Chapter 8

Anticipation and the Dynamics of Expectations

The operationalization of socio-cognitive structures in terms of observables such as texts (e.g., in discourse analysis and scientometrics) or the behavior of agents (e.g., in the sociology of scientific knowledge) may inadvertently lead to reification. The dynamics of knowledge are not directly observable, but knowledge contents can be reconstructed. The reconstructions have the status of hypotheses; hypotheses can be tested against observations. Whereas agent-based modelling (ABM) focuses on observable behavior, simulations based on algorithms developed in the theory and computation of anticipatory systems (CASYS) enable us to visualize the incursive and recursive dynamics of knowledge at the individual level as different from the potentially hyper-incursive dynamics at the intersubjective level. The sciences can be considered as “strongly anticipatory” at this supra-individual level: expectations are discursively reconstructed in terms of next generations of expectations. This reflexive restructuring is embedded in historical dynamics on which it feeds back as a selection environment. The agents and texts entertain discursive models and thus be considered “weakly anticipatory” participants in the communication.

In a lecture entitled “Epistemology Without a Knowing Subject,” the philosopher Karl Popper (1967, 1972) elaborated his argument about “objective knowledge” in “World 3” as follows:

[...] it is important to distinguish between different senses of the word *knowledge*:

- (1) Subjective knowledge which consists of certain inborn dispositions to act, and of their acquired modifications.
- (2) Objective knowledge, for example, scientific knowledge which consists of conjectural theories, open problems, problem situations, and arguments.

All work in science is work directed towards the growth of objective knowledge. We are workers who are adding to the growth of objective knowledge as masons work on a cathedral. (pp. 131f.)

As is probably well-known, Popper distinguished three worlds: World 1 consisting of physical objects and events, including biological phenomena; World 2 as the world

This chapter is based on: Leydesdorff (2015).

of mental processes; and World 3 as containing “objective knowledge.” The three worlds are substantively different and operate in parallel.

Objective knowledge in World 3 is possible because human language has also a descriptive function. A description can be objectified; for example, by writing it on paper. As Popper formulated:

Without the development of an exosomatic descriptive language—a language which, like a tool, develops outside the body—there can be *no object* for our critical discussion. But with the development of a descriptive language (and further, of a written language), a linguistic third world can emerge. (p. 120)

In my opinion, language not only provides the option of exosomatic descriptions, but enables us to shape *discursive knowledge* as a result of interactions among the descriptions. In this context, “discursive” is different from Popper’s use of the word “linguistic.”

From an evolutionary perspective, discursive knowledge can be considered as “speciation” of another medium in the communication. In Chap. 2, I argued for “a communicative turn in the philosophy of science”—beyond Rorty’s 1992 [1967] “linguistic turn.” Synergies based on interactions among *codes in the communication* can induce a knowledge base which operates in terms of expectations. This neo-evolutionary model was elaborated in Chaps. 4 and 5 for the case of the Triple Helix of university-industry-government relations.

With the coding of the communications, the medium has changed into discursive knowledge. A symbolic layer is added to the language. One needs a specific competence to participate in the discourse beyond linguistic competence. Concepts can be symbolically generalized and function in the next-order communication dynamic of interacting codes. The functionally different codes span horizons of meaning that operate as selection environments at arm’s length from the intentions of the language users. The relations between language users and cognitive structures are mediated by language.

Adding a second dimension to language changes the medium into a performative medium. For example, one can use this second dimension first for rank-ordering the communications on a speakers list in a debate; for example, in parliament. However, the second dimension can also be used as a grouping variable. Specific languages usages (e.g., jargons) invoke other symbols. The communication can become richer and more performative because of an additional selection in the second dimension (Broszewski, 2018; Distin, 2010). A restricted discourse can always be elaborated into natural language (Bernstein, 1971; Coser, 1975).

The newly added dynamics (of discourse) feeds back as a modifier on the carrying one of common language. Luhmann (1997a, b, pp. 393f; 2012, pp. 238f.) formulated the dual-layered communication dynamics, as follows:

[...] the medium therefore has to be both condensed and confirmed in a paradoxical because contrary operation of generalization through specification. Media symbols thus generate, we could say, the eigenvalues of their own recursivity. When they are reused, such medium-specific eigenvalues develop—for example, as the value of otherwise valueless money symbols. [...]

In Chap. 1, I discussed Marx' (1867) argument that the mediation of money ("Ware-Geld-Ware" or WGW) is qualitatively different from the exchange of money via the mediation of commodities ("Geld-Ware-Geld" or GWG).¹ The dynamics of money are more abstract than those of commodities; more transactions can be processed. Further codification of money into credit enables us to shop worldwide. Luhmann (1997a, b, pp. 393f.; 2012, pp. 238f.) added that "the issue of generalized acceptance has been discussed particularly with regard to the medium of money. But it concerns all other media as well." The more fine-tuned the medium is, the richer the communication can be. The capacity of the medium to select from the variation is limited by the quality of the reflection in the medium (cf. Ashby, 1958).

When the processing grows more complex than the content of the communications, a change in the relative weights of couplings among subdynamics may restructure the system. In abstract terms, one can consider this as reaction-diffusion dynamics and its bifurcations. Avoiding this abstract vocabulary, Luhmann formulated as follows:

All communication is an operation that takes place concretely under the direction of specific meaning intentions. It is concerned with the truth of certain statements, compliance with certain instructions, the purchase of certain objects, certain signs of love—or indifference. Individual communications of this type are, however, never self-motivating, they draw on a recursive network of reusability of the same medium. In each and every case, the medium therefore has to be both condensed and confirmed in a paradoxical because contrary operation of generalization through specification. Media symbols thus generate, we could say, the eigenvalues (grouping variables, I.) of their own recursivity.

