Bacterial Foraging Optimization Based on Levy Flight for Fuzzy Portfolio Optimization

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Abstract. In this paper, a new kind of bacterial foraging optimization that combines with levy flight (LBFO) is employed to solve a novel portfolio optimization (PO) problem with fuzzy variables and modified mean-semivariance model which includes the transaction fee (including the purchase fee and sell fee), no short sales and the original proportion of the different assets. First of all, a chemotaxis step size using levy distribution takes the place of fixed chemotaxis step size, which makes a good balance between local search and global search through frequent short-distance search and occasional long-distance search. Moreover, fuzzy variables are used to signify the uncertainty of future risks and returns on assets and some constrained conditions are taken into consideration. The results of the simulation show that the model can be solved more reasonably and effectively by LBFO algorithm than the original bacterial foraging optimization (BFO).

Keywords: Bacterial foraging optimization · Levy flight · Fuzzy portfolio optimization

1 Introduction

Portfolio optimization (PO) is a process to decide how to allocate wealth and which assets to invest can gain the maximum returns under the acceptable risk [1]. Markowitz proposed the mean-variance (M-V) model [2] that uses the variance of stock prices to measure the portfolio risk. With the defects of the mean-variance (M-V) model [2], alternative risk measures have been proposed, such as value-at-risk (VaR) [3], sparse and robust mean-variance [4], and vine copula liquidity-adjusted VaR (LVaR) optimization model [5]. To overcome NP-hard problem, swarm intelligence (SI) methodologies are applied to deal with PO, such as particle swarm optimization (PSO) [6], bacterial foraging optimization (BFO) [7], moth search algorithm (MSA) [8], etc.

BFO [9] is a good technique to solve optimization problems. It has been succeeded in solving many real-world problems including flexible job-shop scheduling problem [10], feature selection [11], and supply chain optimization problem [12]. According to its continuous nature and several successful applications in the continuous optimization domain, BFO is considered as a prospective algorithm in solving the PO problem.

Levy flight which simulates the food searching path of numerous animals like deer, bumblebees, and albatross are added to SI algorithms to promote the performance of the algorithms such as PSO [13], cuckoo search algorithm (CSA) [14] and firefly algorithm (FA) [15].

Fuzzy set theory [16] can deal with decision-making problems since it allows us to describe and treat imprecise and uncertain elements. A growing number of researches show that it is hard to predict the future performance of assets exclusively based on historical data. So fuzzy logic [16] is applied to express uncertain knowledge that is suitable for representing the intrinsic uncertainty of the portfolio optimization problem. In a typical project, the return distributions of assets are usually skewed, which means a low possibility of high profits and a high possibility of low profits [24]. Substituting variance with semivariance can avoid sacrificing too much anticipated return in eliminating both high and low return extremes. In our approach, the uncertainty on returns and semivariance is set to trapezoidal fuzzy numbers.

Based on the analysis of the recent literature, the problem about how to improve the ability of original BFO to achieve high-quality solutions to more complicated PO problem attracts our attention. In this paper, we propose a new bacterial foraging optimization based on levy flight (LBFO) to solve a more complicated PO problem. The main contributions of this paper are listed below. First of all, levy distribution replaces the fixed chemotaxis step size to promote the optimization ability and efficiency of BFO through frequent short-distance exploration and occasional long-distance exploration. Next, transaction fee, no short sales and the original proportion of the different assets are used to modify mean-semivariance model. Finally, fuzzy variables are used to simulate the uncertain economic environment.

The rest of the paper is organized as follows. Section 2 is the introduction of some definitions. Section 3 gives a review of BFO and a description of LBFO. Section 4 describes the improved portfolio selection model and the detailed design of encoding and constrained boundary control. In Sect. 5, experimental settings and results are given. In the end, Sect. 6 concludes the paper and presents some future directions.

2 Preliminaries

In this section, we introduce some definitions that are needed in the following section. The assistance of a membership function $\mu_A(X) \rightarrow [0, 1]$ performs the degree of membership of the various elements, which means all elements in the interval between 0 and 1 are mapped.

Definition 1 [17]. A fuzzy number A is called trapezoidal with core interval $[a, b]$, left-width $\alpha > 0$ and right-width $\beta > 0$ if its membership function has the following form:

$$\mu_A(X) = \begin{cases} 1 - \frac{a-X}{\alpha}, & \text{if } a - \alpha \leq X \leq a \\ 1, & \text{if } a \leq X \leq b \\ 1 - \frac{X-b}{\beta}, & \text{if } b \leq X \leq b + \beta \\ 0, & \text{if otherwise} \end{cases} \quad (1)$$

and it is usually denoted by the notation $A = (a, b, \alpha, \beta)$. The family of fuzzy numbers is denoted by F . A fuzzy number A with γ -level set is expressed as $[A]^\gamma = [\alpha_1(\gamma), \alpha_2(\gamma)]$ for all $\gamma > 0$.

