Chapter 8

Beyond the Standard Model

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8.1 Introduction

Starting sometime in 2008/2009 one expects to be able to take a glimpse at physics at the TeV scale. This will be done through the Large Hadronic Collider (LHC) at CERN, Geneva. It will be a result of an unprecedented coordinated international scientific effort. This chapter is written in 2007. It is essentially inviting disaster to spell out in full detail what the current various theoretical speculations on the physics are, as well motivated as they may seem at this time. What I find of more value is to elaborate on some of the ideas and the motivations behind them. Some may stay with us, some may evolve and some may be discarded as the results of the experiments unfold. When the proton antiproton collider was turned on in the early eighties of the last century at Cern the theoretical ideas were ready to face the experimental results in confidence, a confidence which actually had prevailed. The emphasis was on the tremendous experimental challenges that needed to be overcome in both the production and the detection of the new particles. As far as theory was concerned this was about the physics of the standard model and not about the physics beyond it. The latter part was left safely unchallenged. That situation started changing when the large electron positron (LEP) collider experiments also at Cern were turned on as well the experiments at the Tevatron at Fermilab. Today it is with rather little, scientifically based, theoretical confidence that one is anticipating the outcome of the experiments. It is less the method and foundations that are tested and more the prejudices. It is these which are at the center of this chapter. Some claim to detect over the years an oscillatory behavior in the amount of conservatism expressed by leaders in physics. The generation in whose life time relativity and

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quantum mechanics were discovered remained non-conservative throughout their life. Some of the latter developed eventually such adventurous ideas as to form as a reaction a much more conservative following generation. The conservative generation perfected the inherited tools and has uncovered and constructed the Standard Model. They themselves were followed by a less conservative generation. The new generation was presented with a seemingly complete description of the known forces. In order to go outside the severe constraints of the Standard Model the new generation has drawn upon some of the more adventurous ideas of the older generation as well as created its own ideas. In a way almost all accepted notions were challenged. In the past such an attitude has led to major discoveries such as relativity and quantum mechanics. In some cases it was carried too far, the discovery of the neutrino was initially missed as energy conservation was temporarily given up.

The standard model is overall a very significant scientific achievement. It is a rather concise framework encompassing all the known properties of the known basic interactions. It is arguably the most impressive theoretical understanding of a large body of experimental information existing. An understanding backed by precise predictions all verified by high quality experiments. In this context it may seem surprising that one is searching for anything beyond the Standard Model. There are however diverse scientific reasons for the search of the beyond.

In 2007 the scientific community was aware of quite a few gaps in the understanding of the particle interactions. One class of observations posed obvious pressing problems:

- There is a large body of evidence that the so called dark matter should be composed mostly of different particle(s) than those that serve as the building blocks of the standard model. What are they? More recently also what is called a dark energy was needed to explain the data. Its possible origin(s) is under active study.
- A standard model for cosmology is forming and it includes in most cases versions of inflation. Such models seem to require the existence of a new heavy particle(s). What are they? On a more speculative note, models require more detailed understanding of how physical systems behave in big bang/crunch like circumstances and of how and if universes may form.

Another set of observations could be either defined as posing problems requiring an explanation or as pointing to new directions only after being combined with a certain amount of theoretical prejudice (*TP*). In the past some major advances were driven by such combinations.

- Three known interactions, the colored, the weak and the electromagnetic interactions all obey the dictum of quantum mechanics and are all well described by gauge theories but have otherwise very different properties and strengths at the energy scales probed till 2007. A *TP*, a strong and deeply ingrained one, suggests that all interactions, those known and those yet to be discovered should unify at a certain higher energy scale. This is the idea of a Grand Unified Theories (GUTS). Eventually it was found that to realize a particular aspect of this, the

convergence of the all the couplings to a single value at an appropriate energy scale, there should be physics beyond the standard model. In Super Symmetric (SUSY) systems this indeed may occur. In fact the unification is achieved at a distance scale rather close to the Planck scale, the scale at which gravity becomes strong and its quantum effects, if there, should become noted. Thus the fourth force Gravity is naturally added to the unification scheme. An older realization of this *TP* was that all known interactions are but the low energy descriptions of a system containing only the gravitational force and residing in a higher number of space-time dimensions. This had led to Kaluza Klein theories and their variants. This idea has been revisited with the advent of SUSY and string theory.

- The observation of what is called dark energy and its very possible confirmation of a very small, but not vanishing, cosmological constant is viewed by many as a major problem lacking an explanation. The *TP* behind this is that any fundamental quantitative property of nature should be explained and not fine tuned. The absence of any significant amount of *CP* violation in the strong interactions is another such problem. The discovery of a new particle, the Axion, could be a signal of the solution of the latter one. The discovery of another particle, the Dilaton, could indicate a resolution of the former. This issue has led also to the reexamination of the so called entropic principle.
- Many predictions of Classical General Relativity were confirmed experimentally over the years. The *TP* in 2007 is that there should be a quantum theory of gravity. Such a four dimensional quantum theory of gravity is not well defined within only field theory. This has been a driving force in the study of the properties of a theory where the basic constituents of nature are not particles but extended objects, including strings.
- A very successful framework to explain the basic interactions in particle physics is the so called Wilsonian one. It is very powerful when the laws of physics are such that different largely separated scales are essentially decoupled from each other. The physics beneath any energy scale (cutoff) is well described by operators whose scaling dimensions, in d spacetime dimensions, is not much larger than the same d . In many cases there is only a finite number of such operators, i.e. only a finite amount of terms in the Lagrangians describing the Physics below the energy scale. This *TP* has been tested successfully time and again but may eventually be falsified, perhaps in a theory of gravity. But given the validity of this method physical quantities should be only slightly dependent on the cutoff scale, and thus on the unknown physics extending beyond it. Generically the mass of scalar particles is strongly dependent on the cutoff. In particular instead of the scale of such masses being set by the weak interactions scale they will depend strongly on the cutoff. The only known, in 2007, physical scale beyond that of the weak interactions is of the order of the Planck scale. The discovery of Higgs particles whose masses is in the TeV range or lighter will thus require a large amount of fine tuning. The *TP* does not accept that. This is one manifestation of the so called hierarchy problem. This had led to ideas such as technicolor and supersymmetry as properties of beyond the standard model.

- It seems very likely in 2007 that eventually the neutrino masses will be added without caveats to the pantheon of particles, the particle data tables. Some *TPs* point to new physics at a rather high mass scale as the origin of small neutrino masses.

It is interesting that very few of the theoretical ideas and visions used to address the ample set of problems mentioned above were originally created directly for that purpose. They were more tools whose applications were found only well after they were formed.

The theoretical structures discovered and created where an outcome of an urge to question, generalize and unify almost anything.

- In addition to the attempts to unify all symmetries and considering extra dimensions it was suggested that the topology of the extra dimensions may determine the low mass particle spectrum. These extra dimensions were assumed for year to be small but it turned out they could also be large, leading to theories in which large extra dimensions play a key role. A variant of this idea is that there exists a hidden sector where many desired things occur, SUSY is broken, the cosmological constant is (nearly) cancelled to name some. These effects are then communicated to the Standard Model particles by messengers. In many cases the weak gravitational force is designated that role.
- A natural direction of generalization is to question the point particle nature of the basic constituents of nature. Is there a consistent theory of elementary higher dimensional objects such as strings or membranes, this direction of research is developed under the name of string theory. It has led to much more satisfactory theory of gravity. It has success, faces many challenges not least of which is the present lack of experimental evidence and has a touch of magic. This magic has had already a significant impact on Mathematical Physics.
- The never satiated desire for simplicity may suggest to remove even the concept of a point particle which propagates in space time and for that matter to remove even the concept of space time. The idea is that space time is but an emergent, long distance, phenomena. The search for “true” underlying picture is still on.
- The quest for knowledge includes a succession of Copernican like revolutions. After each such step the researcher finds herself even further removed from the Center. A possible prospect for such a revolution is that our universe is but one of many universes. The idea emerges in quite a few contexts not necessarily unrelated. These include the many world interpretation of quantum mechanics, third quantization in quantum gravity, the multiverse in models for inflation and in string theory. In string theory the idea has been refined in the brane world picture, our universe reflects an underlying structure in which different particles reside on different multi dimensional subspaces.
- In the process of quantization it was assumed that the space coordinates commute with each other as do their conjugate momenta. Mathematically one may construct noncommutative manifolds, manifolds whose coordinates do not commute. This had led to the study of physics on such manifolds. This an the

idea that the underlying theory of nature depends only on topology have both been extensively studied but at this stage still more in the realm of mathematics.

The various theoretical ideas mentioned above were extensively studied in the second part of the twentieth century. The respective developments are documented in many books covering thousands of pages. In this review we have made the following choices. The emphasis is on various aspects of SUSY. There has been significant progress in understanding the dynamics of systems which have in some form or another supersymmetric features. These systems have been researched in the weak coupling and the strong coupling regimes, perturbatively and non-perturbatively. The properties uncovered are remarkable in some cases. We also review, more briefly, the attempts to lay out a framework suitable for extracting supersymmetric signatures of nature. This rich area of research will better receive its due rewards after the accumulation of actual experimental data. The more “senior” areas of research such as examining the possibility of the existence of extra dimensions, grand unified theories and superstring theory will be reviewed in a much more descriptive manner. This choice was influenced by the possibility that the LHC will shed some concrete light on what is beyond the standard model.

8.2 Super Symmetry [1]

Super Symmetry (SUSY) embodies several forms of unification and generalization. It joins space time and internal symmetries and it generalizes the meaning of space time by adding fermionic components to the canonical space time bosonic coordinates. An original stated motivation for SUSY in field theory was to have a symmetry which was able to relate the self couplings of bosons, the self coupling of fermions and the Yukawa couplings of fermions to bosons. Eventually SUSY while indeed unifying such couplings has given rise to a multitude of possible non-equivalent ground states. Such a degeneracy of ground states was of a magnitude unknown before. In string theory, SUSY stabilized superstring theories by removing the tachyons which plagued the bosonic string theories. Over the years the motivations have varied and evolved. SUSY was called upon to ameliorate the so called hierarchy problem which will be reviewed below and it was perceived as an omen that in a SUSY theory one was able to arrange that the colored, weak and electromagnetic couplings unify at a single high energy scale. That scale not far from the Planck scale. In addition it was discovered that this enriched structure allows to obtain exact results in situations where only approximations were available before. In particular for four and higher dimensional systems. In its presence a very rich dynamics has been unraveled.