The selective cycling may lead to stabilization of patterns of communication over time and globalization as a pending selection mechanism in the representation of the future. When stability is lost, the communication still has the option of this external hold of globalization:

[...] a medium can use the *future* of its own operations as a *focus for externalizations*. Future is and remains external in that it can never become reality but is always only held in abeyance. In so far as reality is actual, every system always finds itself at the end of its history. However, one can test at every moment, in every present, whether the future still holds what was promised. Whether others are still willing to accept money can, however, be tried out only in the present, but in every present. Lovers swear to be eternally faithful—at a moment for the moment. But here, too, one situation follows another, and we can (however self-destructive this might be) check over and again whether the oath is still valid. Truths can already be revised tomorrow; but if new truths are to be convincing, they must be able to offer an explanation for what, as one now knows, the old truths had wrongly explained, for otherwise there would be not competition for substitutes.

We can accordingly very well assume that media validate themselves with reference to previous states and even derive certain form requirements from this self-validation. We need only a sufficiently subtle theory of time that determines the present as the boundary between past and future.

I shall argue in this and the next chapter that the theory and computation of anticipatory systems as developed by Rosen (1985) and Dubois (e.g., 1998, 2003;

¹"Ware-Geld-Ware" can be translated as "Commodity-Money-Commodity" (Marx, 1995, pp. 66 ff.).

Dubois & Resconi, 1992) can serve us as this “sufficiently subtle theory of time.” In addition to the *recursive* shaping of the codes along the arrow of time (morphogenesis in texts and practices), one can define incursions at the border between past and future, and hyper-incursion in restructuring the order of expectations. Different from recursion on a previous state, incursion operates with reference to the current state and hyper-incursion inverts the arrow of time so that anticipated states can drive the operation. However, the various dynamics cannot sufficiently be distinguished using common (“natural”) languages, since one would have to add time-subscripts to all language usage as in the case of algorithms. Language operates with geometrical metaphors and on the assumption that meanings are relatively stable within a single text or context. Algorithmically, the descriptions, the described, and the meanings can change in strange loops with irregular transitions.²

As we shall see below, the incursive, recursive, and hyper-incursive equations can have very different solutions despite their common background. The equations provide access to different realities because of the involvement of other selection mechanisms (Casti, 1989). What is “true” from one perspective, may be “false” from another. When a routine is evaluated as “false,” the prevailing regime of a “do while true” loop is interrupted and another routine is invoked.

8.1 Popper’s Perspective on the Growth of Knowledge

Can the dynamics of Popper’s “objective knowledge” in World 3 be specified as a selection mechanism? Are incursive and hyper-incursive selection mechanisms different from recursive ones so that we can envisage to answer Luhmann’s (1971, p. 34; 1990a, b, at p. 27) quest for a selection mechanism that does not shrink, but enriches the data? “[W]hat is special about the meaningful or meaning-based processing of experience is that it makes possible *both* the reduction and the preservation of complexity; i.e., it provides a form of selection that prevents the world from shrinking down to just one particular content of consciousness with each act of determining experience.”

In the noted lecture, Popper (1967; 1972, at p. 121) explicated the evolutionary mechanism operating within World 3 as follows:

The autonomous world of the higher functions of language becomes the world of science. And the schema, originally valid for the animal world as well as for primitive man,

$$P_1 \rightarrow TT \rightarrow EE \rightarrow P_2$$

becomes the schema of the growth of knowledge through error-elimination by way of systematic *rational criticism*. It becomes the schema of the search for truth and content by means of rational discussion. It describes the way in which we lift ourselves by our bootstraps. (p. 121)

²The loops can be strange because a stochastic element can be involved.

The scheme $P_1 \rightarrow TT \rightarrow EE \rightarrow P_2$ begins with a problem P_1 which is “tentatively theorized” (TT) and then via “error elimination” (EE) leads to a next problem P_2 which can enter a next cycle in a loop. The cycles can build upon each other, co-evolving into a trajectory.³ According to Popper, this scheme is “originally valid for the animal world as well as for primitive man.” However, the *mechanism* of how this scheme “becomes the world of science” was not further specified. The text suggests that this evolutionary step can be achieved by cumulative and gradual changes.

On the basis of Maturana and Varela's (1984) model of *autopoiesis* (“self-organization”) and Luhmann's elaboration of this model for the sciences as systems of expectations, I argued above that as a consequence of cycling, codes can be developed that function as shortcuts in the communication. The codes can be expected to develop further along the eigenvectors of the communication matrix when this matrix is repeatedly multiplied by itself (von Foerster, 1960). Each code adds a dimension and therefore increases the redundancy. Most cells will initially be empty: the information content is then not affected, but the redundancy is.⁴ Interactions among differently coded communications (e.g., economic, technological, and political) can bootstrap a knowledge-based order into virtual existence. Using the word “virtual” is here intended to signal that the codes and this order (in a vector space) are not “given” but remain a reconstruction, for example, in language (cf. Giddens, 1981, p. 64).

Whereas the cycles continue to loop along trajectories at $\{t_1, t_2, \dots t_n\}$ with the arrow of time—stepwise as in Popper's above scheme $[P_1 \rightarrow TT \rightarrow EE \rightarrow P_2]$ —each solution to a problem *incurs* as a feedback on historical developments and can then trigger discontinuity. When this feedback term prevails, historical trajectories can bifurcate into branches (cf. Sahal, 1985; Waddington, 1957). This mechanism of bifurcation is known as reaction-diffusion dynamics (Turing, 1952; cf. Leydesdorff 2006, pp. 169 ff. for an extensive explanation): the reaction process generates a diffusion dynamic with a different logic. After a bifurcation the diffusion dynamic becomes a selection environment for the reaction process which continues to provide variation. Selection is deterministic and thus this feedback can take control.