Definition 2 [18]. The above trapezoidal fuzzy number A is denoted by the notation

$A = (a, b, \alpha, \beta)$ with core interval $[a, b]$, left-width α and right-width β and the γ -level set of A can be computed as:

$$[A]^\gamma = [a - (1 - \gamma)\alpha, b + (1 - \gamma)\beta] (\gamma \in [0, 1]) \quad (2)$$

The possibilistic mean value of A is given by the following:

$$E(A) = \int_0^1 \gamma [\alpha_1(\gamma) + \alpha_2(\gamma)] d\gamma = \frac{a + b}{2} + \frac{\beta - \alpha}{6} \quad (3)$$

The upper and lower possibilistic semivariance of A is given respectively by the following:

$$Var^+(A) = \int_0^1 \gamma [E(A) - \alpha_2(\gamma)]^2 d\gamma = \left(\frac{b - a}{2} + \frac{\beta + \alpha}{6} \right)^2 + \frac{\beta^2}{18} \quad (4)$$

$$Var^-(A) = \int_0^1 \gamma [E(A) - \alpha_1(\gamma)]^2 d\gamma = \left(\frac{b - a}{2} + \frac{\beta + \alpha}{6} \right)^2 + \frac{\alpha^2}{18} \quad (5)$$

Assuming two fuzzy numbers A and B , with $[A]^\gamma = [\alpha_1(\gamma), \alpha_2(\gamma)]$ and $[B]^\gamma = [b_1(\gamma), b_2(\gamma)]$.

For all $\gamma \in [0, 1]$, the upper and lower possibilistic covariance of A and B are given respectively:

$$Cov^+(A, B) = 2 \int_0^1 \gamma [E(A) - \alpha_2(\gamma)] \cdot [E(B) - b_2(\gamma)] d\gamma \quad (6)$$

$$Cov^-(A, B) = 2 \int_0^1 \gamma [E(A) - \alpha_1(\gamma)] \cdot [E(B) - b_1(\gamma)] d\gamma \quad (7)$$

Lemma 1 [18]. Assuming that A and B are two fuzzy numbers and let μ and λ be positive numbers. Then, the following conclusions can be drawn by using the extension principle:

$$E(\lambda A \pm \mu B) = \lambda E(A) \pm \mu E(B) \quad (8)$$

Theorem 1 [19]. Assuming that $A_1, A_2, \dots, A_n$ are fuzzy numbers, and $\lambda_1, \lambda_2, \dots, \lambda_n$ are real numbers. Then, the following conclusions can be drawn by using the extension principle:

$$Var^+\left(\sum_{i=1}^n \lambda_i A_i\right) = \sum_{i=1}^n \lambda_i^2 Var^+(A_i) + 2 \sum_{i < j=1}^n \lambda_i \lambda_j Cov^+(A_i, A_j) \quad (9)$$

$$Var^-\left(\sum_{i=1}^n \lambda_i A_i\right) = \sum_{i=1}^n \lambda_i^2 Var^-(A_i) + 2 \sum_{i < j=1}^n \lambda_i \lambda_j Cov^-(A_i, A_j) \quad (10)$$

3 BFO and LBFO

3.1 Bacterial Foraging Optimization

BFO, a swarm intelligence algorithm, is based on the biology and physics underlying the foraging behavior of *Escherichia coli*. Four motile behaviors (chemotaxis, swarming, reproduction, and elimination and dispersal) are included in the bacterial foraging process. For more detailed information, please refer to [9].

3.2 Bacterial Foraging Optimization Based on Levy Flight

This part mainly explains the application of the random walk model of the levy flight strategy in algorithm improvement.

3.2.1 Levy Flight in Chemotaxis Step

The levy flight is a statistical description of motion. It is a kind of non-Gaussian random processes whose step length is drawn from levy stable distribution. This special walk strategy of levy flight is subject to the power-law distribution. There is an exponential relationship between the variance of levy flight and time.

$$Levy(s) \sim |s|^{-\lambda}, 1 < \lambda \leq 3 \quad (11)$$

where s is the random step size of levy flight and λ is a real number.

