Chapter 12

Collective Teaching Experiments: Organizing a Systemic Cooperation Between Reflective Researchers and Reflective Teachers in Mathematics Education

The success of any substantial innovation in mathematics teaching depends crucially on the ability and readiness of teachers to make sense of this innovation and to transform it effectively and creatively to their context. This refers not only to the design and the implementation of learning environments but also to their empirical foundation. Empirical studies conducted in the usual style are not the only option for supporting the design empirically. Another option consists of uncovering the empirical information that is inherent in mathematics by means of structure-genetic didactical analyses. In this chapter, a third option is proposed as particularly suited to bridge the gap between didactical theories and practice: collective teaching experiments.

The following five points indicate in a nutshell the line of argument of this paper.

- Mathematics education as a “systemic-evolutionary” design science
- Taking systemic complexity systematically into account: lessons from other disciplines
- Empowering teachers to deal with systemic complexity as reflective practitioners
- Collective teaching experiments: a joint venture of reflective researchers and reflective practitioners
- The role of mathematics in mathematics education.

1 Mathematics Education as a “Systemic-Evolutionary” Design Science

The proposal to consider mathematics education as a design science in Wittmann (1995) was stimulated by the intention to establish a sound methodological basis for a science of mathematics education that would guarantee a firm link between theory and practice and preserve the mathematically founded work achieved in curriculum development and teacher education by mathematics educators in the past (see Sect. 2). This proposal was based on the seminal book by Simon (1970) in which the design sciences were characterized as being concerned with the construction of artefacts that

serve defined purposes. In the design science mathematics education these artefacts are substantial learning environments.

Therefore, the core of this discipline consists of the design, the empirical investigation, and the implementation of substantial learning environments both with respect to boundary conditions set by society and beyond these constraints. It is obvious that there is a basic difference between design sciences, such as mechanical engineering and computer science in which artefacts (cars, computers, etc.) are developed that function according to natural laws in a completely controlled way and can be easily applied by the users, and design sciences such as economics, medicine in which the artefacts (marketing strategies, therapies, etc.) cannot take account of all elements of the environment in which the artefacts are to be used as this environment is simply too complex and also fluid.

Following Malik (1986), these two classes of design science can be distinguished as “mechanistic-technomorph” and “systemic-evolutionary” design sciences. Obviously, mathematics education belongs to the latter class for which a sharp separation between researchers and developers who design artefacts and users who simply apply them is not appropriate. The consequences for mathematics education have been indicated already in Wittmann (1995) and further elaborated in more general terms in Wittmann (2001). In the following, the practical implications of this systemic principle are discussed.

2 Taking Systemic Complexity Systematically into Account: Lessons from Other Disciplines

In the comprehensive literature in which appropriate models for the cooperation between researchers and practitioners in systemic-evolutionary design sciences are developed, Donald Schön’s research on the “reflective practitioner” stands out in depth and in scope (Schön 1983). Schön was mainly concerned with management, architecture, psychotherapy, town planning, and those parts of engineering in which social aspects matter. Later he extended his analyses also to education (Schön 1991). This stimulated other educators to expand on them (cf., for example Wieringa 2011).

Schön describes the traditional relationship between “professionals” and “clients” as follows (Schön 1983, p. 292):

In the traditional professional-client contract, the professional acts as though he agreed to deliver his services to the client to the limits of his special competence ... The client acts as though he agreed, in turn, to accept the professional’s authority in his special field [and] to submit to the professional’s ministrations.

In some parts of some practices ... practitioners can and do make use of the knowledge generated by university-based researchers. But even in these professions, ... large zones of practice present problematic situations which do not lend themselves to applied science. What is more, there is a disturbing tendency for research and practice to follow divergent paths. Practitioners and researchers tend increasingly to live in different worlds, pursue different enterprises, and have little to say to one another.

