Computational Analysis of Third-Grade Liquid Flow with Cross Diffusion Effects: Application to Entropy Modeling

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Abstract. The key goal of this current study is to analyze the entropy generation with cross diffusion effects. The third-grade type non-Newtonian fluid model is used in this study. The current flow problem is modelled with stretching plate. Modified Fourier heat flux is replaced the classical heat flux. The appropriate transformation is availed to convert the basic boundary layers equations into ODEs and then verified by homotopy algorithm. The consequences of various physical quantities on temperature, velocity, entropy and concentration profile are illustrated graphically.

Keywords: Third grade fluid · Linear stretching sheet · Homotopy Analysis Method (HAM) · Soret and Dufour effects · Entropy generation

1 Introduction

Third grade fluid is one of the notable sub kinds of non-Newtonian fluids. The non-Newtonian fluid flow due to the stretching surface is the important area of research due to its broad applications in many industrial and production domains such as, rolling of polymer films, extrusion of metallic sheets, etc. The study on 2nd grade fluid which passed through the stretching sheet is numerically discussed including the variations in thermophysical properties like thermal conductivity, viscosity [1]. It is shown that Eckert number increases the heat transport rate. Hydromagnetic mixed convective heat transfer of 3rd-grade fluid with gyrotactic microorganism is examined [2]. Unsteady flow of power law fluid with uniform velocity is evaluated [3]. With the consideration of heat source and

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heat sink of MHD flow over a oscillatory stretching sheet is numerically studied [4]. With the impact of chemical reaction, the fourth grade fluid through porous plate of MHD radiative fluid is investigated [5]. In addition to MHD nanofluid, the electrically conductive fluid that of second grade with suction parameter is developed [6]. For the application of bio magnetic the third grade fluid is correlated numerically [7]. The modified Fourier heat flux model for the study of carreau fluid is explored numerically [8]. The various features and applications of non-Newtonian fluids are studied in ref's [9–15].

There are several techniques available to solve nonlinear problems. The homotopy analysis method (HAM) is initially constructed by Liao [16] in 1992. Moreover, he altered with a non-zero auxiliary parameter [17] . This parameter shows the way to calculate the convergence rate. It also offers great independence to choose the base functions of the solutions. A few more studies about this technique was seen in previous works [18,19].

Inspired by the above literature surveys, we are constructing a steady 3rd-grade liquid flow with considering radiation, and convective heating effects. Dufour and Soret effects are examined. The system of entropy is discussed briefly for various parameters.

2 Mathematical Formation

The steady third grade incompressible two dimensional chemically reactive fluid flow due to stretchy surface is considered. The sheet to be stretchy by the pair of same and inverse forces with velocity $(u_w = ax), a > 0$, a is known as stretchy rate. The free stream velocity is u_∞. (C_∞) and (T_∞) are the free stream concentration and temperature and in order. The governing equations with boundary conditions are listed below

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\begin{aligned} u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = {} & \mu\frac{\partial^2 x}{\partial y^2} + \frac{\alpha_1^*}{\rho}\left(u\frac{\partial^3 u}{\partial y^2 \partial x} + v\frac{\partial^3 u}{\partial y^3} + \frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2} + 3\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2}\right) \\ & + 2\frac{\partial_2^*}{\rho}\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x \partial y} + 6\frac{\beta^*}{\rho}\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^2 u}{\partial y^2} + g\Big[\beta_T(T - T_\infty) \\ & + \beta_C(C - C_\infty)\Big] \end{aligned} \tag{2}$$

$$\frac{\partial T}{\partial x}u + \frac{\partial T}{\partial y}v = \frac{Q}{\rho c_p}(T - T_\infty) + \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p}\frac{\partial q_r}{\partial y} + \frac{D_m k_m}{c_s c_p}\frac{\partial^2 C}{\partial y^2} \tag{3}$$

$$\frac{\partial u}{\partial x}u + \frac{\partial C}{\partial y}v = D_m\frac{\partial^2 C}{\partial y^2} + \frac{D_m k_m}{T_m}\frac{\partial^2 T}{\partial y^2} - k_m(C - C_\infty) \tag{4}$$