8.2.1 Elementary Particles in SUSY Models: Algebraic Structure

Many successes resulted from the application of symmetry principles to systems such as atoms, nuclei and elementary particles. They have been obtained even in the face of a rather incomplete understanding of the dynamics of these systems. Consequentially evermore higher symmetries were searched for, and in particular unifying ones. Such was the search for a symmetry that would contain in a non-trivial way both the Poincar'e space time symmetry and internal symmetries such as Isospin and flavor SU(3). It was shown that that was impossible, only a trivial product symmetry is allowed. Theorems are proved under assumptions, time and again new important directions emerge once significant loop holes in the assumptions are uncovered. Such was the case for obtaining consistently massive spin one particles and such was the case here. Allowing the algebra of the symmetry generators to be graded, i.e. to include both commutators and anti commutators a new structure containing both Poincar'e and internal symmetries was discovered.

A simple version of the SUSY algebra is given by the following anticommutation relations which obey the following commutation relations:

$$\{Q_\alpha, \overline{Q}_{\dot{\alpha}}\} = 2\sigma_{\alpha\dot{\beta}}^\mu P_\mu, \quad \{Q_\alpha, Q_\beta\} = \{\overline{Q}_{\dot{\alpha}}, \overline{Q}_{\dot{\beta}}\} = 0. \quad (8.1)$$

Where the Q_α are fermionic generators of supersymmetry. P_μ are the generators of space-time translations and the σ matrices are the Pauli matrices.

$$[P_\mu, Q_\alpha] = [P_\mu, \overline{Q}_{\dot{\alpha}}] = [P_\mu, P_\nu] = 0 \quad (8.2)$$

This is called the $N = 1$ supersymmetry algebra.

It can be generalized to include a higher number of supersymmetries. For example in four space-time dimensions there are also $N = 2$ and $N = 4$ supersymmetries:

$$\{Q_\alpha^i, \overline{Q}_{\dot{\alpha}}^j\} = 2\delta^{ij}\sigma_{\alpha\dot{\beta}}^\mu P_\mu + \delta_{\alpha\dot{\beta}} U_{ij} + (\gamma_5)_{\alpha\dot{\beta}} V_{ij}, \quad (8.3)$$

i and j run over the number of supersymmetries, U and V are the central charges i.e. they commute with all other charges, (they are antisymmetric in ij). When they do not vanish they are associated with what are called BPS states such as monopoles. The $d = 4$ realisations have as, $\mu, \nu = 0, 1, 2, 3$ the space-time indices. In four dimensions one has two component Weyl Fermions. Those with α or β indices transform under the $(0, \frac{1}{2})$ representation of the Lorentz group; and those with dotted indices, $\dot{\alpha}$ or $\dot{\beta}$ transform under the $(\frac{1}{2}, 0)$ representation.

The possible particle content of supersymmetric (SUSY) theories is determined by the SUSY algebra.

Consider first the massless representations of $N = 1$ supersymmetry.

The simplest is the called the chiral multiplet. It contains two real scalars and one Weyl Fermion:

$$\left(-\frac{1}{2}, 0, 0, \frac{1}{2}\right) (\varphi, \psi) (2, 2) \quad (8.4)$$

In the above table, first are written the helicities; then the associated component fields, φ denotes a complex scalar and ψ a Weyl Fermion; and finally are the number of physical degrees of freedom carried by the Bosons and Fermions. The massless multiplet containing a spin one boson and a spin one half Fermion is called the vector multiplet. Its content in the case of $N = 1$ is:

$$\left(-1, -\frac{1}{2}, \frac{1}{2}, 1\right) (\lambda_\alpha, A_\mu) (2, 2) \quad (8.5)$$

λ is a Weyl Fermion and A_μ is a vector field.

For $N = 2$ supersymmetry, there is a massless vector multiplet:

$$\left(-1, -\frac{1}{2}, 0, \frac{1}{2}, 1\right) (0) (\varphi, \psi) + (\lambda_\alpha, A_\mu) (4, 4) \quad (8.6)$$

and a massless hypermultiplet which is given by:

$$\left(\begin{array}{c} 0 \\ -\frac{1}{2}, 0, \frac{1}{2} \\ -\frac{1}{2}, 0, \frac{1}{2} \\ 0 \end{array}\right) (\varphi_1, \psi_1) + (\varphi_2, \psi_2) (4, 4). \quad (8.7)$$

For Massive multiplets, in $N = 1$, there is again the chiral multiplet which is the same as the massless multiplet but with now massive fields. The massive vector multiplet becomes:

$$\left(-1, -\frac{1}{2}, 0, \frac{1}{2}, 1\right) (h, \psi_\alpha, \lambda_\alpha, A_\mu) (4, 4) \quad (8.8)$$

Where h is a real scalar field. The massive vector multiplet has a different field content than the massless vector multiplet because a massive vector field has an additional physical degree of freedom. One sees that the massive vector multiplet is composed out of a massless chiral plus massless vector multiplet. This can occur dynamically; massive vector multiplets may appear by a supersymmetric analogue

of the Higgs mechanism. With $N = 4$ supersymmetry, the massless vector multiplet is:

$$\left(\begin{array}{cccc} 0 & & & \\ -\frac{1}{2} & 0 & \frac{1}{2} & \\ -1, & -\frac{1}{2}, & 0 & \frac{1}{2}, \\ & -\frac{1}{2}, & 0 & \frac{1}{2}, \\ & -\frac{1}{2} & 0 & \frac{1}{2} \\ & & 0 & \end{array} \right) (\lambda^a, \phi^I, A_\mu) \quad (8, 8) \quad (8.9)$$

where $I = 1..6, a = 1..4$.

These are the unitary representations of the Super Symmetry algebra whose particle content allows them to participate in renormalizable interactions. Any higher supersymmetry in four dimensions would have to involve non-renormalizable terms. Mostly for particles with spin higher than one.

8.2.2 Supersymmetric Lagrangians

The task of writing down explicit supersymmetric Lagrangians was quite laborious. Originally all the interaction terms had to be written down explicitly. In some cases this had turned out to be much simpler by the introduction what are called superfields. These will be described below and make use of the anticommuting Grassman variables suitable to describe fermions. This is called the Super Space notation. In the spirit of generalization, the mathematical book keeping device has been elevated by some to a generalization of regular space whose coordinates are denoted by commuting numbers to a superspace in which some variables are Grassman variables. This superspace has its own geometrical properties and it was suggested to give it also a life of its own. It is not clear yet how fundamental the superspace description is but adopting this notation leads to considerable simplification. We note here that another generalization of space has been suggested. In regular space the coordinates commute also quantum mechanically, it was suggested to explore the situation that space coordinates not commute in quantizing the theory. This has experimental consequences, as of now this suggestion has no experimental backing.

8.2.2.1 Superspace, Chiral Fields and Lagrangians for Spin Zero and One-half Particles

Returning to superspace, spacetime can be extended to include Grassmann spinor coordinates, $\bar{\theta}_{\dot{\alpha}}, \theta_{\alpha}$. Superfields are functions of the superspace coordinates. Constructing a Lagrangian out of special types of superfields provides a useful way

to construct explicitly supersymmetric Lagrangians. The integration formulas for Grassmann variables are:

$$\int d\theta_\alpha \theta_\alpha = \frac{\partial}{\partial \theta_\alpha} = 1, \quad \int d\theta_\alpha = 0 \quad (8.10)$$

Which results also in:

$$\int d^2\theta d^2\bar{\theta} \mathcal{L} = \int d^2\theta \frac{\partial^2 \mathcal{L}}{\partial \bar{\theta}_1 \bar{\theta}_2} \quad (8.11)$$

The supercharges can be realized in superspace by generators of supertranslations:

$$Q_\alpha = \frac{\partial}{\partial \theta_\alpha} - i\sigma_{\alpha\dot{\alpha}}^\mu \bar{\theta}^{\dot{\alpha}} \partial_\mu, \quad \bar{Q}_{\dot{\alpha}} = -\frac{\partial}{\partial \bar{\theta}_{\dot{\alpha}}} + i\theta^\alpha \sigma_{\alpha\dot{\alpha}}^\mu \partial_\mu. \quad (8.12)$$

To define the concept of (anti) chiral fields one defines a supercovariant derivative:

$$D_\alpha = \frac{\partial}{\partial \theta_\alpha} + i\sigma_{\alpha\dot{\alpha}}^\mu \bar{\theta}^{\dot{\alpha}} \partial_\mu, \quad \bar{D}_{\dot{\alpha}} = -\frac{\partial}{\partial \bar{\theta}_{\dot{\alpha}}} - i\theta^\alpha \sigma_{\alpha\dot{\alpha}}^\mu \partial_\mu. \quad (8.13)$$

A superfield Φ is called “chiral” if:

$$\bar{D}_{\dot{\alpha}} \Phi = 0. \quad (8.14)$$

Anti-chiral fields are defined by reversing the role of the θ and $\bar{\theta}$. One introduces the variable,

$$y^\mu = x^\mu + i\theta\sigma^\mu\bar{\theta} \quad (8.15)$$

in terms of which the expansion of a chiral field is,

$$\Phi(y) = A(y) + \sqrt{2}\theta\psi(y) + \theta\theta F(y) \quad (8.16)$$

The Taylor expansion terminates after just a few terms because of the anticommuting property of the Grassmann coordinates. As a function of the coordinate x the expansion may be written as follows:

$$\begin{aligned} \Phi(x) &= A(x) + i\theta\sigma^\mu\bar{\theta} \partial_\mu A(x) + \frac{1}{4}\theta\theta\bar{\theta}\bar{\theta} * A(x) + \sqrt{2}\theta\psi(x) \\ &- \frac{i}{\sqrt{2}}\theta\theta \partial_\mu\psi(x)\sigma^\mu\bar{\theta} + \theta\theta F(x) \end{aligned} \quad (8.17)$$

The key point is that

$$\mathcal{L} = \int d^2\theta \Phi(x) \quad (8.18)$$

is a invariant under supersymmetric transformations (up to a total derivative).