In a related lecture entitled “Evolution and the Tree of Knowledge,” Popper ([1961] 1972, pp. 262f.) elaborated the analogy with and the difference from biological growth. Like a tree, knowledge “grows,” but “almost in the opposite direction.” The time axis is inverted by the dynamics of knowledge when compared with biological growth:

When we spoke of the tree of evolution we assumed, of course, that the direction of time points upwards—the way the tree grows. Assuming the same upward direction of time, we should have to represent the tree of knowledge as springing from countless roots which grow up into the air rather than down, and which ultimately, high up, tend to unite into one common stem. In other words, the evolutionary structure of the growth of pure knowledge is almost the opposite of that of the evolutionary tree of living organisms, or of human implements, or of applied knowledge.

³One can also consider such a longitudinal series of cycles as a helix.

⁴The maximum entropy $H_{\max}$ is extended from $H_{\max} = \log(n * m)$ to $H_{\max} = \log(n * (m + 1))$. If n is large, this change can have large consequences.

In summary, three elements relevant to my argument were articulated in Popper's philosophy of science: (i) scholarly discourse is constitutive for the development of knowledge at the supra-individual level; (ii) the growth of knowledge operates with another time direction than biological evolution; and (iii) the emerging World 3 exhibits a (quasi-)autonomous dynamic.⁵

I added the possibility of an incursive dynamics at 90° to the plus and minus directions of "with" or "against" the arrow of time. Incursions operate orthogonally (as interventions) on trajectories; regimes operate with one more turn of 90 degrees as feedbacks in the opposite direction; that is, as selections from the perspective of hindsight—against the arrow of time. The codes anchor meanings in the domain of expectations, but some meanings are anchored more than others; the strength of incursive couplings between variation and selection can be expected to vary. The less anchored meanings may be discarded as noise or be forgotten. Whereas the feedback is first shaped against the arrow of time (at 180°), a bifurcation can lead to a second eigenvector at ninety degrees which codifies the dynamics of incursion. However, the relative priorities of the eigenvectors can be expected to change over time, so that the main perspective for the reflection becomes one among two (or more) orthogonal dimensions.

8.2 The Hyper-incursive Order of Expectations

In addition to sequences of events (along trajectories), an event at time t can be provided with meaning at a later moment $t + \Delta t$. In other words, meaning can be provided from the perspective of hindsight to events that have already happened or are happening. Whereas meanings incur on the events, codification operates hyper-incursively on meanings, that is, by grounding the subjective—historically contingent—perspectives on intersubjective layers of control.

In their "sociology of expectations," Brown and Michael (2003) noted a tension between the forward movement along the arrow of time and backward interpretation and control as a balance between "retrospecting prospects and prospecting retrospects." In a similar vein, Latour (1987, at p. 97) argued that "the two versions [...] are not uttered by the same face of Janus."⁶ However, a reflection in the time domain stands orthogonally—i.e., is independent of—substantive reflections at each moment of time. The word "reflexive" can have different meanings in these various contexts.

In a static design, one focuses on latent structures, whereas the development over time can be reconstructed as shaping a trajectory. When the spatial and the temporal reconstructions can operate upon each other, an inter-objective reality of expectations—horizon of meanings—can be generated in a newly emerging dimension.

⁵Maturana and Varela (1984) used the same metaphor of "a tree of knowledge," but two decades later and without a reference to Popper's World 3.

⁶One is reminded of Walter Benjamin's ([1940] 1974) "Angel of History".

However, this construct “exists” only as a structure of expectations; it remains a *cogitatum* or, in other words, a matter about which we remain uncertain. (Popper (1963) preferred to use the word “conjectures.”) Discursive knowledge develops in interactive processes which are “self-organizing” as an evolutionary dynamic including, for example, refutations (at the trajectory level) and crises or paradigm changes (at the regime level). As Popper formulated in *The Logic of Scientific Discovery* ([1935] 1959, at p. 111):

The empirical basis of objective science has thus nothing ‘absolute’ about it. Science does not rest upon solid bedrock. The bold structure of its theories rises, as it were, above a swamp. It is like a building erected on piles. The piles are driven down from above into the swamp, but not down to any natural or ‘given’ base; and if we stop driving the piles deeper, it is not because we have reached firm ground. We simply stop when we are satisfied that the piles are firm enough to carry the structure, at least for the time being.

Although an evolutionary mechanism was envisaged, the evolutionary model was not yet specified by Popper. When he formulated in this quotation that “we simply stop when we are satisfied,” one can raise the question “who are the ‘we’?” Was not “objective knowledge” knowledge without a subject? Are the “we” an aggregate of the “I”s or an interaction term among us? By focusing on “meanings,” the unit of analysis shifts from the constructing agency to the dynamics of “reconstructions and revolutions” in the constructs (Hesse, 1980). It is *no longer the agents or the texts that are updated, but the expectations*. The updates can be reflected by agents and in texts.

8.3 The *Differentia Specifica* of Inter-human Communications

Even if dolphins and monkeys were able to use a kind of language for their communication, human analysts would not have direct access to this (quasi-)language. A biologist can reconstruct and interpret “monkey speech”; for example, when monkeys signal danger to one another. However, the biologist herself using biological discourse (for example, about “monkey speech”) remains a *super-observer*, to be distinguished from the “linguaging” agents under study.

The biologist Maturana (1978, pp. 56 ff.) formulated the specificity of inter-human communications as follows:

Human beings can talk about things because they generate the things they talk about by talking about them. That is, human beings can talk about things because they generate them by making distinctions that specify them in a consensual domain, and because, operationally, talking takes place in the same phenomenic domain in which things are defined as relations of relative neuronal activities in a closed neuronal network.

What is specifically (re)constructed by the languaging among human beings? What is evolving? Unlike biological code (DNA), the codes of expectations are communication-based. The codes enable us to communicate about what is not the

case. *Redundancy* is a measure in the present of these absent possibilities. The future states are analytical specified (in the second contingency), but absent in the first.

