50\text{ }\mu\text{m}$ and 50 pico-seconds. It is to be noticed that these auxiliary quantities allow the experimenter to deduce the particle trajectories in the detection device and to determine the final momentum in the laboratory system from the trajectory of the particle (see below).

To obtain sub-atomic momentum precision (laboratory system) in a single event, the velocity vector (i.e. the total momentum) of the center-of-mass of the single-event collision system must be known from the method of preparation. Conservation laws are therefore of fundamental importance for the implementation of a single-event quantum measurement. An observable can only be measured with sub-atomic accuracy if time-dependent conservation properties are strictly fulfilled during the generally very short duration of the measurement. Total linear momentum, total angular momentum and total energy are conserved but not location. The measured momenta of all fragments can then be corrected for the center-of-mass motion because the total momentum is conserved. For position no conservation law exists, thus a large uncertainty in the location measurement cannot be avoided. Therefore Heisenberg's suggestions that a high resolution position measurement is possible and this position measurement would be even the basis of any quantum measurement completely contradicts real experiments.

Since 1927 numerous papers have been published discussing the consequences of the UR on a quantum measurement within the wave-picture. To the best of our knowledge there is no publication available, where the constraints and the purely classical experimental limits of a single-event quantum measurement are analyzed from the view of an experimenter. Although in the introduction of his paper [1], Heisenberg considered the kinematics and mechanics of a single particle and the measurement of the position and the velocity (momentum) of a single electron "at a given moment", Heisenberg's UR ($\Delta x \cdot \Delta p \geq \hbar$) applies, however, only for the mean statistical fluctuations of a large number of repeated single-event measurements of two conjugate variables and can be viewed to be a prediction of the future particle properties.

We deploy therefor the following two statements:

Statement 1. The UR applies for the statistical distribution of a large ensemble or for repeated measurements but not for the resolution of a single-event measurement.

This statement is in line with previous work, that revisited this discussion, as well. For example, Ballentine [5], Park and Margenau [6] as well as Briggs [7, 8] contradicted the single-event interpretation of Heisenberg and concluded that it applies only to a large ensemble of similarly prepared systems.

Statement 2. A single-event measurement can only provide information on the particle's properties back in the past but never allow a prediction of future properties, since the impact of a particle on the detector changes the particles momentum and position state.

Which parameters of a quantum reaction are measurable and what is the achievable precision? We discuss this question by illustrating the concept of a “real” quantum experiment. As paradigm example for a typical quantum measurement we have chosen the scattering of an electron or ion on a gaseous target atom followed by the coincident detection of all reaction fragments with modern “state-of-the-art” detection devices.

We will discuss the following three findings:

1. **One can measure the final momenta of all emitted charged fragments.** Since each single-event measurement takes some time (from preparation until detection), the conservation with time of the total momentum of the whole scattering system is a crucial property in order to obtain excellent resolution in real measurements. During the short period of measurement the momenta of all particles, are “correlated” due to the law of momentum conservation, i.e. they are even dynamically entangled, for the whole time until they finally impact on a classical detector (see Ref. [7] and comments therein connected to this paper).
2. The angular momentum of a single freely-moving electron emitted in a quantum reaction appears undetectable. However, the quantum states (whose quantum numbers) can be deduced, if the electron kinetic energy can be assigned to a well-defined transition. In an ion-atom collision process, however, a coincidence measurement can provide information on the **angular momentum** of a single particle. In the case of a complete multi-particle coincidence measurement, when the nuclear collision plane is determined, this additional information can be employed in some cases to deduce the angular momentum, as, for example, certain angular momentum states are emitted due to space quantization only into distinct regions like in the Stern-Gerlach experiment (see e.g. data in Fig. 4 of this paper).
3. One can also precisely determine the **amount of the electronic excitation energy** from the measured momenta of all particles in the preparation and final states, because the total energy is also conserved (assumption: projectile and target in the preparation state are in the ground-state). The excitation energy is then the difference between the kinetic energies in the initial and final states.

The UR imposes, in contradiction to Heisenberg's claim, no limit on the achievable momentum (velocity) resolution of a single quantum measurement. The UR affects the resolution of such a measurement only indirectly, as it has an impact on the quality of preparation of the pre-collision states of projectiles and target atoms. This has already been highlighted by Kennard in 1927 [9] who theoretically considered the passage of scattered electrons in a classical detection device and concluded:

„In den hier behandelten Fällen haben wir keinerlei quantentheoretische Abweichung gefunden von den klassischen Ergebnissen. Die einzige quantenhafte Eigentümlichkeit in solchen einfachen Fällen liegt in der durch das Heisenberg'sche Unbestimmtheitsgesetz festgesetzten prinzipiellen Unbestimmtheit der Anfangswerte“ (“In the cases discussed here, we did not find any quantum theoretical deviation from the classically calculated values. The only quantum influence in such cases originates from the effect that the preparation state values are indeterminate in accordance to Heisenberg's Uncertainty Relation.”). Today, the debate over the statistical versus single-event interpretation is still not converged (see [5–16] for proponents of the single-event interpretation and for papers opposing this interpretation).