After the integration some terms will disappear from the expansion of $\Phi(x)$ leaving only:

$$\Phi(x) = A(x) + \sqrt{2}\theta\psi(x) + \theta\theta F(x) \quad (8.19)$$

$A(x)$ will be associated with a complex Boson; $\psi(x)$ will be associated with a Weyl Fermion and $F(x)$ acts as an auxiliary field that carries no physical degrees of freedom. These are called the component fields of the superfield. The product of two chiral fields also produces a chiral field. Therefore, any polynomial, $W(\Phi)$, can be used to construct a supersymmetry invariant as

$$\mathcal{L} = \int d^2\theta W(\Phi) = F_W(\Phi) \quad (8.20)$$

is a supersymmetry invariant. This is used to provide a potential for the chiral field. The kinetic terms are described by:

$$\int d^2\theta d^2\bar{\theta} \bar{\Phi}_i \Phi_j = \bar{\Phi}_i \Phi_j \Big|_{\theta\theta\bar{\theta}\bar{\theta}} \quad (8.21)$$

Φ is an anti chiral field. After expanding and extracting the $\theta\theta\bar{\theta}\bar{\theta}$ term one obtains (up to total derivatives):

$$F_i^* F_f - |\partial_\mu A|^2 + \frac{i}{2} \partial_\mu \bar{\psi} \bar{\sigma}^\mu \psi \quad (8.22)$$

One has thus constructed the following Lagrangian:

$$\mathcal{L} = \bar{\Phi}_i \Phi_i \Big|_{\theta\theta\bar{\theta}\bar{\theta}} + \left[\lambda_i \Phi_i + \frac{1}{2} m_{ij} \Phi_i \Phi_j + \frac{1}{3} g_{ijk} \Phi_i \Phi_j \Phi_k \right]_{\theta\theta} \quad (8.23)$$

$$\begin{aligned} &= i \partial \bar{\psi}_i \bar{\sigma} \psi_i + A_i^* * A_i + F_i^* F_i + \lambda_i F_i + m_{ij} \left(A_i F_j - \frac{1}{2} \psi_i \psi_j \right) \\ &+ g_{ijk} (A_i A_j F_k - \psi_i \psi_j A_k) + h.c. \end{aligned} \quad (8.24)$$

One can eliminate the auxiliary fields F_i , F_i^* in favor of the fields carrying quantum degrees of freedom. The equation of motion for F_k^* is as follows:

$$F_k = \lambda_k^* + m_{ij} A_i^* + g_{ijk}^* A_i^* A_j^* \quad (8.25)$$

This gives:

$$\begin{aligned} \mathcal{L} = & i \partial \bar{\psi}_i \bar{\sigma} \psi_i + A_i^* * A_i - \frac{1}{2} m_{ij} \psi_i \psi_j - \frac{1}{2} m_{ij}^* \psi_i^* \psi_j^* \\ & - g_{ijk} \psi_i \psi_j A_k - g_{ijk}^* \bar{\psi}_i \bar{\psi}_j - A_k^* - F_i^* F_i \end{aligned} \quad (8.26)$$

where the last term leads after integration on the F_i s to a potential for the fields A, A^* ; these are known as the F terms, $V_F(A^*, A)$. (Note, $V_F \geq 0$). It turns out as explained later that at the ground state this must vanish i.e. $V_F(A^*, A) = 0$ if SUSY is not to be spontaneously broken. This in turn implies that $F_i = 0$ for such a symmetric ground state. Although this is a classical analysis so far, in fact it is true to all orders in perturbation theory as there exists a non-renormalization theorem for such an effective potential in SUSY theories. The Lagrangian described above is called the Wess-Zumino Lagrangian (WZ).

So far the scalar fields have been defined over simple flat manifolds. To describe the kinetic terms of supersymmetric Lagrangians of systems containing scalar fields spanning complicated manifolds it is convenient to introduce the following supersymmetry invariant:

$$\int d^4\theta K(\Phi, \bar{\Phi}). \quad (8.27)$$

$K(\Phi, \bar{\Phi})$ is called the Kähler potential, unlike the potential $W(\Phi)$ but similar to the kinetic term introduced above, the Kähler potential depends on both Φ and $\bar{\Phi}$. One may add any function of Φ or $\bar{\Phi}$ to the integrand since these terms will vanish after integration. For the usual kinetic terms, K is taken to be given by $K = \Phi \bar{\Phi}$ which produces the $-\delta_{ij} \partial_\mu A^{*i} \partial^\mu A^j$ kinetic terms for the scalars. For the case of a sigma model with a target space whose metric is g_{ij} ; this metric is related to the Kähler potential by:

$$g_{ij} = \frac{\partial^2 K}{\partial \bar{\Phi}_i \partial \Phi_j}. \quad (8.28)$$

The above supersymmetry invariant (8.27) which previously gave the usual kinetic terms in the action, produces for general K the action of a supersymmetric sigma model, with the target space metric given by Eq. (8.28).

8.2.2.2 Global Symmetries

It is possible to construct Lagrangians which have a global symmetry which does not commute with supersymmetry and thus assigns different quantum numbers to particles in the same supermultiplet. This symmetry already in its discrete form forbids unwanted interaction terms which strongly violate baryon and lepton conservation laws. Such interactions arise due to the bosonic superpartners to

standard model particles carrying Baryonic and Leptonic numbers. The symmetry also in its continuous U(1) version turns out to play a possible role in the possibilities to spontaneously break SUSY. It is called R symmetry.

R -symmetry is a global U(1) symmetry that does not commute with the supersymmetry. Its discrete version is called R parity. The action of the R -symmetry on a superfield Φ with R -character n as follows.

$$R\Phi(\theta, x) = \exp(2in\alpha) \Phi(\exp(-i\alpha\theta), x) \quad (8.29)$$

$$R\bar{\Phi}(\bar{\theta}, x) = \exp(-2in\alpha) \bar{\Phi}(\exp(i\alpha\bar{\theta}), x) \quad (8.30)$$

Since the R -charge does not commute with the supersymmetry, the component fields of the chiral field have different R -charges. For a superfield Φ with R -character n , the R -charges of the component fields may be read off as follows:

$$R(\text{lowest component of } \Phi) = R(A) \equiv n, R(\psi) = n - 1, R(F) = n - 2 \quad (8.31)$$

The R -charge of the Grassmann variables is given by:

$$R(\theta_\alpha) = 1, R(d\theta_\alpha) = -1 \quad (8.32)$$

with, barred variables having opposite R charge. The kinetic term $\bar{\Phi}\Phi$ is an R invariant. ($\bar{\theta}\theta\theta\theta$ is an invariant.) For the potential term,

$$\int d^2\theta W \quad (8.33)$$

to have zero R charge requires that $R(W) = 2$. For the resulting mass term from $W = \frac{1}{2}m\Phi^2$,

$$m\psi\psi + m^2|A|^2, \quad (8.34)$$

to have vanishing R -charge requires

$$R(\Phi) = R(A) = 1, R(\psi) = 0 \quad (8.35)$$

Adding the cubic term:

$$W_3 = \frac{\lambda}{3}\Phi^3 \quad (8.36)$$

produces

$$V = |\lambda|^2 |A|^4 + \lambda A \psi \psi. \quad (8.37)$$

This term is not R -invariant with the R -charges given by (8.35). To restore R -invariance requires λ is assigned an R -charge of -1 . This can be viewed as simply a book keeping device or more physically one can view the coupling as the vacuum expectation value of some field. The expectation value inherits the quantum numbers of the field. This is how one treats for example the mass parameters of Fermions in the standard model. There is also one other global $U(1)$ symmetry, one that commutes with the supersymmetry. All component fields are charged the same with respect to this $U(1)$ symmetry. Demanding that the terms in the action maintain this symmetry requires an assignment of $U(1)$ charges to λ , and m .

The charges are summarized in the following table:

	$U(1)$	$U(1)_R$
Φ	1	1
m	-2	0
λ	-3	-1
W	0	2

These symmetries can be used to prove important nonrenormalisation theorems. In particular it can be shown that the potential:

$$W = \frac{1}{2}m \Phi^2 + \frac{1}{3}\lambda\Phi^3. \quad (8.39)$$

does not change under renormalization.

These non-renormalization theorems play an important role in analyzing the dynamics of supersymmetric systems and in addressing the so called hierarchy problem.

8.2.2.3 Lagrangians for SUSY Gauge Theories

A vector superfield contains spin 1 and spin $\frac{1}{2}$ component fields. It obeys a reality condition $V = \bar{V}$.

$$\begin{aligned} V = & B + \theta \chi \bar{\theta} \bar{\chi} + \theta^2 C + \bar{\theta}^2 \bar{C} - \theta \sigma^\mu \bar{\theta} A_\mu \\ & + i \theta^2 \bar{\theta} \left(\bar{\lambda} + \frac{1}{2} \bar{\sigma}^\mu \partial_\mu \chi \right) - i \bar{\theta}^2 \theta \left(\lambda - \frac{1}{2} \sigma^\mu \partial_\mu \bar{\chi} \right) \\ & + \frac{1}{2} \theta^2 \bar{\theta}^2 (D^2 + \partial^2 B) \end{aligned} \quad (8.40)$$

B, D, A_μ are real and C is complex. The Lagrangian has a local $U(1)$ symmetry with a gauge parameter, Λ an arbitrary chiral field:

$$V \rightarrow V + i (\Lambda - \bar{\Lambda}) \quad (8.41)$$

B, χ , and C are gauge artifacts and can be gauged away. The symmetry is actually $U(1)_{\mathbb{C}}$ as opposed to the usual $U(1)_{\mathbb{R}}$ because although the vector field transforms with a real gauge parameter, the other fields transform with gauge parameters that depend on the imaginary part of Λ .