The cybernetic hypothesis is that a next-order system is constructed bottom-up (by constructing agents), but the construct tends to take control top-down. In the case of language and languaging, the language that is emerging can be expected to structure the use of language by languaging agents. This cultural domain of evolving expectations is specifically human. Giddens (1976, at p. 144) succinctly formulated his critique of using a meta-biological metaphor for studying society, as follows:

The process of learning a paradigm or language-game as the expression of a form of life is also a process of learning what that paradigm is not: that is to say, learning to mediate it with other, rejected, alternatives, by contrast to which the claims of the paradigm in question are clarified.

Given this specific capacity to change the reflexive system under study on the fly, there remains little hope of arriving at the illusion of stable pillars of codified knowledge as, for example, seemingly in physics. The dynamics of communication cannot be stabilized in an experimental setting, since both the analysts and the subjects under study communicate and learn. Within the loops, the *cogitantes* (agency) and *cogitata* (constructs) can “learn” from each other. But one can expect an asymmetrical dynamic in the two directions. Furthermore, the different cycles can reflexively interrupt one another.

In summary, from different perspectives both Popper and Husserl argued against logical positivism that insisted on observations and that non-verifiable statements should be discarded as “metaphysical.” Pieces of the puzzle of a model of cultural evolution were specified by these authors. However, empirical operationalization and problems of the measurement were beyond the scopes of these philosophers. As Popper (1972, pp. 259f.) put it:

I cannot, of course, hope to convince you of the truth of my thesis that observation comes after expectation or hypothesis. But I do hope that I have been able to show you that there may exist an alternative to the venerable doctrine that knowledge, and especially scientific knowledge, always starts from observation.

In my opinion, the subjective “consciousness” of individual actors and the inter-objective “communication” were not sufficiently distinguished by these authors as different units of analysis. The dynamics of communication are different from individual learning.

8.4 The Theory and Computation of Anticipations

The theory and computation of anticipatory systems enable us to simulate cultural (that is, non-biological) evolutions, and thus to take next steps using simulation as a possible mechanism of methodological control. The anticipatory perspective radicalizes the inversion of time into a new paradigm: present states can be considered from the perspective of future states.

Anticipatory systems were first defined by Rosen (1985) as systems that entertain models of themselves. The model provides an anticipatory system with a degree of freedom for entertaining internal representations of other possible states. Dubois (1998; cf. Dubois & Resconi, 1992) proposed to model the representations entertained by the anticipatory systems using incursive and hyper-incursive equations. Using these equations, possible future states can be considered as independent variables counter-intuitively driving the present against the arrow of time.

The possibility of incursion as different from recursion follows analytically from the possibility of evaluating a difference equation forward or backward in discrete time. The differential equation in continuous time—e.g., speed as a function of distance over time; $v(t) = dx/dt$ —can be formulated in general (for any x) as follows:

$$dx/dt = f(x_t) \quad (8.1)$$

The additional option of a backward and a forward mode finds its origin in the possibility to approach the infinitesimal as a limit transition positively from the previous state or negatively from the perspective of the next state, as follows:

$$x(t + \Delta t) = x(t) + \Delta t f(x(t)) \quad (8.2)$$

$$f(x(t)) = (x(t + \Delta t) - x(t))/\Delta t \quad (8.3)$$

Or equivalently backward:

$$x(t - \Delta t) = x(t) - \Delta t f(x(t)) \quad (8.4)$$

$$f(x(t)) = (x(t) - x(t - \Delta t))/\Delta t \quad (8.5)$$

In continuous time, the two tangents can be the same (since $\Delta t \rightarrow 0$); but in discrete time, the one equation is recursive and the other incursive. Drawing the respective tangents, Fig. 8.1 shows that the two approximations may lead to very different results. In other words, there are two pathways for obtaining x_t : one following the arrow of time from the past ($t - \Delta t$) to the present (t), and one developing against the arrow of time from a future state ($t + \Delta t$) to the present.

The recursive equation operates in historical time and the incursive one against the arrow of time, or, in other words, as an intervention. A next state *incurs* on the present one as the expectation of a further selection. This possibility to operate against the arrow of time is akin to the model of information versus redundancy generation discussed above (in Chaps. 4–7). Both redundancy generation and (hyper-)incursive models operate against the arrow of time and therefore with a minus sign, given that the development of entropy in history is by definition positive. The minus sign is needed in order to keep a calculus of redundancy consistent with the Shannon equations: redundancy is generated against the arrow of time (reflexively), whereas entropy is generated with the arrow of time (historically). The algorithmic approach of

Fig. 8.1 Backward and forward evaluation of the infinitesimal transition. *Source* Ling and Langtangen (2020) Fig. 8.4, at p. 214, and 8.22, at p. 245

anticipatory systems elaborates and adds an algorithmic approach to the geometrical model of redundancy generation and synergy measurement at specific moments or during specific periods of time.

8.5 Incursive and Hyper-incursive Equations

The logistic equation—also known as the Pearl-Verhulst equation—can be used, among other things, for modeling growth in a biological system. The model is based on recursion: each next state is a function of the previous one. Following Dubois (1998), this recursive version of the equation serves me here as a baseline for models using incursive and hyper-incursive variants.

In the biological case, the logistic equation is formulated as:

$$x_t = ax_{t-1}(1 - x_{t-1}) \quad (8.6)$$

This model is *recursive*, since each next stage (x_t) builds on its previous state (x_{t-1}). In Eq. 8.6, the time relation is in accordance with the arrow of time in *both* arguments of the equation. For example, a population first grows with each time step ($x_t \rightarrow x_{t+1}$), but then increasingly selection pressure—written as $(1 - x_{t-1})$ in Eq. 8.6—is generated, bending the system's growth curve into the well-known S-shape.

An incursive version of this same equation can be formulated, for example, as follows⁷:

$$x_t = ax_{t-1}(1 - x_t) \quad (8.7)$$

⁷Another incursive equation is $x_t = x_t(1 - x_{t-1})$. This quadratic equation has two roots [$x = (1 - a)/a$ and $x = 0$], which correspond to the steady states of Eq. 8.7 to be discussed below (Leydesdorff & Franse, 2009, at pp. 110f.).