The boundary points stated as $u = (u_w(x) = ax), (v = 0), (-k\frac{\partial T}{\partial y} = h_f(T_f - T_w)), (C = C_w)$ at $(y = 0)$,

$$u(\rightarrow 0), T(\rightarrow T_\infty), (C \rightarrow C_\infty) \text{ as } y(\rightarrow \infty) \tag{5}$$

where u & v(=velocity components along the x & y-direction), μ (=kinematic viscosity), (α_1^*,α_2^* & β_1^*)(=material parameters), ρ (=fluid density), (β_T, β_C) (=coefficient of thermal and concentration expansions), c_p (=specific heat), c_s (=concentration susceptibility), Q (=heat capacity of ordinary fluid), q_r (= radiative heat flux), C (=concentration), C_w (= fluid wall concentration), D_m (=mass diffusion coefficient), k_m (=first order chemical reaction parameter). Incorporating the Cattaneo-Christov heat flux into energy equation, we get

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + 2uv\frac{\partial^2 T}{\partial x \partial y} + \lambda\Big(u^2\frac{\partial^2 T}{\partial x^2} + v^2\frac{\partial^2 T}{\partial y^2} + \left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right)\frac{\partial T}{\partial x}$$
$$+ \left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right)\frac{\partial T}{\partial y}\Big) = \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p}\frac{\partial q_r}{\partial y} + \frac{D_m k_T}{c_s c_p}\frac{\partial^2 C}{\partial y^2} + \frac{Q}{\rho c_p}(T - T_\infty). \tag{6}$$

Consider the transformations given below

$$\eta = \sqrt{\frac{a}{\mu}}y, v = -\sqrt{a\mu}F(\eta), u = axF^{'}(\eta), \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}, \theta(\eta) = \frac{T - T_\infty}{T_f - T_\infty} \tag{7}$$

Using the above mentioned transformations we retrive the ODE system as follows

$$-F^{'2} + FF^{''} + (3\alpha_1 + 2\alpha_2)F^{''2} + 6\beta ReF^{'''}F^{''2} + F^{'''} \quad +\alpha_1(2F^{'}F^{'''} - FF^{iv})$$
$$+\lambda_1(\theta + N\phi) = 0 \tag{8}$$

$$\left(1 + \frac{4}{3}Rd\right)\theta^{''} + Prf\theta^{'} + Q_H\theta - \gamma(FF^{'}\theta^{'} + F^2\theta^{''}) + PrD_F\phi^{''} = 0 \tag{9}$$

$$\frac{1}{Sc}\phi^{''} + F\phi^{'} - C_r\phi + Sr\theta^{''} = 0 \tag{10}$$

Boundary conditions are

$$F^{'}(\eta) = 1, F(\eta) = 0, \theta^{'}(\eta) = -Bi(1 - \theta(\eta)), \phi(\eta) = 1 \text{ at } \eta = 0$$
$$F^{'}(\eta) \rightarrow 0, \theta(\eta) \rightarrow 0, \phi(\eta) \rightarrow 0 \text{ as } \eta \rightarrow \infty \tag{11}$$

where $\alpha_1 = \alpha_1^* a/\rho\mu$,$\alpha_2 = \alpha_2^* a/\rho\mu$ and $\beta = \beta^* a^2/\rho\mu$ (=fluid parameters), $Re = ax^2/\mu$ (=Reynolds number), $Pr = \rho\mu c_p/k$ (= Prandtl number), $Q_H = Q/a\rho c_p$ (=Heat generation), $\gamma = \lambda a$ (=thermal relaxation), $Sc = \mu/D_B$ (=Schmidt number), $C_r = k_m/a$(=Chemical reaction), $\lambda 1 = Gr/Re^2$ (=local buoyancy parameter), $Gr = g\beta_T(T_w - T_\infty)x^3/\mu^2$ (=Grashof number), $N = \beta_C(C_w - C_\infty)/\beta_T(T_w -$

T_∞)(=buoyancy ratio parameter), $Rd = 4\sigma_1 T_\infty^3/kk_*$ (=Radiation parameter), $D_F = D_m k_T/\mu c_s c_p \frac{C_w - C_\infty}{T_w - T_\infty}$ (= Dufour number), $Sr = \frac{D_m k_T}{\mu T_m}\frac{(T_w - T_\infty)}{(C_w - C_\infty)}$ (= Soret number).