It is possible to construct a chiral superfield, W_α , from V as follows

$$W_\alpha = -\frac{1}{4} \overline{D} \overline{D} D_\alpha V, \quad \overline{D}_{\dot{\beta}} W_\alpha = 0 \quad (8.42)$$

One may choose a gauge (called the Wess Zumino gauge) in which B, C and χ vanish and then expand in terms of component fields,

$$\begin{aligned} V(y) &= -\theta \sigma^\mu \bar{\theta} A_\mu + i \theta^2 \bar{\theta} \bar{\lambda} - i \bar{\theta}^2 \theta \lambda + \frac{1}{2} \theta^2 \bar{\theta}^2 D \\ W_\alpha(y) &= -i \lambda_\alpha + \left(\delta_\alpha^\beta D - \frac{i}{2} (\sigma^\mu \bar{\sigma}^\nu)_\beta^\alpha F_{\mu\nu} \right) \theta_\beta + (\sigma^\mu \partial_\mu \bar{\lambda})_\alpha \theta^2 \end{aligned} \quad (8.43)$$

Where A_μ is the vector field, $F_{\mu\nu}$ its field strength, λ is the spin $\frac{1}{2}$ field and D is an auxiliary scalar field. Under the symmetry (8.41), the component fields transform under a now $U(1)_{\mathbb{R}}$ symmetry as:

$$A_\mu \rightarrow A_\mu - i \partial_\mu (B - B^*), \quad \lambda \rightarrow \lambda, \quad D \rightarrow D \quad (8.44)$$

Note, W is gauge invariant. The following supersymmetric gauge invariant Lagrangian is then constructed:

$$\mathcal{L} = \int d^2\theta \left(\frac{-i\tau}{16\pi} \right) W^\alpha W_\alpha + h.c. \quad (8.45)$$

where the coupling constant τ is now complex,

$$\tau = \frac{\theta}{2\pi} + i \frac{4\pi}{g^2}. \quad (8.46)$$

Expanding this in component fields produces,

$$\mathcal{L} = \frac{1}{4g^2} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2g^2} D^2 - \frac{i}{g^2} \lambda \sigma D \bar{\lambda} + \frac{\theta}{32\pi^2} (*F)^{\mu\nu} F_{\mu\nu}. \quad (8.47)$$

D is a non-propagating field. The θ term couples to the instanton number density (this vanishes for abelian fields in a non-compact space). A monopole in the

presence of such a coupling will obtain an electric charge. The supersymmetries acting on the component fields are, (up to total derivatives):

$$\begin{aligned}
 \delta_\epsilon A &= \sqrt{2}\epsilon\psi \\
 \delta_\epsilon \psi &= i\sqrt{2}\sigma^\mu\bar{\epsilon}\partial_\mu A + \sqrt{2}\epsilon F \\
 \delta_\epsilon F &= i\sqrt{2}\bar{\epsilon}\sigma^\mu\partial_\mu\psi \\
 \delta_\epsilon F_{\mu\nu} &= i(\epsilon\sigma_\mu\partial_\nu\bar{\lambda} + \bar{\epsilon}\sigma_\mu\partial_\nu\lambda) - (\mu \leftrightarrow \nu) \\
 \delta_\epsilon \lambda &= i\epsilon D + \sigma^{\mu\nu}\epsilon F_{\mu\nu} \\
 \delta_\epsilon D &= \bar{\epsilon}\sigma^\mu\partial_\mu\lambda - \epsilon\sigma^\mu\partial_\mu\bar{\lambda}.
 \end{aligned} \tag{8.48}$$

One may also add to the action a term linear in the vector field V , known as a Fayet-Iliopoulos term:

$$2K \int d^2\theta d^2\bar{\theta} V = K D = \int d\theta^\alpha W_\alpha + h.c. \tag{8.49}$$

It plays a possible role in the spontaneous breaking of SUSY. The $U(1)$ gauge fields couple to charged chiral matter through the following term

$$\mathcal{L} = \sum_i \int d^2\theta d^2\bar{\theta} \bar{\Phi}_i \exp(q_i V) \Phi_i \tag{8.50}$$

Under the gauge transformation

$$V \rightarrow V + i(\Lambda - \bar{\Lambda}), \quad \Phi_i \rightarrow \exp(-iq_i \Lambda) \Phi_i \tag{8.51}$$

Since there are chiral Fermions there is the possibility for chiral anomalies. In order that the theory is free from chiral anomalies one requires:

$$\sum q_i = \sum q_i^3 = 0. \tag{8.52}$$

Writing out the term (8.50) in components produces:

$$\mathcal{L} = F^* F - \left| \partial_\mu \phi + \frac{iq}{2} A_\mu \phi \right|^2 - i\bar{\phi}\bar{\sigma} \left(\partial_\mu + \frac{iq}{2} A_\mu \right) \psi - \frac{iq}{\sqrt{2}} (\phi\bar{\lambda}\bar{\psi} - \bar{\phi}\lambda\psi) \frac{1}{2} q D \bar{\phi} \phi. \tag{8.53}$$

There are two auxiliary fields, the D and F fields.

Adding the kinetic term (8.47) for the vector field and a potential, $\tilde{W}(\Phi)$ for the matter, gives the total Lagrangian,

$$\mathcal{L} = \int d^2\theta \left(W^\alpha W_\alpha + \int d^2\bar{\theta} \bar{\Phi}^i \exp(q_i V) \Phi_i + \tilde{W}(\Phi) \right) \tag{8.54}$$

this produces the following potential,

$$V = \sum_i \left| \frac{\partial \tilde{W}}{\partial \phi^i} \right|^2 + \frac{q^2}{4} \left((2K + \sum |\phi_i|^2) \right)^2 \quad (8.55)$$

So far only systems with U(1) vector fields were discussed. One can also consider non-Abelian gauge groups. The fields are in an adjoint representation of the group, A^a_μ, λ^a, D^a , the index a is the group index, ($a = 1 \dots \dim(\text{group})$) and $D^a = \sum_i \bar{\phi}_i T^a_{R(\phi_i)} \phi_i$.

8.2.3 *Supersymmetrical Particle Spectrum in Nature?*

The classification of the particles was naturally followed by an attempt to correlate the known particles with the algebraic results. The photon is massless to a very good approximation, the only known fermionic particle that at the time was considered massless as well was the neutrino. It was found that in the standard models the two cannot be members of the same multiplet. As SUSY is broken, it was at the time expected that the breaking be spontaneous and in case of a global symmetry this would lead to a massless spin one half Goldstone particle, a spin half fermion in the case of broken SUSY. It would have also been nice and simple if the neutrino would be at least the Goldstone fermion or as it has become termed a Goldstino. This turned out not to be consistent with experiment as well. Eventually one got resigned to the situation that all known particles, be they bosons or fermions, have supersymmetric partners which are yet to be discovered. The yet to be confirmed spin zero elementary particle, the Higgs, has a spin one half superpartner—the Higgsino. In fact in a supersymmetric model extension of the standard model at least two Higgs fields are required. The SUSY interaction terms can each be composed only of chiral, or only antichiral fields. For such interaction terms a single Higgs field would not permit to construct the Yukawa interaction terms needed to provide masses to all quarks and leptons. Also with only one field the theory would not be consistent as it would suffer what is called an anomaly. The particles carry a U(1) gauge charge, and in the presence of a single Higgsino that gauge symmetry would become invalid once quantum corrections are taken into account. An extra Higgsino, and thus an extra Higgs, is required to restore the gauge invariance at the quantum level. The two mentioned problems get to be resolved by adding the one extra Higgs supermultiplet. The superpartners of the known spin one half quarks and leptons are denoted squarks and sleptons and are required to be spin zero bosons. The superpartners of the various spin one known gauge particles are termed the photino, wino, zino and gluino. They are assigned spin one half and are in the adjoint representation of the gauge group. In the LHC a major effort is planned for observing these particles.

8.2.4 Spontaneous SUSY Breaking: Perturbative Analysis

Any attempt to relate supersymmetry to nature at the level of the known particles requires the symmetry to be broken. It could have been broken explicitly, in this section the currently known mechanisms for its spontaneous breaking are described. This includes the breaking at the classical level in a class of gauge theories (by D terms) and in theories with no gauge particles (by F terms). Also are described the dynamical breaking of Supersymmetry as well as its effective breaking in a metastable vacuum. For SUSY not to be spontaneously broken all the SUSY generators Q^i_α need to annihilate the ground state. As the Hamiltonian is constructed out of positive pairings of the SUSY generators, SUSY preservation occurs if and only if the energy of the ground state vanishes. Conversely, SUSY is broken if and only if the energy of the ground state does not vanish, $Q^i_\alpha | \text{G.S.} \rangle \neq 0$ iff $E_{\text{G.S.}} \neq 0$. As the Hamiltonian of the SUSY system is non-negative, the non-vanishing ground state energy in the case spontaneous SUSY breaking is positive.

As for the actual mechanism for the spontaneous breaking, it turned out that the breaking of supersymmetry requires a somewhat elaborate structure.

8.2.4.1 F -terms

Consider first a system which contains only spin zero and spin one-half particles. In that case the condition for SUSY not to be broken is the vanishing of the potential generated by the F terms. Super Symmetry is thus unbroken when the following equations have a solution:

$$V_F = 0 \iff F_i = 0 \quad \forall i, \quad (8.56)$$

These are n (complex) equations with n (complex) unknowns. Generically, they have a solution. Take the example of the one component WZ model, where

$$F_1 = -\lambda - mA + gA^2. \quad (8.57)$$

This has a solution. There is no supersymmetry breaking classically. The non-renormalization theorem for the F terms ensures this result to be correct to all orders in perturbation theory. The solution is:

$$V = A^* A |gA - m|^2 \quad (8.58)$$

and hence there are actually two supersymmetric vacuum: either at $\langle A \rangle = m/g$ or at $\langle A \rangle = 0$.

Let us examine now how supersymmetry may be spontaneously broken. The following anecdote may be of some pedagogical value. It turns out that a short time after supersymmetry was introduced arguments were published which claimed to

prove that supersymmetry cannot be broken spontaneously at all. Supersymmetry resisted breaking attempts for both theories of scalars and gauge theories. One could be surprised that the breaking was first achieved in the gauge systems. This was done by Fayet and Iliopoulos. The presence in the collaboration of a student who paid little respect to the general counter arguments made the discovery possible. Fayet went on to discover the breaking mechanism also in supersymmetric scalar theories as did O’Raifeartaigh.

We will describe four examples of mechanisms of breaking Super Symmetry. The first will be for theories not containing gauge particles. The second for systems containing gauge particles, the third is of a dynamical breaking of Super Symmetry and the fourth occurs if our universe happens to be in a long lived metastable vacuum of positive kinetic energy.

8.2.4.2 SUSY Breaking in Theories with Scalars and Spin One Half Particles by F Terms

The Fayet-O’Raifeartaigh potential contains three fields this is the minimal number needed in order to break supersymmetry. It is:

$$\mathcal{L}_{\text{Potential}} = \lambda \Phi_0 + m \Phi_1 \Phi_2 + g \Phi_0 \Phi_1 \Phi_1 + \text{h.c.} \quad (8.59)$$

Minimizing the potential leads to the following equations:

$$\begin{aligned} 0 &= \lambda + g \Phi_1 \Phi_1 \\ 0 &= m \Phi_2 + 2g \Phi_0 \Phi_1 \\ 0 &= m \Phi_1 \end{aligned} \quad (8.60)$$

These cannot be consistently solved so there cannot be a zero energy ground state and supersymmetry must be spontaneously broken. To find the ground state one must write out the full Lagrangian including the kinetic terms in component fields and then minimize. Doing so one discovers that in the ground state $A_1 = A_2 = 0$ and A_0 is arbitrary. The arbitrariness of A_0 is a flat direction in the potential, the field along the flat direction is called a moduli. Computing the masses by examining the quadratic terms for component fields gives the following spectrum: the six real scalars have masses: $0, 0, m^2, m^2, m^2 \pm 2g\lambda$; and the Fermions have masses: $0, 2m$. The zero mass Fermion is the Goldstino. We turn next to breaking of supersymmetry theories that are gauge invariant.