In the following chapters we discuss the **purely experimental aspects** and the limits of experimental precision in a single-event measurement of momentum (velocity) and position and present examples:

In Sect. 2: The **scheme and time evolution of a single-event measurement** is discussed beginning with the preparation of the measurement followed by the quantum reaction process and concluding with the detection of the charged fragment in a classical measurement apparatus.

In Sect. 3: The **electron momentum (velocity) measurement by Time-of-Flight (TOF) trajectory imaging** is presented. We consider realistic experimental scenarios for electrons based on experimental results.

In Sect. 4: The determination of the **angular momentum state** of a single electron by a multi-fragment coincidence technique.

In Sect. 5: The **experimental limits for an electron position measurement** are discussed. We also show that Heisenberg's “Gedankenexperiment” on the γ -microscope is not feasible.

In Sect. 6: We consider the **product of precisions in momentum and position measurement of a freely moving single electron**. New experimental techniques for measuring momentum and position of a freely moving electron simultaneously in a one-step approach are provided for the moment of impact on a detector. Within this approach the product of the experimental error bars in electron momentum and detector impact position can be below $\hbar$ by several orders of magnitude.

2 Scheme of a Quantum Measurement

We consider an experiment where a projectile beam intersects in ultra-high vacuum with a gaseous target to ensure controlled single-event conditions i.e. that only one reaction process occurs during each measurement period. Because of the statistical nature of quantum measurements (to yield statistical distributions) one must prepare numerous projectiles in the “nearly identical” pre-collision state and numerous target objects in controlled “nearly identical” momentum and position states. In the preparation of the pre-collision state “nearly identical” means this preparation is still limited by Heisenberg's UR with respect to the large ensemble projectile and target

momentum and position fluctuation widths. E.g. in an ion-atom collision the experimenter cannot precisely adjust the impact parameter to obtain the same deflection angle. The selection of impact parameters is of pure statistical nature. Thus, the experiment has to be repeatedly performed with numerous of such single projectiles and target objects. Finally summing over a huge number of single-events the experimenter obtains a statistical distribution that allows for the retrieval of the final-state fluctuation width (with the help of theory also quantum mechanical properties or properties of the wave function).

2.1 Time Evolution of a Quantum Measurement

In Fig. 1 the scheme of a single-event quantum experiment and the time evolution of such a complete quantum measurement process are shown. The measurement may be separated in three sequential steps: the time of preparation (pre-collision step, zone A), the time of reaction (zone B), and the time after the reaction (post-collision step, zone C) before the reaction products impact on the detector. In the view of the experimenter the momenta and trajectories of the particles in the macroscopic preparation stage A (pre-collision) as well as in the macroscopic spectrometer system C (post-collision) can be treated by the laws of classical physics. The very tiny reaction region B (typically of atomic to micrometer size) is a purely quantum mechanical region and must be treated accordingly. The dynamics in region B cannot be directly observed by the experimenter. The classical behavior in A and C is justified theoretically by the Imaging Theorem of the accompanying papers [7, 8]. This result shows

Fig. 1 Time evolution of a quantum measurement. A indicates the time interval before the interaction of projectile (1) and target (2), B is the very short time interval of the quantum scattering process (3) (occurring at the time t_0) and C the time interval in which the emitted reaction particles (4) are travelling inside the classical detection setup. The particle is finally detected on a detector (6). The detector yields an electronic signal (7) (typically a nanosecond long) providing time information on the quantum scattering event, which is stored electronically in a computer (8)

that, after propagation to or from macroscopic distances, the position and momentum variables of the quantum wave function obey classical relations.

The reaction products emitted in the quantum reaction are interacting with the macroscopic measurement apparatus in zone C. In the macroscopic apparatus they can be treated as classical particles with classically defined momenta (moving on classical trajectories) since they exchange in zone C de facto only momentum with the measurement device due to applied electric or magnetic spectrometer fields. Any interaction of the fragments with the rest gas in the spectrometer can be excluded because of the very low vacuum pressure (typically below 10^{-8} millibar). At the end of the macroscopic detection device position-sensitive detectors measure the impact position in the laboratory system of each fragment and also the time of impact for each fragment separately (if required all fragments can be measured in coincidence).

As Popper pointed out [17], after completion of a measurement the experimenter determines always the kinematical parameters of the “past” for each single event, whereas the UR makes predictions into the future for the outcome of statistical distributions of many repeated single-event measurements.