Heat and mass transfer rate in dimensionless forms are

$$Re^{\frac{-1}{2}} Nu_x = -\left(1 + \frac{4}{3}Rd\right)\theta^{'}(0), Re^{\frac{-1}{2}} Sh_x = -\phi^{'}(0).$$

3 Analytical Procedure and Convergence Study

HAM has been used last twenty years to solve the non-linear system of ODE occurring in various fields. The nonlinear ODE are solved with the aid of HAM algorithm. This algorithm is computed through MATHEMATICA software in our personal computer with 8 GB RAM and 2.30 GHz Processor.

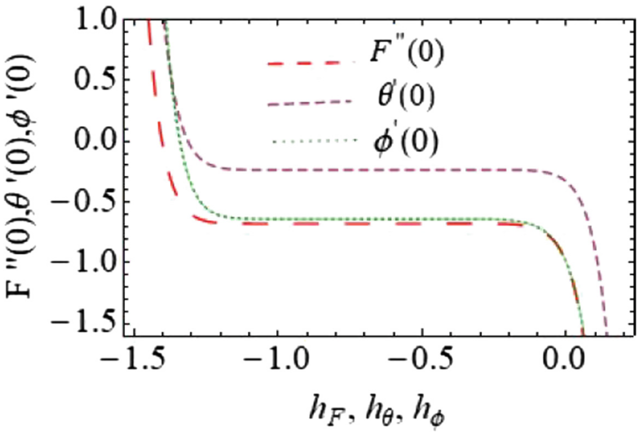

Fig. 1. h-curves for $h_{F,\theta,\phi}$.

Based on the Fig. 1, the auxiliary parameters have the range $-1.2 \leq h_F \leq 0.3, -1.3 \leq h_\theta \leq 0.1, -1.3 \leq h_\phi \leq 0.3$. Approximatley we fixed the h values as $h_F = h_\theta = h_\phi = -0.7$ (Table 1).

4 Entropy Optimization

The entropy minimization optimization for fluid friction, heat and mass transfer irreversibility's are given below:

$$S_{gen}^{'''} = \frac{K_1}{T_\infty^2}\left[\left(\frac{\partial T}{\partial y}\right)^2 + \frac{16\sigma^* T_\infty^3}{3kk^*}\left(\frac{\partial T}{\partial y}\right)^2 + \frac{\mu}{T_\infty} + \left[\left(\frac{\partial u}{\partial y}\right)^2 + \frac{RD}{C_\infty}\left(\frac{\partial c}{\partial y}\right)^2 + \frac{RD}{T_\infty}\left(\frac{\partial T}{\partial y}\right)\left(\frac{\partial c}{\partial y}\right)\right.\right.$$

Table 1. Displays the convergence solutions of HAM in order of approximation when $Pr = 0.9, \alpha_1 = 0.1, Sc = 0.9, D_F = 0.5, Re = 0.1, \beta = 0.1, \alpha_2 = 0.1, \gamma = 0.1, Bi = 0.5, Rd = 0.3, \lambda_1 = 0.2, N = 0.1, Sr = 0.3, Q_H = -0.3, h = -0.7, C_r = 0.1$.

Order	$-F''(0)$	$\theta'(0)$	$-\phi'(0)$
1	0.72116	0.23913	0.72777
5	0.67605	0.23555	0.63669
10	0.67817	0.23648	0.63945
15	0.67803	0.23643	0.63897
20	0.67801	0.23644	0.63901
25	0.67803	0.23644	0.63903
30	0.67803	0.23644	0.63903
40	0.67803	0.23644	0.63903
50	0.67803	0.23644	0.63903

Dimensionless system of entropy generation is defined as:

$$E_G = Re\left(1 + \frac{4}{3}Rd\right)\theta'^2 + Re\left(\frac{Br}{\Omega}\right)f''^2 + Re\left(\frac{\zeta}{\Omega}\right)^2\lambda\phi'^2 + Re\frac{\zeta}{\Omega}\lambda\phi'\theta'$$

5 Validation

In order to validate our numerical procedure, the results are validated with earlier report of Maria et al. [10]. The comparison results are given in Table 2.