For example, the market as a system of expectations does not select commodities, technologies, etc., from among the options provided at a previous moment (that is, using $[1 - x_{t-1}]$); the market selects among options in the present. However, at the same moment the update from x_{t-1} to x_t in the first factor provides the historical (that is, recursive) perspective in Eq. 8.7. In other words: a technology develops historically—that is, with reference to its previous state—but the new technology is selected on the market in the present. We shall see that Eq. 8.7 has solutions that are different from those of Eq. 8.6.

The corresponding hyper-incursive model is:

$$x_t = ax_{t+1}(1 - x_{t+1}) \quad (8.8)$$

I shall argue that (i) the logistic equation (Eq. 8.6) can be used to model a growth process against increasing selection pressure; (ii) the incursive equation (Eq. 8.7) models an instantiation at the present moment t ; (iii) this process is hyper-incursively embedded in the structuring of expectations as modeled in Eq. 8.8.

It may seem that one can reformulate recursive equations into incursive and hyper-incursive ones by changing the temporal subscripts: instead of x_t as a function of x_{t-1} , one can also write x_t as a function of x_{t+1} . However, the consequent solutions of the equations can be very different, and so are their interpretations. In the case of the incursive equation (Eq. 8.7), for example, an anticipatory system x_t builds on its previous state (x_{t-1}), but the selection factor $(1 - x_t)$ operates in the present and not in the past, as does $(1 - x_{t-1})$ in the biological model (Eq. 8.6).

In Eq. 8.8, *history* (x_{t-1}) *no longer plays a role*. This hyper-incursive equation mirrors Eq. 8.6 in terms of the time subscripts. This hyper-incursive system is (re)constructed at $t = t$ in terms of its future states ($t + 1$, $t + 2$, etc.). Such a model without a reference to previous states can be considered *strongly* anticipatory because the expectations are generated internally; the expectations are not in the environment. Weakly anticipatory systems entertain a model to predict future states; a strongly anticipatory one uses future states to reconstruct itself (Dubois, 2002, pp. 112 ff.).

Whereas individuals can be considered as weakly anticipatory systems entertaining a model of themselves reflexively but operating historically (given a life-cycle), systems of rationalized expectations communicate inter-subjectively with reference to horizons of meaning. The communication is continuously restructured in terms of refinements of the expectations, and can be considered as strongly anticipatory while operating at the regime level. Expectations of future states are circulating hyper-incursively in this subdynamic; not the behavior of agents but their expectations are coordinated by these selection mechanisms which are different from “natural selection.” Luhmann (1971; 1990a, b, p. 27) conjectured the possibility of “a form of selection that prevents the world from shrinking down.” Unlike “natural selection,” one can consider this selection mechanism “cultural,” since oriented to generating hitherto unrealized alternatives.

8.6 Solving the Equations

Incursive and hyper-incursive equations can be expected to have solutions that are different from recursive equations. As is well known about the logistic equation (e.g., May, 1976), the bifurcation diagram of x plotted against the so-called bifurcation parameter a is increasingly chaotic when $a \rightarrow 4$, and cannot exist for $a \geq 4$. In Fig. 8.2, this development is depicted as the left half of the figure. In the case of the *incursive* Eq. 8.7, however, this limit value (for $a \rightarrow 4$) loses its relevance. The equation has solutions for $a > 4$.

One can derive on the basis of Eq. 8.7, as follows:

$$x_{t+1} = ax_t(1 - x_{t+1}) \quad (8.7)$$

$$x_{t+1} = ax_t - ax_t x_{t+1} \quad (8.8)$$

$$x_{t+1}(1 + ax_t) = ax_t \quad (8.9)$$

$$x_{t+1} = ax_t / (1 + ax_t) \quad (8.10)$$

By replacing x_{t+1} with x_t in Eq. 8.11, two steady states can be found for $x = 0$ and $x = (1 - a)/a$, respectively, as follows:

Fig. 8.2 The steady state of the weakly anticipatory system x added. *Source* Leydesdorff and Franse (2009), p. 111

$$x = ax/(1 + ax) \quad (8.11)$$

$$1 = a/(1 + ax) \quad (8.11a)$$

$$1 + ax = a \quad (8.11b)$$

$$x = (a - 1)/a \quad (8.11c)$$

These steady states correspond to (i) the non-existence of the system ($x = 0$) and (ii) the brown line penciled into the bifurcation diagram in Fig. 8.2. Note that this incursive system has values in the domain of $a \geq 4$, which is biologically not possible. An expectation itself cannot be a biological given. However, biologically embodied agents—body-mind systems—are needed to entertain these expectations. The body-mind system, for example, has a presence in both the biological and psychological domains.

The line penciled into Fig. 8.2 represents an incursive system which can provide meaning(s) to events by integrating them into both the biological domain ($a < 4$; e.g., bodily perceptions) and the domain of meaning-sharing and processing ($a \geq 4$). The instantiation of the two arguments in a single receiver integrates the information and meaning processing historically (e.g., in action), and thus can function as a linchpin between weakly anticipatory minds and strongly anticipatory communications in the cultural (i.e., non-natural) domain of meaning-processing ($a \geq 4$). As we shall see below, hyper-incursive uncertainty drives a need to take incursive decisions.

The hyper-incursive equation (Eq. 8.8) is quadratic in x_{t+1} and therefore has two possible roots:

$$x_t = ax_{t+1}(1 - x_{t+1}) \quad (8.8)$$

$$x_t = ax_{t+1} - ax_{t+1}^2 \quad (8.12)$$

$$ax_{t+1}^2 - ax_{t+1} + x_t = 0 \quad (8.13)$$

$$x_{t+1}^2 - x_{t+1} + x_t/a = 0 \quad (8.14)$$

$$x_{t+1} = 1/2 \pm 1/2\sqrt{[1 - (4/a)x_t]} \quad (8.15)$$

This system has no real roots for $a < 4$, but it has two solutions for values of $a > 4$. (For $a = 4$, the two roots are equal: $x_1 = x_2 = 1/2$; see Fig. 8.3.)