3 Electron Momentum (Velocity) Measurement by Time-of-Flight (TOF) Trajectory Imaging

3.1 *The Experimental Scheme for Momentum (Velocity) Measurement*

In the following we describe a quantum measurement of charged particles from an ionization process using a momentum-imaging approach. After leaving the reaction zone B (see Fig. 1 at time t_{01n}) the charged fragments begin to move in zone C on “quasi-classical” trajectories (see Refs. [7, 8]) with classically defined momenta, since in zone C they nearly exclusively exchange momentum with the spectrometer via classical forces. The distance d from the reaction point, from where one can neglect quantum mechanical post-collision interaction, can be crudely estimated by comparing the strengths of interacting forces, i.e. the magnitude of momentum exchange. In zone B the force between electron and ion dominates and in zone C the force imposed on the charged particles by the spectrometer fields is dominating. This is because the force between electron and ion depends on their distance d . Assuming the ion is singly charged then the electron-ion force is $F_{\text{ion}} = e^2/d^2$ (in a.u.). For $d = 1000$ a.u. one obtains $F_{\text{ion}} = 10^{-6}$ a.u., for $d = 1 \mu\text{m}$ one obtains $F_{\text{ion}} = 2.8 \times 10^{-9}$ a.u. The strength of the classical force in the fields of the measurement device can be estimated from the electric field strength in the spectrometer. The field is typically larger than 10 V/cm, thus for an electron the acting spectrometer force is $F_{\text{es}} > e \cdot 10 \text{ V/cm} = 1.92 \times 10^{-9}$ a.u. Therefore, for distances d larger than a few tens of micrometers the electron-ion force strength can be neglected and the

Fig. 2 Scheme of trajectory imaging technique for charged quantum particles in a classical spectrometer [4]. The electron momentum vector (blue arrow) is the so-called “final momentum”, with which the electron is emitted from the collision process with respect to the center-of-mass system of the reaction process

emitted fragments are only interacting with the spectrometer field yielding a well-defined classical trajectory due to momentum conservation (charged fragment plus spectrometer are entangled). The momentum change and thus the classical trajectory of the fragment in the spectrometer depend on the electric-magnetic field design and on the final fragment momentum p_{fn} .

A static electric field accelerates electrons and positively charged ionic fragments into opposite directions. The fragments are finally detected by two position- and time-sensitive detectors placed in opposite directions (only one direction is shown in Fig. 2). Since the spectrometer provides for positively and negatively charged particles nearly a 4π -detection efficiency it can capture a complete image of the reaction process in momentum space.

The measurement of the final momentum of an emitted fragment can thus be achieved through a precise determination of the particle trajectory in part C in the classical detection device. To determine the complete classical trajectory of each particle one has to measure only the classical location parameters $r_0 = (x_0, y_0, z_0)$ and $r_{fn} = (x_{fn}, y_{fn}, z_{fn})$ as well as times t_0 and t_{fn} (see Fig. 2). Both time parameters can be determined with a precision of about 50 pico-seconds, t_0 can be measured by using a timed-bunched projectile beam and t_{fn} by using a “state-of-the-art” classical detection device [4]. Target location and position of impact on the detector can be measured with a precision of better than 50 μm (even 10 μm are achievable). Knowing (or calibrating) the electro-magnetic field configuration and measuring the above listed parameters, the final momentum vector of the fragment can easily be deduced by using simple classical equations [4]. **Although all auxiliary parameters**

Fig. 3 Scheme of the C-REMI [4] which can image with 4π solid angle all emitted charged particles (ions: red trajectory, electrons: blue trajectory) in coincidence. A projectile beam intersects in the center of the C-REMI with a super-sonic gas jet (from below) inducing the quantum reaction process. The applied electric field super-imposed by a magnetic field (see the brown coils) projects all charged fragments/electrons on position- and time-sensitive detectors

are measured with macroscopic accuracy only, sub-atomic resolution for the electron and ion momenta (velocities) can be obtained.

The C-REMI [4] is such a “state-of-the-art” momentum-imaging device. In Fig. 3 the scheme of such a detection approach is presented. The reaction takes place within the tiny intersection region of projectile and target beams (e.g. internally very cold super-sonic gas jet). The blue and red curves in Fig. 3 indicate the classical trajectories of ionic fragments (red line) and electrons (blue line) in the spectrometer. With the help of electric and magnetic fields nearly all fragments are projected on position-sensitive detectors yielding a very high multi-coincidence detection efficiency.

Before we discuss a real experimental scenario, we first define “good” and “bad” resolution in a single-event quantum measurement with respect to the standard dimensions in an atomic system. The standard sizes of atomic parameters are defined by the classical features of an electron in a hydrogen atom. The classical K-shell radius is $r_K = 5.29 \times 10^{-9}$ cm, which is used to define the atomic unit of length (a.u.). The classical electron velocity of the electron in the hydrogen K-shell is $v_K = 2.18 \times 10^8$ cm/s, which defines 1 a.u. of velocity. The classical momentum of the electron in the hydrogen K-shell is $p = m_e v_K = 1$ a.u. An atomic unit of time is defined by the ratio of the hydrogen K-shell radius divided by the corresponding electron velocity, or 5.29×10^{-9} cm divided by 2.18×10^8 cm/s yielding 24 attoseconds. Furthermore, the electron charge e and mass are also set to 1 a.u. and hence $\hbar$ results to be 1 a.u., too.

Thus, it appears very reasonable when we define resolution of single-event quantum measurements with respect to these atomic units. “Good” sub-atomic resolution is on the order of a few percent of one a.u. and “very good” resolution is on the order of a per mill or even better. Bad resolution is larger than one a.u.