In an algorithm proposed by Mantegna [20], the method of generating levy random step size is represented by:

$$S = \frac{u}{|v|^{1/\beta}} \quad (12)$$

where β is a real number. u and v are drawn from normal distributions. That is

$$u \sim N(0, \sigma_u^2), v \sim N(0, \sigma_v^2) \quad (13)$$

$$\sigma_u = \left\{ \frac{r(1 + \beta) \sin(\pi\beta/2)}{r[(1 + \beta)/2]\beta 2^{(\beta-1)/2}} \right\}^{1/\beta}, \sigma_v^2 = 1, \beta \in [0.3, 1.99]$$

The improved chemotaxis step size formula is given as follows:

$$C(i) = scale * Levy(s) \quad (14)$$

$$scale = \frac{1}{10p} \sum_{d=1}^p \left| \theta_d^i(j, k, l) - \theta_d^*(j, k, l) \right| \quad (15)$$

In the formula, the *scale* is the scale factor, *Levy* (s) is the levy step size as the parameter, the D -dimensional paratactic value of the position of the i th bacteria is set to $\theta_d^i(j, k, l)$, and the D -dimensional paratactic value of the current optimal bacterial position is set to $\theta_d^*(j, k, l)$. p is the dimension of optimization. In order to make the levy chemotaxis step reach the opportune order of magnitude, the *scale* utilizes the positional relationship between the optimal bacteria and the current bacteria.

4 Bacterial Foraging Optimization for Portfolio Selection Problems

4.1 Portfolio Selection Model

In this section, a model that considers the transaction fee and no short sales in portfolio optimization presented in [21] is introduced. The model can be described as following and the parameters and definitions of PO model are shown in Table 1.

Table 1. Parameters and definitions of PO model

Variables	Definitions
$f(X)$	The function that represents profit and pursuits the maximum value
$g(X)$	The function that represents risk and pursuits the minimum value
d	The number of assets
r_i	The expected yields of asset i
X_i	The proportion of investment of asset i
X_i^0	The initial holding proportion of investment of asset i
μ	$\mu = 1$: buy assets from market $\mu = 0$: sell assets to market
k_i^b	The transaction cost of buying asset i from market
k_i^s	The transaction cost of selling asset i to market
σ_{ij}	The covariance of r_i and r_j

$$\begin{aligned} \text{Min} F(X) &= \min[\varphi g(X) - (1 - \varphi)f(X)] \\ \text{s.t. } &\begin{cases} \sum_{i=1}^n X_i = 1 \\ X_i > 0 \end{cases} \end{aligned} \quad (16)$$

$$\begin{aligned} f(X) &= \sum_{i=1}^d r_i x_i - \sum_{i=1}^n [\mu \cdot k_i^b \cdot (x_i - x_i^0) + (1 - \mu) \cdot k_i^s \cdot (x_i^0 - x_i)] \\ \text{and } \mu &= \begin{cases} 1 & X_i \geq X_i^0 \\ 0 & X_i < X_i^0 \end{cases} \end{aligned} \quad (17)$$

$$g(X) = \sum_{i=1}^d \sum_{j=1}^d X_i X_j \sigma_{ij} \quad (18)$$

where $\varphi \in [0, 1]$, representing risk aversion factors. When φ becomes smaller, investors are willing to take more risks. The bigger the value of φ , the less risks will take.

4.2 Encoding

When LBFO is used to find the solutions of PO model, each particle is regarded as a potentially feasible solution. Three types of information, including the proportion of

investment, the value of corresponding profit and the value of corresponding risk, are carried by each bacterium [22].

$$\theta = [X_1, X_2, X_3 \dots X_d, f(X), g(X)] \quad (19)$$

$$s = \sum_{i=1}^d X_i$$

$$\theta' = \left[\frac{1}{s} (X_1, X_2, X_3 \dots X_d), f(X)', g(X)' \right] \quad (20)$$

Equation (19) illustrates the coding of the bacteria. As is shown in Eq. (20), the proportions of all assets are summed up first and then every proportion of asset is divided by the s so that the sum of all asset proportions is equal to 1.

4.3 Constrained Boundary Control

$$\theta^i(j+1, k, l) = \begin{cases} D_{\min} + \text{rand}(D_{\max} - D_{\min}), & \text{if } P < 0 \\ \theta^i(j, k, l) + C(i) \frac{\Delta(i)}{\sqrt{\Delta^T(i) \Delta(i)}}, & \text{if } P \geq 0 \end{cases} \quad (21)$$

Boundary control guarantees that each proportion is a positive number. Bacteria change direction randomly in the process of chemotaxis, which may lead them to exceed the given area. If bacteria are permitted to leave the prescribed area, the border solutions may be infeasible. Otherwise, the border solutions may not be obtained. The boundary control rules are illustrated in Eq. (12, $D_{\min}$ and $D_{\max}$ are the lower and upper boundaries, separately. And the function $\text{rand}(\cdot)$ is used to generate a random number between 0 and 1.