Table 2. Comparison in absence of $D_F = 0, SR = 0, \omega = 0, Rd = 0, C_r = 0, \lambda_1 = 0, N = 0, Q_H = 0$.

Order	$-f''(0)$		$-\theta'(0)$		$-\phi'(0)$	
	Ref. [10]	Current	Ref. [10]	Current	Ref. [10]	Current
1	0.81450	0.81450	0.72778	0.72778	0.72778	0.72778
5	0.81221	0.81221	0.58070	0.58070	0.64933	0.64933
8	0.81235	0.81235	0.57779	0.57779	0.64835	0.64835
14	0.81235	0.81235	0.57871	0.57871	0.64873	0.64873
17	0.81235	0.81235	0.57878	0.57878	0.64873	0.64873
25	0.81235	0.81235	0.57878	0.57878	0.64873	0.64873
30	0.81235	0.81235	0.57878	0.57878	0.64873	0.64873
35	0.81235	0.81235	0.57878	0.57878	0.64873	0.64873

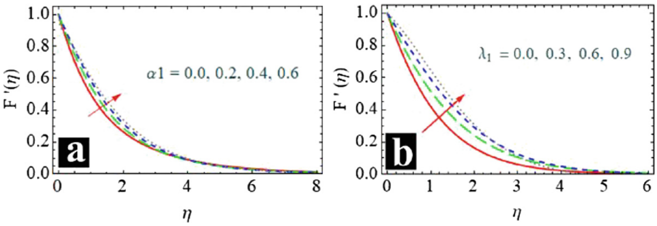

Fig. 2. Variation in $(F^{'}(\eta))$ for α_1 & λ_1.

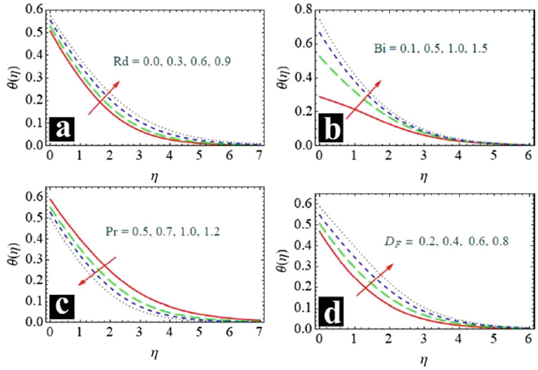

Fig. 3. Variation in $(\theta(\eta))$ for Rd, Bi, Pr and D_F.

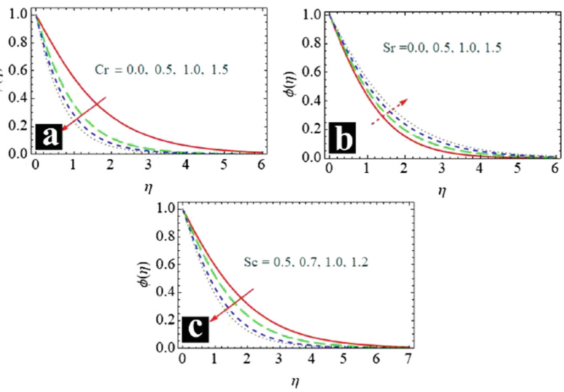

Fig. 4. Variation in $(\phi(\eta))$ for Cr, Sr and Sc.

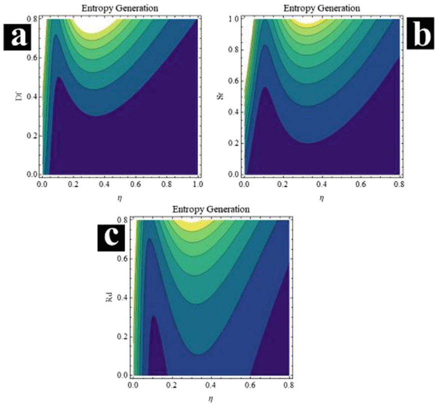

Fig. 5. Variation in (E_G) for D_F, Sr and Rd.