3.2 *Momentum (Velocity) Measurement and Its Achievable Resolution for an Electron*

The achievable experimental precisions for momentum (velocity) are discussed here for two quantum processes. First, the transfer ionization process which is

investigated by Schmidt et al. [18]. This experiment was performed to search for vortices in the electron current which should be visible in the velocity/momentum distribution of the emitted electrons. To visualize such effects in the electron momentum distribution, a high experimental momentum resolution ($\delta p = 0.01$ a.u.) in a single event is required. Additionally, a coincidence measurement with the ejected ions is necessary in order to determine the orientation of the quasi-molecule during the collision. This was achieved with the C-REMI approach. During such slow collisions quasi-molecular orbitals are formed and electrons are promoted to the continuum via a few selected angular momentum states.

In Fig. 4 the measured electron-momentum distributions are shown together with the achieved single-event resolution δp (black square) and with one example of a momentum fluctuation width $\langle \Delta p \rangle$ (varies with electron energy). It is to be noticed that in this experiment of Schmidt et al. the electron-detector distance from the intersection region (gas jet-projectile beam) was only 3 cm due to other experimental requirements. This short distance limits the momentum resolution, because of the very short TOF. Nevertheless a resolution of 0.01 a.u. was obtained. The resolution can be improved by increasing this distance.

To demonstrate the high resolving power for electron momenta of the C-REMI a numerical example, a kind of “*Gedankenexperiment*”, i.e. the process of electron impact ionization of He

is discussed here. Today such an experiment would be feasible.

In Appendix A the preparation of the required electron beam quality is described which enables the high required momentum accuracy of the projectiles. To yield the required excellent “Time-of-Flight” TOF resolution the detectors should be located as far as possible from the zone B, i.e. the spectrometer should be as large as possible. For a trajectory length inside region C (from zone B to the electron detector surface)

Fig. 4 **a** Measured electron velocity distribution in the nuclear collision plane in units of the projectile velocity $v_p = 0.63$ a.u. for small nuclear scattering angles < 1.25 mrad. **b** Perpendicular to the nuclear collision plane. **c**, **d** corresponding theoretical predictions. An electron moving with the projectile velocity $v_p = 0.63$ a.u. has a momentum of 0.63 a.u. [18]. The experimental resolution in a single event of $\delta p = 0.01$ a.u. corresponds to an energy resolution of approximately 1 meV

of 2 m the angular resolution of the trajectory measurement is of the order of the sum of the intersection width of projectile beam and target beam and detector position resolution divided by the trajectory length. This ratio is about $2 \cdot 50 \mu\text{m}/200 \text{ cm} \approx 0.5 \times 10^{-4}$. This geometrical ratio limits the transverse momentum resolution in x- and y-direction. The longitudinal momentum resolution (in z-direction) depends on the TOF resolution. An electron moving with 2 a.u. momentum has a velocity of $4.38 \times 10^8 \text{ cm/sec}$ and its total TOF inside C is $200 \text{ cm}/4.38 \times 10^8 \text{ (cm/sec)} = 450 \text{ ns}$. Thus the relative TOF resolution $\Delta\text{TOF}/\text{TOF}$ is about 10^{-4} yielding an overall momentum precision for an electron of 2 a.u. momentum of 2×10^{-4} a.u..

We would like to notice, that in C-REMI the velocities and masses of moving particles are measured, which yield directly the momenta. The velocities are macroscopically large and therefore directly measurable with macroscopic classical TOF devices. Heisenberg considered the measurement of the velocity (momentum) of an electron bound in an atom too. His approach will be discussed in Appendix B together with the possibility of momentum measurements of bound electrons via the process of Compton scattering.

4 Measurement of Angular Momentum of a Single Electron

Any bound electron usually has an orbital angular momentum in addition to its own spin. Due to the spin-orbit coupling, all electrons in an atom form one unit providing a quantized total angular momentum. If an experimenter can only measure the momentum of only one emitted electron (so-called single parameter measurement), then an experimenter can hardly make any statement about the quantum state in which the electron was originally bound. In case of single parameter measurement only from the electron momentum distribution of a large amount of identical ionization processes one can make a statement about the type of multipole distribution and thus on the angular momentum transfer involved. Thus the angular momentum of a single freely-moving electron emitted in a quantum reaction appears undetectable.

However, if the electron kinetic energy can be assigned to a single transition between well-defined quantum states, whose quantum numbers can be deduced. Furthermore in an ion-atom collision process and in the case of a complete multi-particle coincidence measurement, when the nuclear collision plane is determined too, this additional information can be employed in some cases to deduce the angular momentum states of a single ejected electron. In a slow ion-atom collision process, quasi-molecular electronic orbitals are formed during the collision, which are sharply angularly quantized with respect to the nucleus-nucleus scattering plane. Thus different angular momentum states are emitted due to space quantization only into distinct regions like in the Stern-Gerlach experiment (see e.g. data in Fig. 4 of this paper). If e.g. in a transfer-ionization process an electron passes over from these quasi-molecular states into the continuum [18] then the electrons in the x-y plane perpendicular to the nucleus-nucleus scattering plane are emitted with discrete transverse momenta (Fig. 4) and the different quasi-molecular orbitals e.g. 1 and 2 in Fig. 4 can clearly distinguished. Just as in the Stern-Gerlach experiment, these discrete transverse momenta correspond to certain angular momentum states which can be discerned in a coincidence measurement.