8.2.4.3 SUSY Breaking in Supersymmetric Gauge Theories

The potential for the supersymmetric gauge theory was obtained to be:

$$V = \sum_i \left| \frac{\partial \tilde{W}}{\partial \phi^i} \right|^2 + \frac{q^2}{4} \left(2K + \sum |\phi_i|^2 \right)^2 \quad (8.61)$$

The first term is the F -term and the second is the D -term. Both these terms need to vanish for supersymmetry to remain unbroken.

Some remarks about this potential are in order:

Generically, the F -terms should vanish since there are indeed n equations for n unknowns. If $\langle \varphi_i \rangle = 0$, that is if the $U(1)$ is not spontaneously broken then supersymmetry is broken if and only if $K_{F,1} \neq 0$. When $K = 0$ and the F -terms have a vanishing solution then so will the D -term and there will be no supersymmetry breaking.

These ideas are demonstrated by the following example. Consider fields Φ_1, Φ_2 with opposite $U(1)$ charges and Lagrangian given by:

$$\mathcal{L} = \frac{1}{4} (W^\alpha W_\alpha + h.c.) + \overline{\Phi}_1 \exp(eV) \Phi_1 + \overline{\Phi}_2 \exp(-eV) \Phi_2 + m \Phi_1 \Phi_2 + h.c. + 2KV \quad (8.62)$$

This leads to the potential:

$$V = \frac{1}{2} D^2 + F_1 F_1^* + F_2 F_2^* \quad (8.63)$$

where

$$\begin{aligned} D + K + \frac{e}{2} (A_1^* A_1 - A_2^* A_2) &= 0 \\ F_1 + m A_{2*} &= 0 \\ F_2 + m A_1^* &= 0 \end{aligned} \quad (8.64)$$

Leading to the following expression for the potential:

$$V = \frac{1}{2} K^2 + \left(m^2 + \frac{1}{2} eK \right) A_1^* A_1 + \left(m^2 - \frac{1}{2} eK \right) A_2^* A_2 + \frac{1}{8} e^2 (A_1^* A_1 - A_2^* A_2)^2 \quad (8.65)$$

Consider the case, $m^2 > \frac{1}{2} eK$. The scalars have mass, $\sqrt{m^2 + \frac{1}{2} eK}$ and $\sqrt{m^2 - \frac{1}{2} eK}$. The vector field has zero mass. Two Fermions have mass m and one Fermion is massless. Since the vector field remains massless then the $U(1)$ symmetry remains unbroken. For $K \neq 0$, supersymmetry is broken as the Bosons and Fermions have different masses. (For $K = 0$ though the symmetry is restored.) The massless Fermion (the Photino) is now a Goldstino. Note that a trace of the underlying supersymmetry survives as one still has $\text{Tr} M_B^2 = \text{Tr} M_F^2$ even after the breaking of supersymmetry. M_B and M_F are the bosonic and fermionic mass matrices respectively.

Next, consider the case, $m^2 < \frac{1}{2} eK$; at the minimum, $A_1 = 0, A_2 = v$ where $v^2 \equiv 4 \frac{\frac{1}{2} eK - m^2}{e^2}$.

The potential expanded around this minimum becomes, with $A \equiv A_1$ and $\tilde{A} \equiv A_2 - v$:

$$\begin{aligned} V = & \frac{2m^2}{e^2} (eK - m^2) + \frac{1}{2} \left(\frac{1}{2} e^2 v^2 \right) A_\mu A^\mu \\ & + 2m^2 A^* A = \frac{1}{2} \left(\frac{1}{2} e^2 v^2 \right) \left(\frac{1}{\sqrt{2}} (\tilde{A} + \tilde{A}^*) \right)^2 \\ & + \sqrt{m^2 + \frac{1}{2} e^2 v^2} (\psi \tilde{\psi} + \overline{\psi} \overline{\tilde{\psi}}) + 0 \times \lambda \bar{\lambda} \end{aligned} \quad (8.66)$$

The first term implies that supersymmetry is broken for $m > 0$. The photon is massive, $m_\gamma^2 = \frac{1}{2} e^2 v^2$ implying that the U(1) symmetry is broken as well. The Higgs field, $\frac{1}{\sqrt{2}} (\tilde{A} + \tilde{A}^*)^2$ has the same mass as the photon. Two Fermions have non-zero mass and there is one massless Fermion, the Goldstino.

In the above example there is both supersymmetry breaking and U(1) symmetry breaking except when $m = 0$ in which case the supersymmetry remains unbroken.

Next consider a more generic example where there is U(1) breaking but no supersymmetry breaking, Φ is neutral under the U(1) while Φ_+ has charge +1 and Φ_- has charge -1. The potential is given by:

$$\mathcal{L} = \frac{1}{2} m \Phi^2 + \mu \Phi_+ \Phi_- + \lambda \Phi \Phi_+ \Phi_- + h.c. \quad (8.67)$$

There are two branches of solutions to the vacuum equations (a denotes the vacuum expectation value of A, etc.):

$$a_+ a_- = 0, \quad a = -\frac{\lambda}{m} \quad (8.68)$$

which leads to no U(1) breaking and

$$a_+ a_- = -\frac{1}{8} \left(\lambda - \frac{m\mu}{g} \right), \quad a = -\frac{\mu}{g} \quad (8.69)$$

which breaks the U(1) symmetry.

Note, the presence of a flat direction:

$$a_+ \rightarrow e^\alpha a_+, \quad a_- \rightarrow e^{-\alpha} a_- \quad (8.70)$$

leaves $a_- a_+$ fixed and the vacuum equations are still satisfied.

Typical generic supersymmetry breaking requires that some of the equations for the vanishing of the potential to be redundant. Such could be the case if the system had an extra symmetry such as an R symmetry mentioned before. In the examples above containing only scalar fields this was indeed the case. The absence of a zero energy solution led also to a spontaneous breaking of the R -symmetry. This should lead to the presence of a Goldstone Boson corresponding to the broken

$U(1)_R$. Inverting this argument leads to the conclusion that supersymmetric breaking in nature is not easy to obtain since we do not observe even such a particle. This argument was revisited in recent years.

So far only systems with $U(1)$ vector fields were discussed. To have a non-vanishing Fayet-Iliopoulos term there needs to be a $U(1)$ factor in the gauge group. To obtain a breaking of SUSY in a gauge system which does not have a $U(1)$ factor one needs to consider dynamical SUSY breaking. This is a more complex problem and new tools were developed to explore this possibility. The functional form of such a breaking allows to obtain a scale of SUSY breaking which is naturally much smaller than the relevant cutoff of the problem. This is reflected by the relation:

$$M_{\text{SUSY-breaking}} = M_{\text{cutoff}} \exp(-c/g(M_{\text{cutoff}})). \quad (8.71)$$

8.2.5 Dynamics of SUSY Gauge Theories and SUSY Breaking

The description of the mechanism of dynamical breaking of SUSY is preceded by a survey of the general possible phase structure of gauge theories as well as its concrete realizations in SUSY gauge theories.

8.2.5.1 Phases of Gauge Theories

The phase structure of gauge theories can be introduced by analyzing them as statistical mechanical systems regulated by a finite lattice. The basics can be illustrated by considering $D = 4$ lattice gauge theories, in particular those for which the gauge fields which are Z_N valued. The system has a coupling g .

The effective “temperature” of the system is given by, $T = \frac{Ng^2}{2\pi}$.

For a given theory there is a lattice of electric and magnetically charged operators. The electric charge is denoted by n and the magnetic charge by m . An operator with charges (n, m) is *perturbative*, i.e., it is an irrelevant operator and weakly coupled to system, so long as the free energy, $F > 0$, i.e.,

$$n^2 T + \frac{m^2}{T} > \frac{C}{N}, \quad (8.72)$$

however, when the free energy is negative for the operator (n, m) , it condenses indicating the presence of a relevant operator and hence an infra-red instability, this occurs when,

$$n^2 T + \frac{m^2}{T} < \frac{C}{N}. \quad (8.73)$$

Keeping N, C fixed and vary T .

Fig. 8.1 Different possible phases

The system has three phases depending which operators condense. At small T , there is electric condensation which implies that there is electric charge screening, magnetic charges are confined, and the log of the Wilson loop is proportional to the length of the perimeter of the loop. (This is called the Higgs phase).

At high T , magnetic condensation occurs, this is the dual of electric condensation. Magnetic charges are screened, electric charges are confined and the log of the Wilson loop is proportional to the area. (This is called the confinement phase.) For intermediate values of T it is possible that there is neither screening of charges nor confinement, this is the Coulomb phase (Fig. 8.1).

In the presence of a θ parameter, an electric charge picks up a magnetic charge and becomes dyonic.

$$n' = n + \frac{\theta}{2\pi} m \quad (8.74)$$

This leads to a tilted lattice of dyonic charges and one may condense dyons with charges (n_0, m_0) . This leads to what is called oblique confinement with the charges commensurate with (n_0, m_0) being screened and all other charges being confined.

These ideas relate to the gauge theories of the standard model. For QCD it was suggested that confinement occurs due to the condensation of QCD monopoles. This is a magnetic superconductor dual to the electric one which describes the weak interactions. It is difficult to study this phenomenon directly. The Dirac monopole in a U(1) gauge theory is a singular object; however by embedding the monopole in a spontaneously broken non-Abelian theory with an additional scalar field one may smooth out the core of the monopole and remove the singularity. One may proceed analogously, by enriching QCD; adding scalars and making the theory supersymmetric one can calculate the condensation of monopoles in a four dimensional gauge theory.