For $a > 4$, two expectations are generated at each time step: one on the basis of the plus and one on the basis of the minus sign in Eq. 8.15. After N time steps, 2^N future states are possible if this system were to operate without historical retention by making decisions. Thus, the system of expectations needs a mechanism for making choices between options, because otherwise the system would rapidly become overburdened with options. In other words, in short order the communication cannot be

Fig. 8.3 The system of expectations (x) as a result of hyper-incursion. *Source* Leydesdorff and Franse (2009, at p. 113)

further developed without agent(s) able to make choices between options, because of the continuous proliferation of uncertainty by the hyper-incursive mechanism.

Decisions by agents anchor the hyper-incursive anticipations historically in instantiations. Reasoning in another (sociological) context, Luhmann (2000) also suggested considering decisions as the structuring mechanism of organizations. Beyond single decisions, the organization of meaning can also be achieved by institutional agency, using *decision rules* as a codification of decision-making (cf. Achterbergh & Vriens, 2009). From this perspective, the individual can perhaps be considered as the minimal unit of reflection for making choices (Habermas, 1981; Leydesdorff, 2000). Both agency and organizations—institutional agents—are able to *integrate* perspectives by reflexively making choices (and taking action on that basis).

If decisions are socially organized—for example, by using decision rules instead of individual preferences—an institutional layer can increasingly be shaped. The institutional layer provides a retention mechanism for a next round of developing expectations (Aoki, 2001). Thus, the system can be considered as dually layered: (i) as a forward-moving retention mechanism, and (ii) as sets of possible expectations which flow through the networks in the opposite direction, that is, against the arrow of time.

Note that expectations can proliferate much faster than their retention in *res extensa*. Unlike action-based instantiations, “horizons of meaning” are not material or given, but continuously in flux and undergoing reconstructions. While the agents and the texts can both be part of the recursive retention mechanism, the agents as minds can also partake incursively as *cogitantes* in *res cogitans*.⁸

⁸In the semiotic tradition—actor-network theory and the sociology of translation—a distinction is made between agents in sociology and “actants” in the narrative (e.g., Latour, 1996).

8.7 Simulations of Incursive and Hyper-incursive Equations

Hitherto, x was not yet specified. The advantage of this abstractness is that x can be anything about which one is able to specify and entertain an expectation (x_{t+1}). How can one move from these very abstract bifurcation diagrams (in Figs. 8.1, 8.2 and 8.3) to modeling the sciences operating as strongly anticipatory systems? The technique of cellular automata for simulations enables me to develop and illustrate my argument (see Leydesdorff & Van den Besselaar, 1998a, 1998b). The simulation results will be used to refine the theoretical reasoning (in the next chapter.)

8.7.1 Cellular Automata

A cellular automaton is a grid of cells with rules for the interactions among the cells resulting in updates for each cell. For example, one can assume that each cell at the coordinates $\{x, y\}$ on a screen can be influenced by the four cells (above, below, to the right, and to the left) in its so-called Von Neumann environment. The coordinates of these four neighbors are: $\{x, y + 1\}$, $\{x, y - 1\}$, $\{x + 1, y\}$, and $\{x - 1, y\}$. In Table 8.1, a routine is provided simulating an environment in which the presence of at least three of the four neighbors in this Von Neumann environment induces adaptation in terms of the color of the pixel. I use a simple form of Basic (*QBasic*) in this example for the introduction of the technique.

In the routine of Table 8.1 an array “scrn(321, 201)” is first declared (in line 40) with the same size as a window opened on the screen: 320 cells horizontally and 200 cells vertically. The array is filled randomly with the values +1 and -1 (in lines 120–140). The correspondence between the values in the array and the colors on the

Table 8.1 Simulation of Von Neumann Neighborhoods in QBasic

```

10 SCREEN 1: WINDOW (0, 0)-(320, 200): CLS
20 RANDOMIZE TIMER

30 ' $DYNAMIC
40 DIM scrn(321, 201) AS INTEGER

50 FOR I = 1 TO 1000000
60   y = INT(RND * 200)
70   x = INT(RND * 320)
80   IF (x = 0 OR y = 0) GOTO 120           ' prevention of errors
90   z = scrn(x - 1, y) + scrn(x + 1, y) + scrn(x, y - 1) + scrn(x, y + 1)
100  IF z = 0 GOTO 120                     ' random attribution
110  IF z > 0 THEN GOTO 130 ELSE GOTO 140
120  IF RND < .5 THEN GOTO 130 ELSE GOTO 140
130   PSET (x, y), 1: scrn(x, y) = 1: GOTO 150
140   PSET (x, y), 2: scrn(x, y) = -1
150 NEXT I
151 SLEEP 600
160 END

```

screen enables us to study both the micro operations and macro effects in the same passes.

In each run, a pixel is randomly drawn (in lines 60 and 70) from the set of pixels horizontally ($0 < x < 320$) and vertically ($0 < y < 200$). The corresponding cell value in the array is evaluated in line 90 as the sum of the positive and negative values of its four neighbors $\{x, y + 1\}$, $\{x, y - 1\}$, $\{x + 1, y\}$, and $\{x - 1, y\}$. If the value of z resulting from this summation is positive, the pixel is set to “+1” in line 130; and otherwise to “-1” in line 140. If $z = 0$, the attribution is random (in lines 100 and 120). The system loops, for example, one million times from line 50 to line 150. The routine changes a randomly distributed screen in two colors on the left side of Fig. 8.4 into a pattern as on the right side of this same figure. In other words, a structure is always generated.