Schön replaces the unproductive traditional roles of researchers and practitioners with a picture in which the responsibilities are to some extent shared. Researchers act as “reflective researchers” and practitioners as “reflective practitioners” (Schön 1983, p. 323):

In the kinds of reflective research I have outlined, researchers and practitioners enter into modes of collaboration very different from the forms of exchange envisaged under the model of applied science. The practitioner does not function here as a mere user of the researcher’s product. He reveals to the reflective researchers the ways of thinking that he brings to his practice, and draws on reflective research as an aid to his own reflection-in-action. Moreover, the reflective researcher cannot maintain distance from, much less superiority to, the experiences of practice. ... Reflective research requires a partnership of practitioner-researchers and researcher-practitioners.

However, Schön is far from denying researchers a special status: “Nevertheless, there are kinds of research which can be undertaken outside the immediate context of practice in order to enhance the practitioner’s capacity for reflection-in-action” (Schön 1983, p. 309).

Schön distinguishes four types of this “reflective research” (Schön 1983, p. 309ff.):

Frame analysis: This type of research deals with general attitudes that provide practitioners with general orientations for their work.

Repertoire-building research: The focus here is on practical solutions of exemplary problems (“cases”) that provide guidance not only in routine cases but also when it comes to dealing with similar new problems.

Research on fundamental methods of inquiry and overarching theories: This type is closely connected to both types mentioned above. It is directed to developing “springboards for making sense of new situations” for which no standard solution is available.

Research on the process of reflection-in-action: Here the emphasis is on stimulating and reinforcing practitioners to engage in reflective practice.

In recent years, the paradigm of applied science with its typical separation of responsibilities has been challenged also from another side. In his sociological studies of the ways technological tools (nuclear power stations, pesticides, vaccines, etc.) are developed, tested, and implemented and how these tools affect natural and social systems, the French philosopher Bruno Latour equally rejected the traditional separation between research and applications and introduced the concept of “collective experiment”:

In this new constellation, the expert is more and more disappearing. . . . The expert has been responsible for the mediation between the producers of knowledge and the society concerned with values and ends. However, in the collective experiments in which we are intrinsically caught up, exactly this separation of different roles has disappeared. So the position of the expert has been undermined. [It has] been proposed that the extinct concept of “expert” be replaced by the comprehensive concept of “co-researcher.” (Latour 2001, p. 32, transl. E. Ch. W.)

Obviously the educational system is a “collective experiment, in which we are intrinsically caught up”. A separation between researchers who provide professional knowledge and teachers who simply use this knowledge is not appropriate.

Neither Schön's nor Latour's analyses provide practical solutions for mathematics education. However, they stimulate ideas for addressing the issue of managing complexity in this field.

3 Empowering Teachers to Cope with Systemic Complexity as Reflective Practitioners

In the first part of this section proposals are made how the collaboration between mathematics educators as reflective researchers and teachers as reflective practitioners can be filled with life. In the second part these proposals are examined in the light of the preceding section.

A good general orientation for this section is given by John Dewey's view on the role teachers can play as "investigators". This view bears witness to the systemic sensibility of this farsighted author:

It seems to me that the contributions that might come from classroom teachers are a comparatively neglected field; or, to change the metaphor, an almost unworked mine. ... There are undoubted obstacles in the way. It is often assumed, in effect if not in words, that classroom teachers have not themselves the training that will enable them to give effective intellectual cooperation. This objection proves too much, so much so that it is almost fatal to the idea of a workable scientific content in education. For these teachers are the ones in direct contact with pupils and hence the ones through whom the results of scientific findings finally reach students. They are the channels through which the consequences of educational theory come into the lives of those at school. I suspect that if these teachers are mainly channels of reception and transmission, the conclusions of science will be badly deflected and distorted before they get into the minds of pupils. I am inclined to believe that this state of affairs is a chief cause for the tendency, earlier alluded to, to convert scientific findings into recipes to be followed. The human desire to be an "authority" and to control the activities of others does not, alas, disappear when a man becomes a scientist. (Dewey 1929/1988, 23–24)

As stated in Sect. 1 the core of mathematics education as a design science consists of the design, the empirical investigation and the implementation of substantial learning environments with respect to boundary conditions set by society and beyond. So it has to be examined in which way teachers can be enabled and encouraged to act as reflective practitioners in these three areas.

