Analytical Study of Radiative Casson Nanoliquid Flow with Heat Absorption

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Abstract. The divergence of thermally radiative MHD flow of a Casson nanofluid over a stretching paper alongside heat absorption. The governing non linear equations are remodeled into a nonlinear ODE's. The HAM is adopted to find the series solution. The changes of pertinent parameters are analyzed with diagrams and tables. The fluid velocity is controlled by suction and it develops with injection. The local Nusselt number rapidly suppresses with increasing the magnetic field parameter in heat generation case.

Keywords: Casson nanoliquid · Heat absorption · Magnetic field · Thermal radiation

1 Introduction

Most of the engineering and industrial processes depend on heat transfer mechanism, because they have cooling and heating processes. In general, the ordinary fluids are transfer less amount heat because they owing poor thermal conductivity. Various researchers are tried to increase the fluid thermal conductivity in different ways. One of the simplest method is nanosized particles are suspended into an ordinary fluids to raise the fluid thermal conductivity. Applications of nanofluids are investigated by many authors for both the nanofluids with Newtonian or non-Newtonian base with different geometrical shapes. One of the base fluid is Casson fluid and which posses yield stress. After applying the shear stress, Casson fluid performs as a solid when low shear stress and it moves when higher shear stress compared to the yield stress. Example of these fluids are soup, blood, jelly, tomato sauce, etc. Some important studies in this directions are [1–10].

2 Governing Equations

The flow system is modeled with

1. Incompressible flow
2. Casson nanoliquid
3. thermal radiation
4. Magnetic field
5. Buongiorno nanofluid model
6. Stretching sheet with linear velocity.
7. heat absorption

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = v \left(1 + \frac{1}{\beta} \right) \frac{\partial^2 u}{\partial y^2} - \left(\frac{\sigma B_n^2}{\rho} \right) \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_T \frac{\partial^2 T}{\partial y^2} - \frac{16\sigma_s T_\infty^2}{3k_e \rho C_p} + \frac{\rho^* C_p^*}{\rho C_p} \left[D_B \frac{\partial C}{\partial y} \frac{\partial T}{\partial y} + \frac{D_T}{T_\infty} \frac{\partial T^2}{\partial y} \right] + \frac{Q}{\rho C_p} (T - T_\infty) \quad (3)$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_B \frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty} \frac{\partial^2 T}{\partial y^2} \quad (4)$$

The boundary points of the above system are:

$$u = u_w(x) = ax, \quad v = v_w, \quad \text{at } y = 0 \quad (5)$$

$$u \rightarrow 0, \quad v \rightarrow 0, \quad T \rightarrow T_\infty, \quad C \rightarrow C_\infty, \quad \text{at } y \rightarrow \infty \quad (6)$$

where β (=Casson fluid parameter), σ (=electrical conductivity), ρ (=density of fluid), α_T (=thermal diffusivity), C_p (=specific heat), D_B (= Brownian diffusion), D_T (+ thermophoretic diffusion coefficient), Q (=heat absorption/generation coefficient).

Consider the transformations:

$$\eta = \sqrt{\frac{a}{\nu}} y, \quad v = -\sqrt{a\nu} f(\eta), \quad u = ax f'(\eta), \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}; \quad \phi = \frac{C - C_\infty}{C_w - C_\infty} \quad (7)$$

The following ODEs are retrieved from the governing system using above transformations,

$$\left(1 + \frac{1}{\beta} \right) f'''(\eta) + f''(\eta)f(\eta) - f'^2(\eta) - M f'(\eta) = 0 \quad (8)$$

$$\left(1 + \frac{4}{3} Rd \right) \theta''(\eta) + Pr \theta'(\eta) f(\eta) + Pr Nb \theta'(\eta) \phi'(\eta) + Pr Nt \theta'^2(\eta) + Pr Hg \theta = 0 \quad (9)$$

$$\phi''(\eta) + Sc(f\phi') + \frac{Nt}{Nb} \theta''(\eta) = 0 \quad (10)$$

The boundary points of f, θ, ϕ becomes

$$f(0) = f_w, f'(0) = 1, f'(\infty) = 0, \theta(\infty) = 0, \phi(\infty) = 0 \tag{11}$$

where $M = \frac{\sigma B_a^2}{\rho}$ [=magnetic field parameter], $Pr = \nu/\alpha_T$ [=Prandtl number], $Nb = (\frac{\rho^* c_p^*}{\rho c_p} D_B (C_W - C_\infty))/\nu$ [=Brownian motion] parameter, $Nt = (\frac{\rho^* c_p^*}{\rho c_p} D_T (T_W - T_\infty))/(\nu T_\infty)$ [=thermophoresis parameter], $Hg = Q/\rho c_p a$ [=heat generation/absorption parameter], $Sc = \frac{\nu}{D_B}$ [= Schmidt number], $Rd = \frac{4}{3} \frac{\sigma_s T_\infty^3}{k_e \rho c_p \alpha_T}$ [=radiation parameter].