In general the possible phase structure of gauge theories and their actual realizations were obtained by using various approximation schemes. While supersymmetry has not yet disclosed if it is part of nature in some form or another, in its presence, the

Fig. 8.2 Possible phases of gauge theories (g_1 and g_2 are some relevant couplings)

phase structure of gauge theories was exactly obtained in some cases. Most earlier exact important results in field theories, such as asymptotic freedom, were obtained in circumstances in which the couplings are weak. SUSY enables to obtain also results in the strong coupling regime. The analytic control seems to become larger the more supersymmetries the system possesses. This has been achieved in four dimensions for supersymmetric gauge theories with $N = 1, 2, 4$ supersymmetries. There are many new methods that have been utilized and the phase structures of these theories have been well investigated. Novel properties of these theories have been discovered such as new types of conformal field theories and new sorts of infra-red duality. To these we turn next (Fig. 8.2).

8.2.5.2 SUSY QCD: The Setup

The goal will be to examine theories that are simple supersymmetric extensions of QCD. Consider the case of a $N = 1$ vector multiplet with gauge group $SU(N_C)$, and N_F chiral multiplets in the fundamental representation of $SU(N_C)$, and N_F chiral multiplets in the antifundamental representation. The Lagrangian is:

$$\begin{aligned} \mathcal{L} = & \int (-i\tau) \text{Tr} W^\alpha W_\alpha d^2\theta + h.c. \\ & + Q_F^+ \exp(-2V) Q_F + \tilde{Q} \exp(2V) Q_F^+ |_{\overline{\theta\theta}\theta\theta} + m_F \tilde{Q}_F Q |_{\theta\theta} \end{aligned} \quad (8.75)$$

where the coupling is:

$$\tau = \frac{\theta}{2\pi} + i \frac{4\pi}{g^2}. \quad (8.76)$$

Apart from the local $SU(N_C)$ gauge symmetry, the fields are charged under the following global symmetries.

$$\begin{array}{ccccc}
 & SU(N_F)_L & \times SU(N_F)_R & \times U(1)_V & \times U(1)_A & \times U(1)_{RC} \\
 Q_a^i & N_F & 1 & 1 & 1 & 1 \\
 \tilde{Q}_i^a & 1 & \overline{N}_F & -1 & 1 & 1 \\
 W_\alpha & 1 & 1 & 0 & 0 & 1
 \end{array}$$

When $N_C = 2$, because $2 \sim \overline{2}$, the global flavor symmetry is enhanced to $SO(2N_F)_L \times SO(2N_F)_R$.

There is an anomaly of the $U(1)_A \times U(1)_R$ symmetry. A single $U(1)$ symmetry survives the anomaly. This is denoted as $U(1)_R$ and is a full quantum symmetry. The adjoint Fermion contributes $2N_C \times R(\lambda)$ to the anomaly. The Chiral Fermions contribute, $2N_F \times R_F$. $R(\lambda) = 1$, while R_F is now chosen so that the total anomaly vanishes,

$$2N_C + 2R_F N_F = 0. \quad (8.77)$$

This leads to

$$R_F = -\frac{N_C}{N_F}. \quad (8.78)$$

The Bosons in the chiral multiplet have an R -charge one greater than the Fermions in the multiplet. Thus,

$$R_B = 1 - \frac{N_C}{N_F} = \frac{N_F - N_C}{N_F}. \quad (8.79)$$

The non-anomalous R -charge, is given by:

$$R = R_C - \frac{N_C}{N_F} Q_A, \quad (8.80)$$

where R_C is the classical R -charge and A is the classical $U(1)_{rMA}$ charge. Following non-anomalous global charges: This leads to the

$$\begin{array}{ccccc}
 & SU(N_F)_L & \times SU(N_F)_R & \times U(1)_V & \times U(1)_R \\
 Q_a^i & N_F & 1 & 1 & \frac{N_F - N_C}{N_F} \\
 \tilde{Q}_i^a & 1 & \overline{N}_F & -1 & \frac{N_F - N_C}{N_F} \\
 W_\alpha & 1 & 1 & 0 & 1
 \end{array}$$

One is now ready to identify the classical moduli space.

8.2.5.3 The Moduli Space

The classical moduli space is given by solving the D -term and F -term equations:

$$\begin{aligned} D^a &= Q_F^\dagger T^a Q_F - \tilde{Q}_F^\dagger T^a \tilde{Q}_F \\ \overline{F}_{Q_F} &= -m_F \tilde{Q} \\ \overline{F}_{\tilde{Q}_F} &= -m_F Q \end{aligned} \quad (8.81)$$

For $N_F = 0$ or for $N_F \neq 0$ and $m_F \neq 0$, there is no moduli space. Note, the vacuum structure is an infra-red property of the system hence having $m_F \neq 0$ is equivalent to setting $N_F = 0$ in the deep infrared.

Consider the quantum moduli space of the case where $N_F = 0$. One can show that the number of zero energy states of the system is no smaller than The Witten index, $\text{Tr}(-1)^F = N_C$ i.e. the rank of the group $+1$. This number is larger than zero and thus there is no supersymmetry breaking in these systems. There are $2N_C$ Fermionic zero modes (from the vector multiplet). These Fermionic zero modes break through instanton effects the original $U(1)_R$ down to Z_{2N_C} . Further breaking occurs because the gluino two point function acquires a vacuum expectation value which breaks the symmetry down to Z_2 . This indeed leaves N_C vacua. The gluino condensate is:

$$\langle \lambda\lambda \rangle = \exp\left(\frac{2\pi i k}{N_C}\right) \Lambda_{N_C}^3 \quad (8.82)$$

where Λ_{N_C} is the dynamically generated scale of the gauge theory and $k = 1, \dots, N_C - 1$ label the vacua. Chiral symmetry breaking produces a mass gap. Note, because chiral symmetry is discrete there are no Goldstone Bosons. Further details of quantum moduli spaces will be discussed later.

Consider the case where $m_F = 0$ and $0 < N_F < N_C$. The classical moduli space is determined by the following solutions to the D -term equations:

$$Q = \tilde{Q} = \begin{pmatrix} a_1 & 0 & & 0 & \dots & 0 \\ 0 & a_2 & & 0 & \dots & 0 \\ & & \ddots & 0 & \dots & 0 \\ & & & \ddots & 0 & \dots & 0 \\ & & & & \ddots & 0 & \dots & 0 \\ & & & & & a_{N_F} & 0 & \dots & 0 \end{pmatrix}_{N_F \times N_C} \quad (8.83)$$

Where the row indicates the flavor and the column indicates the colour. There are N_F diagonal non-zero real entries, a_i . (To validate this classical analysis the vacuum expectation values must be much larger than any dynamically generated scale, i.e., $a_i \gg \Lambda$. The gauge symmetry is partially broken:

$$SU(N_C) \rightarrow SU(N_C - N_F). \quad (8.84)$$

This is for generic values of a_i . By setting some subset of a_i to zero one may break to a subgroup of $SU(N_C)$ that is larger than $SU(N_C - N_F)$. Also, if $N_F = N_C - 1$ then the gauge group is complete broken. This is called the Higgs phase.

The number of massless vector Bosons becomes

$$N_C^2 - \left((N_C - N_F)^2 - 1 \right) = 2N_C N_F - N_F^2, \quad (8.85)$$

the number of massless scalar fields becomes,

$$2N_C N_F - \left(2N_C N_F - N_F^2 \right) = N_F^2. \quad (8.86)$$

The matrix

$$M_{ij} \equiv \tilde{Q}_i Q_j \quad (8.87)$$

forms a gauge invariant basis. The Kahler potential is then,

$$K = 2\text{Tr}\sqrt{(M\overline{M})}. \quad (8.88)$$

When singularities appear, i.e. $\det M = 0$, it signals the presence of massless particles as well as of enhanced symmetries.

When $N_F \geq N_C$, one has the following classical moduli space,

$$Q = \begin{pmatrix} a_1 & 0 & & & \\ 0 & a_2 & & & \\ & & \ddots & & \\ & & & \ddots & \\ & & & & a_{N_C} \\ 0 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \dots & \vdots & 0 \\ 0 & 0 & \dots & 0 & 0 \end{pmatrix}_{N_F \times N_C}, \quad \tilde{Q} = \begin{pmatrix} \tilde{a}_1 & 0 & & & \\ 0 & \tilde{a}_2 & & & \\ & & \ddots & & \\ & & & \ddots & \\ & & & & \tilde{a}_{N_C} \\ 0 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \dots & \vdots & 0 \\ 0 & 0 & \dots & 0 & 0 \end{pmatrix}_{N_F \times N_C} \quad (8.89)$$

with the constraint that

$$|a_i|^2 - |\tilde{a}_i|^2 = \rho. \quad (8.90)$$

Generically the $SU(N_C)$ symmetry is completely broken. However, when $a_i = \tilde{a}_i = 0$ then a subgroup of the $SU(N_C)$ can remain.

We will now consider some special cases, first the classical moduli space for $N_F = N_C$. The dimension of the moduli space is given by:

$$2N_C^2 - (N_C^2 - 1) = N_C^2 + 1 = N_F^2 + 1 \quad (8.91)$$

There are N_F^2 degrees of freedom from M_{ij} and naively one would have two further degrees of freedom from:

$$B = \epsilon_{i_1 \dots i_{N_C}} Q_{j_1}^{i_1} \dots Q_{j_{N_F}}^{i_{N_C}}, \quad \tilde{B} = \epsilon_{i_1 \dots i_{N_C}} \tilde{Q}_{j_1}^{i_1} \dots \tilde{Q}_{j_{N_F}}^{i_{N_C}}. \quad (8.92)$$

There is, however, a classical constraint:

$$\det M - B\tilde{B} = 0 \quad (8.93)$$

which means M , B and $\tilde{B}$ are classically dependent. This leaves only $N_F^2 + 1$ independent moduli.

Generically, as well as the gauge symmetry being completely broken, the global flavor symmetry is also broken. There is a singular point in the moduli space where $M = 0 = B = \tilde{B}$.

Next, consider the case, $N_F = N_C + 1$, again there are N_F^2 moduli from M_{ij} . There are also, $2(N_C + 1)$ degrees of freedom given by:

$$B_i = \epsilon_{ii_1 \dots i_{N_C}} Q_{j_1}^{i_1} \dots Q_{j_{N_F}}^{i_{N_C}}, \quad \tilde{B}_{\bar{i}} = \epsilon_{i_1 \dots i_{N_C}} \tilde{Q}_{j_1}^{i_1} \dots \tilde{Q}_{j_{N_F}}^{i_{N_C}}. \quad (8.94)$$

However, there are again the classical constraints:

$$\begin{aligned} \det M - M_{\bar{i}j} B^i B^{\bar{j}} &= 0 \\ M_{\bar{j}i} B_i &= M_{\bar{i}j} B_{\bar{j}} = 0 \end{aligned} \quad (8.95)$$

giving again an $N_F^2 + 1$ dimension moduli space. (The moduli space is not smooth). There is a generic breaking of gauge symmetry. In all these cases the potential has flat directions of zero energy, SUSY is unbroken. The non-renormalization theorem extends this result to all order in perturbation theory, it will be the non-perturbative effects which will lift the vacuum degeneracies in some cases and will lead to dynamical breaking of SUSY.