In terms of design: In the author's view the most important service mathematics educators can render to teachers is to provide them with elaborated substantial learning environments together with the structure-genetic didactical analyses on which the design has been based. The language in which substantial learning environments are communicated is meaningful to teachers. So reflective practitioners have good starting points to transform what is offered to them into their context and to adapt, extend, cut, and improve it accordingly. In a recent paper Chun Ip Fung has demonstrated teachers' creative work in this area by means of a striking example and has shown that in this way a constructive dialogue between researchers and teachers can be established (see Sect. 3).

In terms of implementation: Individual learning environments and curricula cannot be implemented successfully without teachers' support. The implementation requires again teachers' creative powers in taking the local conditions into account and in adapting the proposed materials correspondingly. It is a triviality that teachers will engage more in the implementation of contents, objectives, or methods, the more these are meaningful to them. Reflective researchers have to keep this in mind.

In terms of empirical evidence: This is a particularly important issue. In the author's view teachers can best act as reflective investigators if empirical studies are attached to substantial learning environments and the results are communicable in a language that is understandable. Under these conditions teachers can cooperate in these studies and contribute to communicating the findings to practice.

However, empirical studies of the ordinary type are not the only way to get empirical evidence for the feasibility and the effectiveness of substantial learning environments. Another source are structure-genetic didactical analyses of the subject matter. Mathematics, well understood, provides not only the subject matter of teaching, but also methods of learning and teaching as it is itself the result of learning processes (see Sect. 4 in Wittmann 2018, with references to the fundamental paper by Dewey 1977). As these analyses imply empirical information on "staging" learning environments in the interaction with students, it is justified to call them empirical research "of the first kind," in distinction from ordinary empirical studies, the empirical research "of the second kind." Both structure-genetic didactical analyses and ordinary empirical studies are conducted either by researchers alone or determined by them. As teachers who collaborate with researchers in a research team are provided with additional information, have access to additional material, and enjoy support in various ways, they work under conditions that do not reflect the real practice. So for systemic reasons another type of empirical study seems promising: "collective teaching experiments." This empirical research "of the third kind" is obviously derived from Latour's "collective experiments." It is conducted by "freelancing" teachers in their daily practice, as will be discussed in some detail in the following section.

To conclude the present section, the above proposals for the interaction between reflective researchers and reflective teachers are examined against Schön's (1983) four types of "reflective research."

In terms of frame analysis: In order to provide teachers with an orientation beyond substantial learning environments, it is useful to summarize basic knowledge about mathematics, learning and teaching mathematics in didactical principles. One principle, for example, is "orientation on fundamental mathematical ideas." This principle is based on Alfred N. Whitehead's view on mathematical education (Whitehead 1929), Jean Piaget's epistemology (e.g. Piaget (1972), and Hans Freudenthal's work, in particular Freudenthal (1983). This principle can be communicated to teachers best by linking it to series of learning environments in which this principle is a leading one.

In terms of repertoire-building research: Elaborated substantial learning environments form a repertoire for teaching par excellence. They contain the essential information for teaching. The reflective teacher, however, will not stick to this repertoire but use it as a springboard for exploring other learning environments.

In terms of research on fundamental methods of inquiry and overarching theories: In close connection with the two types of research discussed before this type of research is directed to introducing teachers into methods of inquiry inherent in mathematics and into elementary mathematical theories of subject matter that are relevant for teaching.

In terms of research on the process of reflection-in-action: The proposals that have been made for the design, the empirical study, and implementation of a substantial learning environment are well suited to stimulating teachers to act as reflective practitioners.

It is obvious that both pre-service and in-service teacher education play a key role in educating reflective practitioners. Therefore the reflective mathematics educator is well advised to link his research to teacher education including mathematics education and at least elementary mathematics.

4 Collective Teaching Experiments: A Joint Venture of Reflective Teachers and Reflective Researchers

The idea to encourage teachers to become researchers of their own practice is not new at all. It is particularly manifest in the Japanese tradition of lesson studies (Stigler and Hiebert 1999). In lesson studies, a group of teachers collaborates over a period of time on the design, the empirical investigation, and the implementation of learning environments. The lessons are given by teachers in actual classrooms, observed, discussed and refined in several rounds until an acceptable result has been reached. A striking example is the recent Japanese research on elements of knot theory (Kawauchi and Yanagimoto 2012).