This clearly proves (Fig. 4: comparison of experiment and theory) that in a coincidence experiment the directional quantization of the quasi-molecular states becomes measurable and thus in selected collision systems the angular momentum states of single emitted electrons can be determined too.

5 Electron-Position Measurement and Achievable Resolution

Heisenberg described the position measurement of single electrons at a given moment as the foundation of any parameter measurement. He proposed to measure the velocity by detecting the electron positions at two succeeding moments. He explained his view on position measurements by thought experiments: *“If one wants to understand, what the definition of ‘position of a particle’, e.g. of the electron (relative to the reference system of measurement) means, one must describe well-defined experimental approaches, how the ‘position of an electron’ can be measured; otherwise the definition of position is meaningless. He continued: “There is no shortage of such experimental approaches, which can measure the ‘position of an electron’ with unlimited precision.”* (page 174) [1]. Therefore he viewed a trajectory as a discontinuous path because of discontinuous observations. On page 185 he continued: *“I believe that the appearance of a classical trajectory is manifested by its observation”*.

Heisenberg proposed to use a so-called γ -microscope to measure the position of a quantum object, e.g. an electron at a given moment. He ascertained [1]: *“The resolution of the light microscope is only limited by the wave length of the light. Using short wave length x-rays the resolution should have no limitation.”* The scheme of such a photon microscope measurement can only be explained in the wave-picture (thus many photons must be detected). But one has to make sure that the object is not changing its position during the exposure time of the measurement. With the help of such a microscope (combination of lenses) one can magnify tiny objects and project their image on a detector, e.g. photo plate. There is an one-to-one correspondence between position on the object and the position on the detector (only valid in the transverse plane). Thus with the help of lenses relative positions on very small quantum objects can be enlarged and thus become observable. It should be noted, that a “microscope” device for magnifying the geometrical size of an atom (about 10^{-8} cm diameter) and also magnifying the relative positions of atoms in a molecule to the macroscopic size of 1 mm must have a magnification factor of more than 10^6 .

Heisenberg was convinced that the position of an electron at a given time could be measured even with “ultimate” precision using the technique of such a light microscope if the wavelength of the light would be small enough to resolve sub-atomic structure.¹ At a “given time” means always an exposure time period in which

¹ Several reasons prevent that Heisenberg’s so-called γ -microscope can measure the position of one selected electron inside an atom at a given time with a required resolution of 10^{-10} cm or even better: First: Since the focus of the γ -pulse is of macroscopic size (larger than $1 \mu\text{m}^2$) the scattered photons of the γ -pulse interact with different electrons in the atom or molecule and the measurement can on principal not identify which photons were scattered on the one special electron. Second: The Compton cross sections for scattering photons with a wave length of 10^{-10} cm (or $h\nu = 1.2$ MeV) on an electron are smaller than 10^{-25} cm². Therefore it requires per attosecond pulse more than 10^{19} photons in a focus of $1 \mu\text{m}^2$ to scatter about 100 photons on this electron. Such a photon pulse carries an energy power equivalent with 1% of the total energy emission of the sun. Third a technical reason: The γ -microscope needs a high precision lens system for 1 MeV photons to magnify the $1 \mu\text{m}^2$ focus size to make the different electrons on a macroscopic detector distinguishable.

the location of a moving electron must be considered as “frozen”. Such a time period for an electron detection must be shorter than one attosecond.

Therefore, in order to obtain an image of the position of an electron with sub-atomic resolution using a γ -microscope, one would have to scatter on the **same electron** numerous photons in a one attosecond “exposure” time period (since the electron is moving with a typical velocity of 1 % of the speed of light). Because these γ -scattering cross sections (Compton scattering) are of nuclear size the photon pulse intensity in one attosecond must exceed 10^{19} photons per pulse in a focus of 1 μm diameter. A further problem in such a measurement is that the experimenter has no control on which electron in the target atom or molecule the photons are scattered. Both effects make such a γ -microscope measurement physically not feasible.

Furthermore, each Compton scattering process, as mentioned above, is destructive for the electronic state, thus the electronic state changes immediately. This disturbing effect of momentum transfer to the electron and thus changing the electron’s position subsequently was already realized by Bohr [19]. These arguments show that Heisenberg’s γ -microscope is not suited to measure the position of an electron at a given moment.

In one attosecond exposure time because of the tiny cross sections at most one photon might be scattered on the same electron. Thus the only information the experimenter obtains with Compton scattering is the detection of only one single photon providing one momentum vector. Even if this photon momentum vector is measured with sub-atomic precision the location of the reaction can never be deduced from this one vector with a precision better than the preparation of the target position before the scattering.