Thus, one can both numerically (in the array declared at line 40) and visually (on the screen) follow how the rules affect each element at both the individual pixel level and the aggregate level. The rules can also be made dynamic; for example, by specifying thresholds for the introduction of new routines. In Table 8.1, the drawing of pixels is random (lines 60–70) and structure is emerging. However, this can be defined differently in non-biological models. One can change the perspective without consequences for this methodology.

A cellular automaton allows for intersections among loops, including “strange loops” based on incursions within recursive loops. Such an intersection is not allowed in a formal calculus because loops can be created. Action at one place, for example, may cause an avalanche of changes at other places. In principle, cellular automata enable us thus to simulate the “fractional manifolds” discussed in previous chapters. Different mechanisms and time horizons can be combined into these simulations.

Fig. 8.4 Simulation of Von Neumann Neighborhoods in QBasic on the Basis of Table 8.1

8.7.2 *Modelling of Expectations Using Cellular Automata*

Cellular automata have been used in the social sciences for modelling bottom-up processes in so-called agent-based modelling (ABM), but the method is more encompassing and abstract, allowing also for units of analysis other than individual agents (Von Neumann & Burks, 1966). ABM has become popular in the social sciences since the publication of Epstein & Axtell's *Growing Artificial Societies: Social Science from the Bottom Up* (1996).⁹ On the basis of his work with ABMs, Epstein (2006) formulated what he called a “generative” research program for the social sciences: one cannot explain a social phenomenon until one has “grown” it by simulating the phenomena under study as emerging from the bottom up (cf. Hedström, 2005).

This agent-based research program accords with the strong program in the sociology of science: individuals and their aggregates in institutions—agency—are considered as the units that generate the dynamics of the sciences (Edmonds, Gilbert, Ahrweiler, & Scharnhorst, 2011). The focus is on the “bottom-up” genesis of patterns and not on the validity of the resulting constructs as reconstructions of selection environments. While micro-founded at the level of individuals taking action (or not), the sciences are thus considered as community-based beliefs attributed to agents who can be driven by a blend of socio-epistemic interests (Axelrod, 1997); the intellectual organization of the sciences is considered as an attribute of their social organization; content is defined in terms of the individual cognition of the interacting agents (Payette, 2012; Sun, Kaur, Milojević, Flammini, & Menczer, 2013).¹⁰ Although the agents may be able to perceive and understand the intellectual dimensions of their activities, the interpretation of the results of their interactions remains agent-based (cf. Bloor, 1976).

Edmonds et al. (2011), for example, stated that “science is substantially a social phenomenon.” Furthermore, these authors claimed that “agent-based simulations of social processes are able to incorporate lessons from qualitative social science studies of what scientists actually do on a day-to-day level as well as insights from the more naturalistic philosophers of science” (cf. Scharnhorst, Börner, & Van den Besselaar, 2012). McGlade (2014, at p. 295) noted that such an “agent-based ontology” entails problems when simulating mental processes.

How would one be able to visualize the intellectual organization of cognitions? Sun et al. (2013, p. 4) noted that “[f]uture ‘science of science’ studies have to gauge the role of scientific discoveries, technological advances, and other exogenous events in the emergence of new disciplines against the purely social baseline.” Along these lines, Gilbert (1997), for example, replaced the agents at the nodes with interacting “kenes”—the knowledge-based equivalents of genes. This allowed him to show how these analytically hypothesized units of analysis can drive the evolution of the sciences.

⁹Biological applications of cellular automata, such as in Artificial Life, preceded this development by more than a decade (e.g., Allen, 1988; Langton, 1989).

¹⁰Given this focus on agency, references and citations can be understood as rhetorical devices in practices (e.g., Cozzens, 1989; Gilbert, 1977; Gilbert & Woolgar, 1974).

8.7.3 Visualizations of Anticipations

Using communications as units of analysis evolving in cellular automata, let me illustrate the operation of the three layers (A, B, and C) distinguished in Chap. 4 by elaborating an example. Figure 8.5 shows Van Gogh's well-known "Langlois Bridge at Arles" that I will use as an exemplary representation in the routines here below. The height of this reproduction was set to 308 pixels and the width to 400 pixels. However, Visual Basic counts the screen in twips, which are fine-grained and screen-independent. In the simulation of the bridge at Arles, the equivalent of $308 * 400$ pixels is $(3322 * 4200 =) 13,952,400$ twips.

Analogously to Table 8.1, the computer code in Table 8.2 provides an example (in Visual Basic) for a recursive subroutine (Eq. 8.6) of the larger routine to be discussed here below.

Two pictures are first distinguished: PicFrom(0) and PicTo(0). Horizontal (x) and vertical (y) values are attributed to PicFrom(0) in lines 2 and 3, and PicTo(0) in lines 35 and 36. PicTo(0) serves for the reconstruction of the picture after each cycle.

One can find the logistic equation (Eq. 8.6) in lines 17–19 for the red, green, and blue components at each specific position (x, y). I use the traditional red-green-blue (RGB) decomposition for the colors. Since the logistic equation requires values for x between zero and one, the color values (between 1 and 256) are first divided by 256 in lines 11–13, then transformed (lines 17–19) and renormalized into integers before picturing the results in lines 29–31.