Collective teaching experiments are a modification of lesson studies in the following way: The reflective researchers offer research problems publicly and invite teachers to investigate them in their daily practice. There is only a loose connection with researchers who collect the feedback and turn it into the improvement of the design and the implementation.

The following example is to illustrate this proposal:

In the past decades, German math teaching at the primary level has undergone a development away from standard procedures towards flexible strategies that reflect the true nature of mathematics. The curriculum developed in the project Mathe 2000 is based on fundamental ideas of mathematics that can be developed over the grades. The arithmetical laws represent such a fundamental idea. The commutative and associative law of addition are implicitly introduced even at the kindergarten level and applied in a consequent and consistent manner at the primary and secondary level. The laws leave space for applying them in different ways, and it is important that teachers and children become aware of this freedom by being offered different strategies.

In adding two digit numbers there are essentially three basic strategies. None of them causes problems (see Fig. 1).

First plus tens, then plus ones		Tens plus tens, ones plus ones		Auxillary problem	
$47 + 35 = 82$	$65 + 28 = 93$	$47 + 35 = 82$	$65 + 28 = 93$	$47 + 35 = 82$	$65 + 28 = 93$
$47 + 30 = 77$	$65 + 20 = 85$	$40 + 30 = 70$	$60 + 20 = 80$	$45 + 35 = 80$	$63 + 30 = 93$
$77 + 5 = 82$	$85 + 8 = 93$	$7 + 5 = 12$	$5 + 8 = 13$		

Fig. 1 Basic strategies for addition problems

All three strategies can be transferred to subtraction. However, the second strategy causes a problem when the ones in the subtrahend exceed the ones in the minuend (see Fig. 2).

First minus tens, then minus ones		Tens minus tens, ones minus ones		Auxillary problem	
$87 - 35 = 52$	$65 - 28 = 37$	$87 - 35 = 52$	$65 - 28 = 37$	$87 - 35 = 52$	$65 - 28 = 37$
$87 - 30 = 57$	$65 - 20 = 45$	$80 - 30 = 50$	$60 - 20 = 40$	$85 - 35 = 50$	$65 - 25 = 40$
$57 - 5 = 52$	$45 - 8 = 37$	$7 - 5 = 2$	$5 - 8 = -3$		$40 - 3 = 37$

Fig. 2 Basic strategies for subtraction problems

Experience shows that many children transfer the strategy “tens plus tens, ones plus ones” blindly to the strategy “tens minus tens, ones minus ones” and arrive at wrong results. For the problem $65 - 28$, for example, they calculate $60 - 20 = 40$, $8 - 5 = 3$ and get 43. So teachers, supported by textbook authors, reject and avoid this strategy either by prescribing the first subtraction strategy or by modifying the critical one as follows: $50 - 20 = 30$, $15 - 8 = 7$, so $65 - 28 = 37$.

For us such didactic compromises are no option. We believe that it is better not to avoid the critical strategy also for its long-term importance. As early as 1977, the Dutch computer scientist Sytze van der Meulen, after his talk in our colloquium at our Institute for Development and Research in Mathematics Education in Dortmund, left a message in our guestbook that has since been a continuous reminder to us:

When a boy answers the question “how much is $7 - 4$ ” with 3, he is not a genius when his age is 7. When this boy answers the question how much is “ $4 - 7$ ” with “there are three missing” he shows some intelligence, but still is not a genius at the age of 7. The tragedy of our school-education is that this boy at the age of 11 may have difficulties with the concept of negative numbers. The tragedy of his teacher is that he missed 4 years of the boy’s development!

Over the years we have taken several steps to overcome teachers’ scruples concerning the subtraction strategy “tens minus tens, ones minus ones,” and we have stimulated teaching experiments on a small scale. Since 1995 we have been using any opportunity to explain this strategy to teachers and to ask them to try it out with their students.