In contrast, position-measurements of heavy nuclei or atoms can be performed with a γ -microscope, since the velocities of atoms or nuclei are typically a factor of 10.000 smaller. Thus, the heavy particle position can be considered as “frozen” even for an exposure time of a few femtoseconds. Such relative position measurements of heavy atoms in molecules are now routinely performed with FEL X-ray pulses [20], where a lateral position resolution of about 5 Å is achieved. A slightly better resolution of about 3 Å is achieved with CRYO-electron microscopy [21].

One may expect that when performing a multi-coincidence measurement, i.e. measuring the momentum vectors of several fragments of the same reaction with excellent resolution, one could deduce the position, where the reaction took place,

Heisenberg proposed also to use energetic α -particles as scattering projectiles (because of their even shorter de Broglie wave-length) for a super high-resolution microscope and estimated even a position resolution of 10^{-12} cm as possible. He wrote on page 175: “*When two very fast particles succeeding each other scatter in a very short time distance Δt on the same electron, then the distance between the positions of both collisions is Δl . From the scattering laws, which has been observed for α -particles, we can conclude, that Δl can be made as small as 10^{-12} cm, if Δt can be made sufficiently small and the α -particles fast enough.*”

For a several MeV α -particle beam this requires a relative distance of the α -particles in the beam of about 10^{-11} cm. This relative distance of the α -particles is about one thousand times smaller than the normal inter-nuclear distance in a hydrogen molecule. α -particles with a relative distance of 10^{-11} cm would repel each other with a huge Coulomb repulsion force creating huge non-controllable transverse momenta of the α -beam.

by reconstructing the intersection point of all momentum vectors. Even if the impact positions of all fragments on the detector could be measured with atomic position resolution, the momentum vectors have still a finite angular uncertainty limited again by the target preparation. Because of the macroscopic dimensions of the detection device even a tiny angular uncertainty of these vectors would spoil any precise position measurement of the reaction region within the laboratory frame.

6 Product of Precisions in Momentum and Precision in Position in a Real Measurement of a Freely Moving Single Electron

The paradigmatic demonstration experiment for the UR given in textbooks for measuring simultaneously position and momentum of an electron (wave picture) is the scattering of this “wave” on a narrow slit (first step). The scattered wave yields an interference pattern of the electron wave on a screen (second step). According to most of the textbooks position and momentum of these electrons can only be measured in such a **two step-approach**, where in the first step the position is measured by the slit width and in the second-step by the interference pattern on the screen the momentum. The electron is theoretically described by wave functions which are different before and after the slit: before passing the slit the electron is described as plane wave with well-defined momentum eigenvalue but not localized in x -position; just after the slit the electron is described as a wave packet with some distribution in position and momentum. Thus on its way to the screen the electron is in a state which is not an eigenstate of the momentum operator. In both of the two time steps the UR is fulfilled. This is a result of the fact that the two operators do not commute. Thus the wave function is disturbed and a conceptually unavoidable uncertainty in the second-step measurement (momentum measurement) is generated. From the interference structure in the transverse momentum distribution of many single-event measurements the de Broglie wave length λ and thus electron momentum p can be determined.

We will now estimate how small the product of the two precision widths $\Delta x \cdot \Delta p_x \geq \hbar$ can be made in a **single-event process** by using a modern state-of-the-art detection device. We are in particular interested in whether the product of the experimental position resolution times the experimental momentum resolution can be made smaller than $\hbar$. With today’s detection technique the two-step detection scheme can be replaced for single electron detection by a quasi **one-step detection approach**, where the narrow slit is “upgraded” to a very small pixel detector, which measures position and time of impact too. Thus we consider the momentum and position at the time when a single moving electron impacts on the position-sensitive detector. One can construct detectors which can measure the impact position of the electron on the detector with a few a.u. precision $\delta x_y = 2$ a.u. (see Appendix C).

In a single event this detector provides, at the instant of electron impact, also a very fast electronic signal (time resolution < 50 pico-seconds) which yields precise

information on the electron velocity. If furthermore the location of the interaction region and the interaction time, from where this electron is emitted, are known with macroscopic precision, one can determine the electron Time-of-Flight TOF. Knowing precisely the distance d between emission point to detector (e.g. $d = 2$ m and $\Delta d = 0.1$ mm) one can precisely calculate the electron velocity: Assuming the measured TOF is e.g. 456.6 ± 0.1 ns and d is 200.00 ± 0.01 cm the electron velocity is then $v_e = 2 \cdot 200.00 \text{ cm} / 456.6 \text{ nanosecond} = 4.3800 \times 10^8 \text{ cm/s}$ with an error bar of $\pm 0.025\%$. Transforming the velocity in a.u. we obtain for $v_e = p_z = 2.0021 \pm 0.001$ a.u. (p_z is the electron momentum in flight direction). In perpendicular direction the errors in momentum are $\delta p_{x,y} = (0.1 \text{ mm} / 2000 \text{ mm}) \cdot 2.0021 \text{ a.u.} \approx 10^{-4} \text{ a.u.}$ Thus, in case of a single event measurement the product of the experimental error bars in the momentum and position measurement can be made $\delta p_{xy} \cdot \delta x_{xy} \approx 10^{-4} \text{ a.u.} \cdot 2 \text{ a.u.} = 2 \times 10^{-4} \text{ a.u.}$ which is much smaller than $\hbar$.