8.2.5.4 Quantum Moduli Spaces/Dynamical SUSY Breaking

A rich structure emerges. One is required to examine on a case by case basis the role that quantum effects play in determining the exact moduli space. Quantum effects

Fig. 8.3 Potential for $1 < N_F < N_C$, it has no ground state

both perturbative and nonperturbative can lift moduli. In what follows the quantum moduli space is examined for the separate cases: 1

$$\leq N_F \leq N_C - 1, N_F = N_C, N_F, N_C + 1, N_C + 1 < N_F \leq \frac{3N_C}{2},$$

$$\frac{3N_C}{2} < N_F < 3N_C, N_F = 3N_C, \text{ and } N_F > 3N_C.$$

We start with the study of the quantum moduli space for $0 < N_F < N_C$.

Classically, the dimension of the moduli space is N_F^2 from $Q, \bar{Q}$. The following table summarizes the charges under the various groups.

	$SU(N_C)$	$SU(N_F)_L$	$SU(N_F)_R$	$U(1)_V$	$U(1)_A$	$U(1)_{R_{cl}}$	$U(1)_R$
Q_a^i	N_C	N_F	1	1	1	1	$\frac{N_F - N_C}{N_F}$
$\bar{Q}_i^a$	$\bar{N}_C$	1	$\bar{N}_F$	-1	1	1	$\frac{N_F - N_C}{N_F}$
$\Lambda^{3N_C - N_F}$	1	1	1	0	$2N_F$	$2N_C$	0
M	1	N_F	$\bar{N}_F$	0	2	2	$2 - \frac{2N_C}{N_F}$
$\det M$	1	1	1	0	$2N_F$	$2N_F$	$2(N_F - N_C)$

Λ , the dynamically generated QCD scale is assigned charges as m and g were before. The power $3N_C - N_F$ is the coefficient in the one loop beta function. There is no Coulomb phase so W_α does not appear.

The symmetries imply, the superpotential, W , has the following form:

$$W = \left(\Lambda^{3N_C - N_F} \right)^a (\det M)^b c \quad (8.96)$$

a, b are to be determined. c is a numerical coefficient, If c does not vanish, the classical moduli space gets completely lifted by these nonperturbative effects (Fig. 8.3).

One examines the charges of W under the various symmetries. Automatically, the charges of W for the flavor symmetries, $SU(N_F)_L \times SU(N_F)_R$ and the $U(1)_V$ vanish.

If one requires the $U(1)_A$ charge to vanish then this implies $a = -b$. Requiring the $U(1)_R$ charge to vanish implies that $b = \frac{1}{N_F - N_C}$. These restrictions fix:

$$W = c \left(\frac{\Lambda^{3N_C - N_F}}{\det M} \right)^{\frac{1}{N_C - N_F}} \quad (8.97)$$

For non-vanishing c , all the moduli are now lifted and there is no ground state.

What is the value of c ? This is difficult to calculate directly unless there is complete higgsing. For $N_F = N_C - 1$ there is complete symmetry breaking and one can turn to weak coupling. From instanton calculations one calculates that $c \neq 0$ and the prepotential for the matter fields is

$$W \sim \left(\frac{\Lambda^{2N_C + 1}}{\det M} \right). \quad (8.98)$$

One may now go to $N_F < N_C - 1$ by adding masses and integrating out the heavy degrees of freedom. This produces

$$\langle M^i_j \rangle_{\min} = \left(m^{-1} \right)^i_j \left(\Lambda^{3N_C - N_F} \det m \right)^{\frac{1}{N_C}}. \quad (8.99)$$

Thus in this region the dynamical effects remove the supersymmetric vacuum. In fact the system has no ground state and supersymmetry is broken dynamically. One can go one step further and consider the possibility that the effective potential above is modified so as to have a local minimum with positive vacuum energy. It is possible to construct many such examples and consider that supersymmetry seems broken as a result of the universe being for the (long) time being in the metastable state. Eventually the system may tunnel to another lower energy vacuum and perhaps eventually will reach a SUSY vacuum. In the analysis of such systems one needs also to take into account gravity. In the cases which follow supersymmetry is not broken, they are described to illustrate the rich structure which emerges once one can treat the system when it strongly interacts. Perhaps also structures like those of the dyonic condensates will yet play a role somewhere in nature (Fig. 8.4).

We now turn to study the dynamics and quantum moduli space for the $N_F \geq N_C$ case. In this situation there is a surviving moduli space. In the presence of a mass matrix, m_{ij} for matter one obtains

$$\langle M^i_j \rangle = \left(m^{-1} \right)^i_j \left(\Lambda^{3N_C - N_F} \det m \right)^{\frac{1}{N_C}}. \quad (8.100)$$

Fig. 8.4 Potential with finite masses has a ground state.

$M_{\min} \rightarrow \infty$ as $m \rightarrow 0$

Previously, for the case of $N_F < N_C$, it turned out that $m \rightarrow 0$ implied $\langle M^i_j \rangle \rightarrow \infty$ thus explicitly lifting the classical moduli space. For $N_F \geq N_C$ it is possible to have $m \rightarrow 0$ while keeping $\langle M^i_j \rangle$ fixed.

Consider the case where $N_F = N_C$. Quantum effects alter the classical constraint to be:

$$\det M - B\tilde{B} = \Lambda^{2N_C}. \quad (8.101)$$

This has the effect of resolving the singularity in moduli space. The absence of a singularity means there will not be additional massless particles.

The physics of this theory depends on the position in moduli space of the vacuum. For large, $M/B/\tilde{B}$ one is sitting in the Higgs regime; however, for small $M/B/\tilde{B}$ one is in the confining regime. Note that M cannot be taken smaller than Λ . As the system has particles in the fundamental representations there is no actual phase transition between these two regimes, the transition is of a quantitative nature. In addition global symmetries need to be broken in order to satisfy the modified constraint equation.

Consider some examples: with the following expectation value,

$$\langle M^i_j \rangle = \delta^i_j \Lambda^2, \quad \langle B\tilde{B} \rangle = 0, \quad (8.102)$$

the global symmetries are broken to:

$$SU(N_F)_V \times U_B(1) \times U_R(1), \quad (8.103)$$

and there is chiral symmetry breaking. When,

$$\langle M^i_j \rangle = 0, \quad \langle B\tilde{B} \rangle \neq 0 \quad (8.104)$$

Fig. 8.5 Phases of super QCD

then the group is broken to:

$$SU(N_F)_L \times SU(N_F)_R \times U_R(1) \quad (8.105)$$

which has chiral symmetry and also has confinement. This is an interesting situation because there is a dogma that as soon as a system has a bound state there will be chiral symmetry breaking (Fig. 8.5).

The dynamics for the case $N_F = N_C + 1$ brings about some different dynamics. The moduli space remains unchanged. The classical and quantum moduli spaces are the same and hence the singularity when $M = B = \tilde{B} = 0$ remains. This is not a theory of massless gluons but a theory of massless mesons and baryons. When, $M, B, \tilde{B} \neq 0$ then one is in a Higgs/confining phase. At the singular point when, $M = B = \tilde{B} = 0$ there is no global symmetry breaking but there is “confinement” with light baryons.

Fig. 8.6 Two systems with a different ultra-violet behavior flowing to the same infra-red fixed point.

There is a suggestion that in this situation, $M, B, \tilde{B}$ become dynamically independent. The analogy is from the nonlinear sigma model, where because of strong infrared fluctuations there are n independent fields even though there is a classical constraint. The effective potential is:

$$W_{\text{eff}} = \frac{1}{\Lambda^{2N_C-1}} \left(M^i{}_j B_i \tilde{B}^j - \det M \right) \tag{8.106}$$

the classical limit is taken by:

$$\Lambda \rightarrow 0 \tag{8.107}$$

which in turn imposes the classical constraint.

For higher values of the number of flavors N_F the plot thickens even more as new duality emerge. Infrared dualities which in some circumstances uncover new types of dynamical structures.

8.2.5.5 Infra-red Duality

Two systems are called infra-red dual if, when observed at longer and longer length scales, they become more and more similar (Fig. 8.6).

It has been observed that the following set of $N = 1$ supersymmetric gauge theories are pairwise infra-red dual called Seiberg dualities.

System		Dual System		
Gauge Group	#flavors	Gauge Group	#flavor	#singlets
$SU(N_C)SO(N_C)Sp(N_C)$	$N_F N_F 2N_F$	$SU(N_F - N_C)$ $SO(N_F - N_C + 4)$ $Sp(N_F - N_C - 2)$	$N_F N_F 2N_F$	$N_F^2 N_F^2 N_F^2$

For a given number of colors, N_C , the number of flavors, N_F , for which the infrared duality holds is always large enough so as to make the entries in the table meaningful. Note that the rank of the dual pairs is usually different. Lets explain why this result is so powerful. In general, it has been known for quite a long time that two systems which differ by irrelevant operator have the same infra-red behavior.