This example is only a subroutine of a larger program in which the bifurcation parameter a can be provided interactively by the user. The bifurcation parameter is here labeled "parameter" in lines 17–19. The DoEvents in lines 40–41 makes the program sensitive to switching to other (sub)routines or exiting. The program runs in two loops in order to capture the horizontal variation x (line 32) and the vertical variation y (line 34), respectively; all the pixels are repainted in each cycle. After a

Fig. 8.5 Van Gogh's "Langlois Bridge at Arles" to be used as input to the routines. This image is in the public domain; see at http://commons.wikimedia.org/wiki/File:Vincent_Van_Gogh_0014.jpg

Table 8.2 Transformation of a representation using the logistic equation (Eq. 8.6 above)

```

1 DO
2   For Y = 0 To PicFrom(0).ScaleHeight
3     For X = 0 To PicFrom(0).ScaleWidth
4
5       ' Get the source pixel's color components
6       ' (red, green, and blue)
7       clr = PicFrom(0).Point(X, Y)
8       r = clr And &HFF
9       g = (clr \ &H100) And &HFF
10      b = (clr \ &H10000) And &HFF
11      ' scale the color values between zero and one
12      rt = r / 256
13      gt = g / 256
14      bt = b / 256
15
16      'transform recursively using the logistic equation
17      rt = (parameter * rt) * (1 - rt)
18      gt = (parameter * gt) * (1 - gt)
19      bt = (parameter * bt) * (1 - bt)
20      r = Int(rt * 256)
21      g = Int(gt * 256)
22      b = Int(bt * 256)
23
24      ' possible source of error
25      If r > &HFF Then r = &HFF
26      If g > &HFF Then g = &HFF
27      If b > &HFF Then b = &HFF
28
29      ' Write the new color of the pixel
30      clr = RGB(r, g, b)
31      PicTo(0).PSet (X, Y), clr
32
33    Next X
34    DoEvents
35  Next Y
36  ' Make the changes permanent.
37  ' PicTo(0).Picture = PicTo(0).Image
38  PicFrom(0).Picture = PicTo(0).Image
39  cmdGo.Enabled = False
40  MousePointer = vbDefault
41
42  DoEvents
43  If param2 = 0 Then Exit Do
44 Loop While True

```

cycle, the original picture (PicFrom) is replaced by the newly generated one (PicTo). For example, the representation can be expected to erode in a number of steps towards chaos for values of $a > 3.57$ when using the logistic equation recursively (Eq. 8.6).

In summary, Fig. 8.6 is based on a routine which can be run interactively using the program available at <http://www.leydesdorff.net/simulation.2015/netsci.exe>. The recursive, incurive, and hyper-incurive routines are combined in a single context so that they can be visually distinguished and compared in terms of their effects.

Figure 8.6 shows the different states of the system after a number of runs when the bifurcation parameter a is set, for example, at $a = 3.6$. Since for $a > 3.57$, the “natural” representation is decaying using the logistic equation in the left-top (PicFrom) and middle-top (PicTo) representations that alternate; after each loop PicTo becomes defined as the next PicFrom, etc. (Table 8.2, line 38).

Fig. 8.6 Recursion, incursion, and hyper-incursion in cases of using the logistic equation with Van Gogh's "Langlois Bridge at Arles." Interactive at <http://www.leydesdorff.net/simulation.2015/netsci.exe>

I have added two reflexive observers using incurive routine. The first observer is generated in the left-bottom screen observing directly the original picture (PicFrom) in the left-top screen.¹¹ The observer generated in the right-bottom screen, however, does not observe the original picture directly, but only its transformation using the hyper-incursive equation operating in the screen box in the middle at the bottom. Whereas the representation in this latter box seems almost to have disappeared, the receiver at the right-bottom is nevertheless able to regenerate the picture.

The results (in Fig. 8.6) show the possibility of operationalizing reflexive transmissions without invoking a social process. This is a communication process making a specific selection on the underlying social process. The state of mind of the local observers and their social contexts are not relevant to the reception which, instead, is determined by the communication dynamics. The specificity of this process is not the social, but a multi-layered communicative dynamics.

8.8 Sociological Implications and Concluding Remarks

The focus in this chapter has been on how an evolving system of expectations can be simulated without an a priori sociological interpretation. In the next chapter, I shall show the relevance of incurive and hyper-incursive variants of the logistic equation for addressing long-standing problems in sociology, such as how to operationalize “double contingency,” the organization of meaning and knowledge, and their further self-organization. These problems could hitherto not be addressed because the conceptualization of the social system of interhuman communications as strongly anticipatory was lacking.

I have argued in this chapter against reification of the cognitive process, but as against Husserl and Luhmann, I argue in favor of *operationalization and measurement*. As Luhmann (1995, at p. 164; 1984, p. 226) formulated: “*communications cannot be observed directly, only inferred.*” (italics in the original). Whereas communications cannot be observed, they can excellently be measured using Shannon’s (1948) information theory and simulated using cellular automata. A focus on observations without prior specification of expectations, however, has hitherto blocked this perspective.

The variants of the logistic equation used in Fig. 8.6 enrich the models by showing their limitations. Different ontologies are indicated by using the various time-subscripts in the equations. Whereas the *recursive* version of the logistic equation can be used, for example, to model the development of a biological population in history, the observer at the left-bottom of Fig. 8.4 can be considered as an individual mind using an *incurive* routine for the observation. Among other things, incursion can be used to model the coupling of biological presence with mental representations

¹¹For a more extensive discussion of the generation of an observer and observers observing each other, see also Leydesdorff (2006a) and Von Foerster (1982).

of present and future states. Incurive models are needed at the organizational level since like individual action, institutional action remains historical.

The *hyper-incurive* routines refer to evolutionary dynamics that are no longer necessarily historical. This order of conjectures can be elaborated into hypotheses which can be entertained reflexively and further informed by observations. The reference is to an intersubjective communication domain of expectations. In other words, this social order of communications does not *exist* (in history); it remains an order of expectations operating on expectations to which human beings can have reflexive access.

As Popper (1972, pp. 262f.) noted: the knowledge-base is rooted upward. In empirical terms, one can expect both upward and downward dynamics to be continuously invoked as subroutines of cultural evolution. The theory and computation of anticipatory systems provides the tools needed to take a further step: the simulation of structures in systems of expectations. Given my own criteria, I have to complement this “bottom up” genesis with a validation. The genesis shows only the historical process; the validation of these equations and their mutual relations is the purpose of the next chapter.

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