One could argue, however, that the detection plus preparation is still a two-step measurement. But nevertheless in a single event the product of precisions can be made much smaller than $\hbar$. Thus, once the particle has been detected, the trajectory, that the particle has travelled on in the past, can be defined such that the product of precisions of momentum and position measurement of this freely moving single electron is not limited by $\hbar$.

7 Conclusion

We have shown that in **a single-event quantum measurement** the momenta of emitted electrons or ions can be measured with high sub-atomic precision and the limits of precision for the momentum measurement are not restricted by Heisenberg's UR if one assigns trajectories to particles that have been detected. The precision in measuring positions in a single event can never approach or being better than 1 a.u. in a single-event measurement because the two conjugate parameters position and momentum do not have the apparent physical symmetry suggested by the UR, i.e., there exists a disparity in momentum compared to position measurement. The fundamental reason is: in a single event momenta are conserved with time (i.e. they are dynamically entangled), but positions are not conserved. This fundamental difference between momentum and position measurement as function of time in a quantum reaction is also apparent from the wave description (see Appendix D). The position wave functions broaden with increasing time even during a very short single-event measurement.

For a single freely moving particle in the moment of impact on the detector momentum and also position on the detector can be simultaneously detected in a single-step approach using position-sensitive detectors combined with a time-of-flight measurement. The product of the experimental momentum resolution δp times position resolution δx on the detector can be made much smaller than $\hbar$.

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Appendix A

Electrons with a very well-defined momentum $p_z = 2.000$ a.u. can be created by photoionization. The primary photon energy $h \cdot \nu$ is chosen to be $78.988 \text{ eV} \pm 0.007 \text{ eV}$ (linearly polarized photons). These photons can singly ionize the He atom. In our example a single electron with a kinetic energy of $54.392 \pm 0.007 \text{ eV}$ and He-recoil ion with a kinetic energy of 7.4 meV are emitted back-to-back. In the He center-of-mass system both freely moving particles, electron and recoil ion, have the identical momentum with opposite direction ($p_e = -p_{\text{rec}} = 2.000$ a.u. with a precision of about 10^{-3} a.u.). The angular distribution of electrons emitted by photon ionization and recoil ions is a perfect dipole distribution.

Since the ionized He atom is not at rest in the laboratory system at the moment of ionization, one must correct the electron momentum vector for the motion of the He-atom in the Lab frame. The final electron velocity is about 5000 km/sec and the internal velocity spread of the cold super-sonic He jet is below 50 m/sec . Because the He jet is moving along the negative y direction, the correction for the electron in the z -direction can be performed with sufficient precision. Furthermore, the very small momentum kick of the incoming photon ($p_{\text{photon}} \approx 0.021$ a.u.) to the center-of-mass of the He atom changes the He velocity by only about 10 m/sec . Thus the absolute value of the electron momentum is known with about 10^{-3} a.u. precision. The direction of the final electron beam can now be defined by a collimation system (collimation in x and y direction). Using a double slit system at 2 m distance in the z -direction with slit widths of 10 microns each (in x - and y - direction) the so-collimated electron beam has an angular divergence of $\delta = 20/(2 \times 10^6) \text{ rad} = 10^{-5} \text{ rad}$. The momentum exchange with electrons inside the slits due to image-charge formation is also insignificant. Thus the momenta of the electron beam have a width $\Delta p_{x, y, z} < 10^{-3} \text{ a.u.}$, i.e. each electron in this electron beam has a momentum of 2.000 a.u. in the z -direction (with a precision of about 10^{-3} a.u. in all three dimensions x , y , and z). By changing the primary photon energy, the electron beam's momentum can be varied. Similar momentum resolution results can be also obtained for photon beams and slow ion beams, too.

Appendix B

Heisenberg described also a way to measure the velocity (momentum) of an electron bound in an atom. Heisenberg's approach to measure the velocity of a bound electron

was copied from classical physics where the velocity of a particle is determined from the quotient of measured distance divided by measured time difference. Heisenberg's concept was to detect by γ -photon scattering at two instants in time the electron locations (for a bound electron separated by less than 10^{-11} cm). This would require a very precise simultaneous measurement of location and time within sub-attosecond time separation. As we discussed in Sect. 4 such a measurement of position and time with the required resolution is physically impossible to perform. Furthermore, Heisenberg worried that the electron velocity (momentum) just before and just after the photon (Compton) scattering process is not the same one due to the momentum kick by the photon and thus the velocity measurement before the kick seemed to be not accessible.

In 1927, Heisenberg was not aware that one could measure nevertheless both—the electron momenta just before and after the scattering by performing a **coincidence measurement of the scattered Compton-photon and the ejected electron**.