The surface drag force and heat transfer rate can be defined as:

$$\frac{1}{2} C_f \sqrt{Re} = f''(0) \left(1 + \frac{1}{\beta}\right) \text{ and } \frac{Nu}{\sqrt{Re}} = -\theta'(0) \left(1 + \frac{4}{3} Rd\right)$$

Table 1. Order of approximations

Order	$-F_{\eta\eta}(0)$	$-\theta_\eta(0)$	$-\phi_\eta(0)$
1	-1.875	-0.229472	-0.825641
5	-2.07069	-0.235657	-0.631038
10	-2.07751	-0.234869	-0.61575
15	-2.07737	-0.235076	-0.621188
20	-2.07737	-0.235057	-0.620607
25	-2.07737	-0.235014	-0.620104
30	-2.07737	-0.235064	-0.620437
35	-2.07737	-0.235032	-0.620348
40	-2.07737	-0.235041	-0.620326

3 Results and Discussion

The present nonlinear system was solve through HAM technique. The HAM was computed via MATHEMATICA software. Figure 1 sketched for the convergent solutions of the current study. Appoximation orders of HAM is shown in Table 1. The examinations are complete for various range of the relevant parameters intricate in this study. It is clear from Fig. 2 the velocity profile enhances for magnetic parameter (M) whereas it reduces for suction/ injection parameter

Table 2. The surface drag force and heat transfer rate values of β , F_w , M, Nb, Nt, Rd, Hg

β	F_w	M	Nb	Nt	Rd	Hg	$\frac{1}{2}C_f\sqrt{Re}$	$\frac{Nu}{\sqrt{Re}}$
0.3	0.2	0.3	-0.2	0.1	0.2	0.2	-2.23314	-0.329693
0.1							-1.96458	-0.329552
1.5							-1.81318	-0.328395
0.1							-1.75965	-0.327874
0.5	0.2	0.3	0.1	0.1	0.3	-0.2	-1.74059	-0.275054
	-0.5						-1.83052	-0.312842
	0.5						-1.92547	-0.354811
	0.1						-1.97484	-0.400887
0.5	0.2	0.3	0.1	0.1	0.3	-0.2	-1.83493	-0.640482
		0					-1.91934	-0.633721
		0.5					-2.15183	-0.633721
		0.1					-2.22367	-0.615345
			0				-1.87965	-0.621188
			2				-1.81318	-0.610898
			0.1				-1.75965	-0.602256
0.5	0.2	0.3	0.1	0.1	0.3	-0.2	-0.329107	-0.791791
				0			-0.328067	-0.621188
				2			-0.327535	-0.451693
				0.1			-0.326996	-0.28316
0.5	0.2	0.3	0.1	0.1	0.3	-0.2	-0.434391	-0.607522
				0			-0.45963	-0.636994
				2			-0.64889	-0.639523
				0.1			-0.67117	-0.64451
0.5	0.2	0.3	0.1	0.1	0.3	-0.2	-0.340084	-0.616351
						-0.5	-0.333235	-0.627037
						0.5	-0.329107	-0.634088
						0.1	-0.271261	-0.65124

(f_w). In Fig. 3 temperature ($\theta(\eta)$) increases for the radiation (Rd), Brownian constant (Nb), heat absorption constant (Hg), thermophoresis constant (Nt) and Casson parameter (β) and it decays for suction/injection parameter (f_w) and Prandtl number (Pr). The concentration($\phi(\eta)$) rises with higher (β) and Nt but it diminishes with upsurge in (f_w) and Sc (see Fig. 4) (Table 2).

Fig. 1. h-curves for $h_{f,\theta,\phi}$

Fig. 2. Influence of f_w and M on velocity

Fig. 3. Influence of Rd , Nb , Nt , β , Hg , Pr and f_w on temperature

Fig. 4. Influence of β , Nb , f_w and Sc on concentration

4 Conclusion

The key features of the present study is given below:

- Temperature profile enhances while increasing Rd , Nb , Nt , and Hg .
- Casson parameter (β) enhances the concentration and temperature profiles.
- Higher range of Prandtl number (Pr) deduce the thermal boundary.

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