5 Experiments and Discussions

In order to verify the performance of LBFO algorithm, the original BFO and PSO are selected for comparison. Both the algorithms shown in this paper were coded in MATLAB language.

5.1 Definition of Experiments

In the experiments, we assume that there are five assets to invest and the original proportion in every asset X_i^0 equals 0.2. Besides, the sum of the initialized ratio of each asset equals 1. Since we suppose that the future returns of the assets are trapezoidal fuzzy numbers, the left and right width and the core interval of the fuzzy numbers are needed to estimate. The estimation method proposed by Vercher *et al.* [23] is used to calculate the trapezoidal fuzzy return rates of the proposed model. According to Vercher *et al.*, the historical returns are regarded as samples and the core and spreads of the trapezoidal fuzzy returns on the assets are approximated by sample percentiles. So the core of the fuzzy return r is set as the interval [40th, 60th], left-width (40th, 5th) and right-width

Table 2. The probability distribution of returns

The asset i	1	2	3	4	5
α	-0.32266	-0.18761	-0.29708	-0.0477	-0.14339
a	0.266462	0.303832	0.137969	0.176914	0.122531
b	0.687169	0.519906	0.37341	0.380465	0.375195
β	1.432592	1.107447	1.816847	0.926154	0.918654

(60th, 95th]. We consider the yearly returns over the examined period between January 2010 and December 2019. The probability distribution of returns is shown in Table 2.

In this paper, we assume that investors are risk-averse. Thus, the lower possibility semivariance is used to describe the risk. According to Theorem 1 and Definition 2, the risk of investment is shown as follows:

$$\begin{aligned} Var^-\left(\sum_{i=1}^d x_i r_i\right) &= \sum_{i=1}^d x_i^2 Var^-(r_i) + 2 \sum_{i < j=1}^d x_i x_j Cov^-(r_i, r_j) \\ &= \left[\sum_{i=1}^d x_i \left(\frac{a_i - b_i}{2} + \frac{\alpha_i + \beta_i}{6} \right) \right]^2 + \frac{1}{18} \left(\sum_{i=1}^d x_i \alpha_i \right)^2 + 2 \sum_{i < j=1}^d x_i x_j Cov^-(r_i, r_j) \end{aligned} \quad (22)$$

$$\begin{aligned} Cov^-(r_i, r_j) &= \left(\frac{b_i - a_i}{2} + \frac{\beta_i - \alpha_i}{6} \right) \left(\frac{b_j - a_j}{2} + \frac{\beta_j - \alpha_j}{6} \right) + \frac{\alpha_j}{3} \left(\frac{b_i - a_i}{2} + \frac{\beta_i - \alpha_i}{6} \right) \\ &\quad + \frac{\alpha_i}{3} \left(\frac{b_j - a_j}{2} + \frac{\beta_j - \alpha_j}{6} \right) + \frac{\alpha_i \alpha_j}{6} \end{aligned} \quad (23)$$

According to Eq. (3) and Eq. (14), the asset's probability means of return that is regarded as the expected yields of assets and the covariance of each asset's return is as follows:

$$r = [0.769357567, 0.627711275, 0.608011358, 0.440998792, 0.425870775]$$

$$\begin{aligned} \sigma &= [0.162079, 0.106683, 0.152003, 0.098973, 0.103597; \\ &\quad 0.106683, 0.070256, 0.100059, 0.065359, 0.068279; \\ &\quad 0.152003, 0.100059, 0.142555, 0.092867, 0.097176; \\ &\quad 0.098973, 0.065359, 0.092867, 0.061722, 0.063802; \\ &\quad 0.103597, 0.068279, 0.097176, 0.063802, 0.066444] \end{aligned}$$

Other parameter settings are the same as reference [1]. Three risk aversion factors that equal to 0.15, 0.5 and 0.85 respectively are used to identify three different kinds of investors. The transaction cost of buying asset $k_i^b = 0.00065$ and the transaction cost of selling asset $k_i^s = 0.00075$. In BFO, the number of chemotactic $N_c = 1000$. The number of elimination-dispersal $N_{ed} = 2$. The number of reproduction $N_{re} = 5$. The number of swimming $N_s = 4$. The elimination-dispersal frequency $P_{ed} = 0.25$. The swimming length $C = 0.2$. In PSO, inertia weight $w = 1$ and $c_1 = c_2 = 2$.