Both electron momenta just before and just after scattering can be determined with very high resolution ($\delta p = 10^{-2}$ to 10^{-3} a.u.) due to momentum conservation during the scattering process. This is possible at high photon energies and large photon momentum transfers where the “impulse approximation” is well justified [22]. In the “impulse approximation” for Compton ionization the ejected electron is treated as a quasi-unbound electron and thus the momentum change of the whole atom by Compton ionization is rather small and the remaining ion acts only as a spectator. The coincidence measurement allows therefore for a precise determination of $\mathbf{P}_{\text{e ionized}}$ which is the momentum vector of the ejected electron after the Compton scattering and $\mathbf{P}_{\text{e bound}}$, which is the unknown momentum vector of the bound electron just before scattering (see vector equation in Fig. 5). When Heisenberg wrote his paper in 1927, the experimental techniques to study quantum processes were in an “archaic” state compared with today. Precise timing measurements in the nanosecond regime required for coincidence measurements were beyond imagination. The first generation of coincidence technique was just invented in 1924 by Bothe and Geiger [23], but Bothe's and Geiger's time resolution was limited to a fraction of a millisecond. Instead of using high-energetic photon impact one can perform the same kind of momentum spectroscopy of bound electrons by using very fast ion [24] or electron impact [25].

Heisenberg also considered a velocity measurement of a bound electron by using the Doppler-effect (wave length-shift) of scattered red light. Heisenberg wrote on page 177 [1]: *“The velocity of a particle can easily be defined by a measurement, if the particle velocity is constant (no acting forces). One can scatter red light on the particle and measures by the Doppler shift its velocity. The measurement becomes more precise, as longer the wave length of the light is, since then the velocity change of the particle per photon becomes smaller. The determination of position becomes accordingly more unprecise predicted by equation (1) (UR). To measure the velocity at a given moment, the Coulomb forces of the nucleus and the of the other electrons must suddenly disappear, to ensure from this moment a constant velocity, which is necessary to perform the measurement.”* The Doppler-effect approach would, however, only allow the determination of the particle velocity component in the

Fig. 5 Momentum vector diagram for Compton scattering. The dotted vectors $P_{Sum \text{ initial}}$ and $P_{Sum \text{ final}}$ represent the sum vectors in the initial state of impacting photon P_{γ} (prepared and precisely known) and of bound electron $P_{e \text{ bound}}$ (unknown) as well as in the final state of ejected electron $P_{e \text{ ionized}}$ and of scattered photon $P_{\gamma'}$ (both precisely measured)

direction of the incoming red light. Furthermore, the red light photon would be scattered on the whole atom and not on a single electron and would thus probe the atom velocity only.

Appendix C

Today, in principle one could build a macroscopic position-sensitive electron or ion detector with better than 10 a.u. position and 50 pico-seconds timing resolution. This detector can be a very small pixel detector or a large area position sensitive detector. Such a detector can be reassembled from two components: a commercially available position-sensitive channel-plate detector with a standard position resolution of 50 μm in x and y direction, respectively and a very thin (nanometer thickness) mask of regularly positioned holes of <10 a.u. diameter each, where electrons and ions can only be detected when they impact into such a hole (distance mask to detector surface a few nano-meter only). Then they induce in the detector an electronic signal. From this single electronic signal simultaneously position and timing information is obtained. The distance from hole to hole is 100 μm , thus the detection device is able to determine each particle impact on an absolute scale in the laboratory system

for the position measurement with 10 a.u. precision. Therefore, one can detect the location of this particle impact in x and y direction with $\delta x \approx \delta y \approx 10$ a.u. resolution.

Appendix D

Time-dependent conservation laws are of fundamental importance for the implementation of a high-resolution single-event quantum measurement. This conservation is valid for the total linear momentum, for the total angular momentum and for the total energy, but not for the location. This is obvious for a particle picture, but also in the wave picture. In Fig. 6 one can see that the spatial wave function widens linearly over time, but the momentum wave function maintains its narrow width, which is determined by the preparation of the measurement. If several fragments are emitted in the reaction process, momentum conservation applies to each of the fragments on their flight to the detector. The momentum exchange with the classical electro-magnetic fields of the measuring apparatus can be determined from the classically measured trajectory of each fragment and thus corrected with high resolution.

The asymmetry as function of time for position and momentum space of the freely moving particle is also apparent in the wave approach and is due to the time propagation with $U = \exp(\Delta T \cdot E)$ with $E(p) = p^2/2$ that breaks the symmetry of position vs. momentum space.

Fig. 6 Fourier limited wave function in position **(a)** and momentum **(b)** space. The phase is flat in position and momentum space and the amplitude is a Gaussian distribution. Wave function **c** from **a** after propagation for the time $\Delta T = 10$ a.u. The resulting wave function **d** in momentum space is $\Psi_2 = \Psi_1 \cdot \exp(\Delta T \cdot E)$ with $E(p) = p^2/2$. Ψ_2 has the same amplitude in momentum space as in **b** but a quadratic phase. In position space the amplitude distribution broadens compared to **a**

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