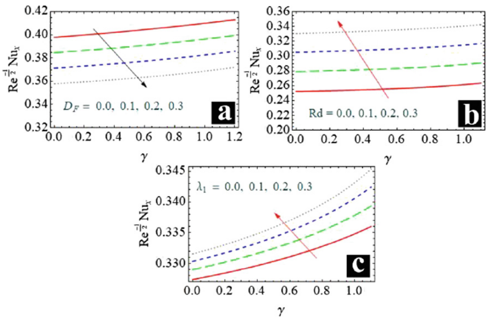

Fig. 6. Effect of Nu_x for values of D_F, Rd and λ_1.

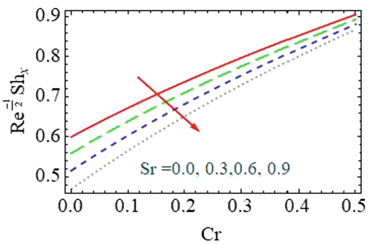

Fig. 7. Effect of Sh_x for values of Cr and Sr.

6 Results and Discussion

In this section, we examine the impacts of physical quantities on temperature($\theta(\eta)$), velocity ($F^{'}(\eta)$), entropy(E_G) and concentration profiles($\Phi(\eta)$) with the fixed values $Pr = 0.9$, $Sc = 0.9$, $Re = 0.14$, $\alpha_1 = 0.1$, $\alpha_2 = 0.1$, $\beta = 0.1$, $\gamma = 0.1$, $D_F = 0.5$, $Sr = 0.3$, $Bi = 0.5$, $Rd = 0.3$, $\lambda_1 = 0.2$, $N = 0.1$,$Q_H = -0.3$, $h = -0.7$ and $Cr = 0.1$.

Figure 1 describes the effect of velocity profile ($F^{'}(\eta)$) on material fluid parameter (α_1) and mixed convection parameter (λ_1). From Fig. 1(a & b), we have seen an increase in α_1 and λ_1 the velocity profile ($F^{'}(\eta)$) rises. Figure 2 revels the temperature profile ($\theta(\eta)$) for different parameters. In Fig. 2(a, b, d), we note that, the ($\theta(\eta)$) enhances for the augmentation in Rd, Bi and D_F and it diminishes for the higher values of Prandtl number Pr as shown in Fig. 2c. Figure 3 depicts the different effects of Cr, Sr, and Sc on concentration profile. From Fig. 3(a) and Fig. 3(c) concentration profile ($\Phi(\eta)$) is inversely proportional to the higher Cr and Sc. Whereas in Fig. 3(b), it is found that ($\Phi(\eta)$) increases with augments in Sr. Figures 4(a–c) shows the effects of D_F, Sr and Rd on E_G (entropy generation profile). From these plots we obtain that the system of entropy enhances for the larger values of D_F, Sr and Rd.

From Fig. 5(a) we note that the Nusselt number (Nu_x) decreases with increases in D_F. Also, in Fig. 5(b) and Fig. 5(c), it is noted that Nu_x decrease with upsurge in Rd and γ. By increasing the D_F, fluid resits to move the hotter side of the sheet that subsequently Nu_x decreases. In addition, it is noted that as we increase the Rd and γ the Nu_x enhances. Figure 6 exposes the mass transfer rate for the combined parameters Sr and Cr. We noted that decreasing trend in mass transfer for larger Sr and mass transfer rate enhance for Cr.

7 Conclusion

The salient outcomes of 3^{rd}-grade fluid flow with Soret and Dufour effects along with entropy calculation is stated as follows:

1 Higher range of mixed convection parameter(λ_1) and fluid parameter(α_1) intensifying the velocity profile.
2 Entropy of the system enhances with radiation, Sored and Dufour numbers.
3 Mass transfer rate rises with chemical reaction and reduces with Soret number.

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