We recommend to explain $5 - 8 = -3$ as follows:

We have 5 and have to take away 8. First we take away 5, and then we have to take away 3 more. In order not to forget this, we note it down as “-3”. Finally we take away 3 by breaking up one ten into 10 ones, and remove 3 of them.

By means of bars of ten and counters, this procedure can be well demonstrated step by step.

We also tell teachers that this strategy has an important advantage: The calculations are easier in comparison with the first strategy, so this strategy seems particularly suited for weaker students despite the first impression that it might not be appropriate for them.

One teacher did a small study and communicated it to us: After she had taught subtraction in the hundreds space in the traditional way without the critical strategy, she administered a test to her class. Then she introduced this strategy and repeated the test. It turned out that the results were no better and no worse. As she had a class with many weak students who had difficulties with this strategy, her recommendation was to avoid this strategy. Nevertheless, we continued our “propaganda” for this strategy and improved our proposal how to explain it to students.

One year later an “unforced” e-mail arrived from this teacher that read as follows:

Last year my students did have difficulties with the strategy “tens minus tens, ones minus ones” because of the negative numbers. Now I would like to report on my latest experiences with this strategy in grade 3. In the introductory lesson, I wrote down the problem $629 - 263$ without any repetition of the calculations from the year before and without any explanations from my side. Apart from very few exceptions the children calculated $20 - 60 = -40$. For them it was obvious: “The result is -40 , exactly as last year with the ones.” I would emphasize that my class is not a superclass, and that I have very many weak students. For them, calculations with negative numbers do not cause any problem. I am strongly in favour of introducing this strategy already in Grade 2.

As many teachers have still reservations against this strategy, we have refined our explanation. We recommend now to let students distinguish between the cases when there are enough ones in the minuend and those where more ones have to be taken away than are available. We recommend to propose packages of subtraction problems to students and asking them to mark those where a minus sign appears in ones calculation with an asterisk before they perform the actual calculations. The latest improvement in teaching this strategy is to explicate $5 - 8 = -3$ in more detail: $5 - 5 - 3 = 0 - 3 = -3$. We do not have enough feedback from teachers as this moment; however, we are confident that this step will increase the acceptance of this strategy.

These experiences and similar ones with other issues have led us to a far-reaching conclusion: Our main publication, a handbook for teaching arithmetic at the primary level (Wittmann and Müller 1990/1992) will be rewritten soon with the explicit invitation to teachers to conduct collective teaching experiments. All learning environments collected in this new book will belong to the standard curriculum. They will be accompanied not only with the general recommendation to read them critically and to test them in their classroom but also to participate in conducting collective teaching experiments in cooperation with other teachers. We will also create a platform for an exchange about the experiences with these experiments.

Issues that are of particular interest for us are operative proofs, the use of our course on mental arithmetic, and the use of new digital means of representation.

5 Closing Remarks: The Role of Mathematics in Mathematics Education

It is important to realize that the research and development program that has been described in this paper heavily depends on resources that are offered by “well-understood” mathematics. “Well-understood” means that mathematics is seen as a social organism that has developed in history and it still developing with strong relations to many areas of human life, and that also the mathematical knowledge of the individual is seen as an organism in its genesis from tiny seeds to a more or less extensive body. Doing mathematics is learning mathematics and learning mathematics should also be firmly linked to doing mathematics. Therefore, the interaction between teachers and students and between teachers and researchers can greatly profit from relying on the adaptability of elementary mathematical structures with respect to students’ individual cognitive levels and on the processes inherent in vital mathematics.

When once asked what his motives as a mathematician were for engaging in mathematics education Hans Freudenthal replied: “I want to understand better what mathematics is about.” The reverse also holds: mathematics educators who want to understand better what mathematics education should be about are well advised to study elementary mathematical structures thoroughly. It is highly rewarding to “unfreeze” the educational material that is “deep-frozen” in polished presentations of mathematics, as they are common in higher mathematics. After all “well-understood” mathematics is the best common reference for all involved in teaching and learning mathematics: researchers, teachers and students. “Theories of mathematics education” like those collected in Sriraman and English (2010) are far from being suited for establishing a systemic cooperation between reflective researchers and reflective teachers.

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