5.2 Experimental Results

Experimental results obtained by three algorithms of different λ and the final portfolio selection results are showed in Tables 3, 4 and 5. Figure 1 shows the convergence curves with various λ generated by BFO, LBFO and PSO.

Table 3. Experimental results of $\varphi = 0.15$

	BFO	LBFO	PSO
Min	-4.5740	-4.8613	-3.2783E-01
Max	0	0	-2.7206E-01
Mean	-4.5740E-01	-4.8613E-01	-2.9714E-01
Std.	1.4464	1.5373	2.8308E-02
X ₁	3.4661E-01	1.5205E-01	9.2883E-01
X ₂	2.7350E-01	2.1958E-01	3.1142E-13
X ₃	2.0433E-01	1.7316E-01	7.1166E-02
X ₄	9.4721E-02	1.9758E-01	6.7232E-17
X ₅	8.0839E-02	2.5764E-01	1.8993E-18
Return	6.3847E-01	5.5684E-01	7.5686E-01
Risk	1.1110E-01	8.9945E-02	1.0211E-01

Table 4. Experimental results of $\varphi = 0.5$

	BFO	LBFO	PSO
Min	-5.2977E-01	-5.7620E-01	-5.0209E-02
Max	0	0	-3.1582E-02
Mean	-5.2977E-02	-5.7620E-02	-3.9313E-02
Std.	1.6753E-01	1.8221E-01	9.7086E-03
X ₁	1.5109E-01	1.1588E-01	9.6977E-01
X ₂	2.4915E-01	2.2208E-01	4.5337E-21
X ₃	2.0202E-01	3.2296E-01	3.0691E-01
X ₄	3.1492E-01	2.5108E-01	1.9870E-05
X ₅	8.2821E-02	8.7998E-02	5.3115E-14
Return	5.6938E-01	5.7285E-01	7.1899E-01
Risk	9.1322E-02	9.7371E-02	1.1718E-01

Table 5. Experimental results of $\varphi = 0.85$

	BFO	LBFO	PSO
Min	-2.7749E-02	-2.8004E-02	-3.7393E-02
Max	0	0	7.1282E-02
Mean	-2.7749E-03	- 2.8004E-03	-1.1155E-03
Std.	8.7750E-03	8.8557E-03	6.2698E-02
X ₁	7.7191E-02	2.1459E-01	4.8588E-11
X ₂	3.8539E-01	2.0830E-01	9.8150E-10
X ₃	1.3545E-01	1.8445E-01	4.9773E-01
X ₄	2.9850E-01	1.9818E-01	5.9825E-17
X ₅	1.0347E-01	1.9448E-01	5.0227E-01
Return	5.5896E-01	5.7819E-01	7.2298E-02
Risk	8.1073E-02	9.6231E-02	6.9586E-02

Fig. 1. The convergence curve of three algorithms

According to the tables and the figures, we can find that:

- (1) The fitness values grow up with the increase of the risk aversion factor φ . The results illustrate that high returns involve increased risk.
- (2) With the different φ , the proportion of the five assets is different. Different investors have different degrees of risk aversion, so they will make different investment decisions when facing the same assets and market conditions.
- (3) By analyzing the tables and the convergence graphs, it is obvious that LBFO usually produces the best performance among these three algorithms and BFO is better than PSO. Detailed differences are listed as follows:
 - a) On account of random search directions and fixed chemotaxis step size, the original BFO algorithm has a restrained global search capability and a weak convergence performance. Comparing to BFO, LBFO algorithm utilizes levy flight to balance global search and local search. The results illustrate that LBFO has higher convergence rate and solution accuracy.
 - b) Due to the defects of algorithm performance, PSO algorithm shows worse results than BFO in this model. PSO algorithm is caught in local optimum through successive iterations and slow convergence rate appearing, which limits the accuracy of the algorithm. On the contrary, BFO algorithm has a more effective search capability, so it is easy to jump out of local optimum and find better results.
 - c) It can be seen that LBFO can always find the best solution from the data of the mean value.

6 Conclusions and Future Work

In this paper, we employ original BFO and LBFO algorithms to solve PO problem with different risk aversion factors. The obtained results indicate that LBFO algorithm is a better choice to solve the difficult PO problem due to the levy flight strategy, whose characteristic is the frequent short-distance exploration and occasional long-distance exploration is utilized to adjust the chemotaxis step size.

Further work may consider more useful conditions such as inflation, fundamental analysis and focus on multi-period or dynamic objectives to satisfy demands from the real market to propose a new PO model.

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