In these cases the UV structure of the Infrared dual theories is very different, the dual systems have a different numbers of colors. The common wisdom in hadronic physics has already identified very important cases of infra-red duality. For example, QCD, whose gauge group is $SU(N_C)$ and whose flavor group is $SU(N_F) \times SU(N_F) \times U(1)$, is expected to be infra-red dual to a theory of massless pions which are all color singlets. The pions, being the spin-0 Goldstone Bosons of the spontaneously broken chiral symmetry, are actually infra-red free in four dimensions. We have thus relearned that free spin-0 massless particles can actually be the infra-red ashes of a strongly-interacting theory, QCD whose ultraviolet behavior is described by other particles. By using supersymmetry, one can realize a situation where free massless spin- $\frac{1}{2}$ particles are also the infra-red resolution of another theory. This duality allows for the first time to ascribe a similar role to massless infra-red free spin-1 particles. Massless spin-1 particles play a very special role in our understanding of the basic interactions. This comes about in the following way: Consider, for example, the $N = 1$ supersymmetric model with N_C colors and N_F flavors. It is infra-red dual to a theory with $N_F - N_C$ colors and N_F flavors and N_F^2 color singlets. For a given N_C , if the number of flavors, N_F , is in the interval $N_C + 1 < N_F < \frac{3N_C}{2}$, the original theory is strongly coupled in the infra-red, while the dual theory has such a large number of flavors that it becomes infrared free. Thus the infra-red behavior of the strongly-coupled system is described by infrared free spin-1 massless fields (as well as its superpartners), i.e. infrared free massless spin-1 particles (for example photons in a SUSY system) could be, under certain circumstances, just the infra-red limit of a much more complicated ultraviolet theory. This is the first example of a weakly interacting theory in which spin one particles that in the infra-red may be viewed as bound states of the dual theory. The duality has passed a large number of consistency checks under many circumstances. The infrared duality relates two disconnected systems. From the point of view of string theory the two systems are embedded in a larger space of models, such that a continuous trajectory relates them. An additional new consequence of this duality follows for the case

$$\frac{3N_C}{2} < N_F < 3N_C$$

the two dual theories are both asymptotically free in the UV and are describe by the same nontrivial conformal field theory in the infrared. The panorama of these structures is given in Fig. 8.8. Finally a class of examples was found among $N = 2$ SUSY conformal systems for which there are two very different Lagrangian descriptions as far as the local symmetries are involved which are actually identical at all distance scales.

8.2.5.6 More General Matter Composition of SUSY Gauge Theories

One can enrich the structure of the theory by adding N_a particles in the adjoint representation. At first one has no matter in the fundamental representation and scalar multiplets which are adjoint valued. The potential for the scalars, φ_i is given by:

$$V = ([\varphi, \varphi])^2. \quad (8.108)$$

This potential obviously has a flat direction for diagonal φ . The gauge invariant macroscopic moduli would be $\text{Tr}\varphi^k$. Consider the non-generic example of $N_C = 2$ and $N_a = 1$, the supersymmetry is now increased to $N = 2$. There is a single complex modulus, $\text{Tr}\varphi^2$. Classically, $\text{SU}(2)$ is broken to $\text{U}(1)$ for $\text{Tr}\varphi^2 \neq 0$. One would expect a singularity at $\text{Tr}\varphi^2 = 0$. The exact quantum potential vanishes in this case.

Naively, one could have expected that when $\text{Tr}\varphi^2$ is of order Λ or smaller, one would expect that the strong infra-red fluctuations would wash away the expectation value for $\text{Tr}\varphi^2$ and the theory would be confining. The surprising thing is that when $\text{SU}(2)$ breaks down to $\text{U}(1)$, because of the very strong constraints that supersymmetry imposes on the system, there are only two special points in moduli space and even there the theory is only on the verge of confinement. Everywhere else the theory is in the Coulomb phase. At the special points in the moduli space, new particles will become massless.

One can examine the effective theory at a generic point in moduli space where the theory is broken down to $\text{U}(1)$. The Lagrangian is given by,

$$\mathcal{L} = \int d^2\theta \text{Im} \left(\tau_{\text{eff}} \left(\text{Tr}\varphi^2, g, \Lambda \right) W_\alpha W^\alpha \right) \quad (8.109)$$

The τ_{eff} is the effective complex coupling which is a function of the modulus, $\text{Tr}\varphi^2$, the original couplings and the scale, Λ . This theory has an $\text{SL}(2, \mathbb{Z})$ duality symmetry. The generators of the $\text{SL}(2, \mathbb{Z})$ act on τ , defined by (8.76), as follows:

$$\tau \rightarrow -\frac{1}{\tau}, \quad \tau \rightarrow \tau + 1 \quad (8.110)$$

This is a generalization of the usual $\text{U}(1)$ duality that occurs with electromagnetism to the case of a complex coupling. Recall the usual electromagnetic duality for Maxwell theory in the presence of charged matter is:

$$E \rightarrow B, B \rightarrow -E, e \rightarrow m, m \rightarrow -e. \quad (8.111)$$

This generalizes to a $\text{U}(1)$ symmetry by defining:

$$E + iB, e + im. \quad (8.112)$$

The duality symmetry now acts by:

$$E + iB \rightarrow \exp(i\alpha) (E + iB), e + im \rightarrow \exp(i\alpha) (e + im). \quad (8.113)$$

For the $SU(2)$ case the moduli are given by $u = \text{Tr}\varphi^2$, for $SU(N_C)$ the moduli are given by $u_k = \text{Tr}\varphi^k$, $k = 2, \dots, N_C$. The classical moduli space is singular at times, there are no perturbative or nonperturbative corrections.

The dependence of τ on the moduli coordinate u was found. The vast number of results and literature on this will not be described here. Briefly:

The following complex equation,

$$y^2 = ax^3 + bx^2 + cx + d \quad (8.114)$$

determines a torus. The complex structure of the torus, τ_{torus} will be identified with the complex coupling τ_{eff} . a, b, c, d are known holomorphic functions of the moduli, couplings, and scale, and so will implicitly determine τ_{torus} .

When $y(x)$ and $y'(x)$ vanish, for some value of x , τ is singular. Therefore,

$$\tau_{\text{eff}} = i\infty, g_{\text{eff}}^2 = 0 \quad (8.115)$$

and the effective coupling vanishes. This reflects the presence of massless charged objects. This occurs for definite values of u in the moduli space. These new massless particles are monopoles or dyons. The theory is on the verge of confinement. For $N = 2$ supersymmetry that is the best one can do. The monopoles are massless but they have not condensed. For condensation to occur the monopoles should become tachyonic indicating an instability that produces a condensation. One can push this to confinement by adding a mass term: $\tilde{m}\text{Tr}\varphi^2$, or generally for $SU(N_C)$ the term:

$$\delta W = g_k u_k. \quad (8.116)$$

This breaks $N = 2$ supersymmetry down to $N = 1$. The effective prepotential is now:

$$W = M(u_k) q\tilde{q} + g_k u_k \quad (8.117)$$

then

$$\frac{\partial W}{\partial u_k} = 0, \quad \frac{\partial W}{\partial (q\tilde{q})} = 0 \Rightarrow M(<u_k>) = 0, \quad \partial_{u_k} M(<u_k>) <q\tilde{q}> = -g_k \quad (8.118)$$

Since generically,

$$\partial_{u_k} M(<u_k>) \neq 0 \quad (8.119)$$

There will be condensation of the magnetic charge, confinement has been demonstrated to be indeed driven by monopole condensation. A monopole is usually a very heavy collective excitation. It is only the large amount of SUSY which allows one to follow the monopole as it becomes massless and even condenses.

8.2.6 *Dynamics of SUSY Gauge Theories with $N > 1$ SUSY*

8.2.6.1 $N = 4$ Supersymmetry

In the presence of this large amount of supersymmetry in four space-time dimensions the particle content was described in an earlier section. It consists of spin one, spin one half and spin zero particles. The particles are all in the adjoint representation of the gauge group. They fall into representations of the $SU(4)$ global symmetry group as well. The full Lagrangian is fully dictated by the symmetry. The large symmetry leads to several properties which can be demonstrated.

- The theory is scale invariant quantum mechanically. This was shown to all orders in perturbation theory as well as non-perturbatively. This served as an example of non-trivial four dimensional scale invariant theories.
- The theory has thus a meaningful coupling constant on which the physics truly depends (unlike massless QCD in which dimensionless quantities do not depend on the coupling). Moreover the coupling can be complexified by adding the θ parameter. (In the absence of a chiral anomaly the theory truly depends also on θ even though massless fermions are present). The theory is invariant under the modular group $SL(2, Z)$, this group relates in particular small and large values of the coupling.
- The theory has flat directions along the scalar fields for any value of the coupling. The different points along the flat directions are not generically related by any symmetry. Each point characterizes a different vacuum choice for the system. These different vacua are called moduli. There is a special point in the moduli space and that is the point at the origin of field space where all scalar fields obtain a zero expectation value. At that point the theory is realized in a scale invariant manner. The massless fields rendering the theory with a rather complex analytic structure. Choosing different vacua along the moduli space leads to different residual gauge symmetries, as the scalar fields are in the adjoint representations the residual gauge group is at least $U(1)^r$ where r is the rank of the group. In each of these vacua the scale symmetry is spontaneously broken leading to a presence of a dilaton, the Goldstone Boson of broken scale invariance, in the spectrum. The vacuum energy is the same in all the phases associated with the different vacua choice. It has no dependence on any of the expectation values of the scalar fields. The spectrum includes massless and massive gauge particles.
- In the broken phases of the theory the conditions are appropriate for the presence of various solitons in the system. In general they contain particles which

have both electric and magnetic charges, called dyons. The massess of many of these particles, called BPS states is protected by the large SUSY to the extent that the mass dependence on the coupling is exactly known. This has applications in the counting microscopically the entropy of some black holes.

- Moreover, a hidden wish of theoreticians is that not only will one theory describe all physical phenomena but that that theory be exactly solvable. Given the complexity of four dimensional field theory this hope was suppressed. It had resurfaced when it was uncovered that quite a few properties of the $N = 4$ theory are exactly calculable. The system seems to have a large number of conserved quantities and this results in many so called integrability features.
- As will be discussed in the section on string theory, there is a large body of evidence that $N = 4$ theory encodes in it the information of special string theories which include gravity and black holes. These are string theories for strings propagation on a manifold part of which has a negative curvature and a negative cosmological constant.

To conclude, supersymmetric gauge theories have a very rich phase structure and many outstanding dynamical issues can be discussed reliably in the supersymmetric arena that are hard to address elsewhere.

8.2.7 *Gauging Supersymmetry*

During most of the second part of the twentieth century local symmetries were at center stage. The model unifying the electromagnetic and weak interactions actually united them mainly by using the concept that gauge theories do describe both. The stronger unification using only one non-semi simple group is yet to be achieved. The gauge theories have allowed to make precise calculable predictions for experimentally measurable quantities. Yet in the last year of that century some scientists suggested to move on and accept that gauge symmetries are simply redundant descriptions (choosing not to emphasize that this description allows for a local description of the theory). Be that as it may, at the time it was natural to promote global symmetry into a local symmetry. The result was very rewarding, it turned out that the “gauge particle” of local supersymmetry is a massless fermion of the spin $3/2$, called the gravitino, whose partner in the same supermultiplet is a massless spin two particle, the graviton. Local supersymmetry led to general coordinate invariance and the presence of gravity. This is called supergravity (SUGRA).

Lagrangians invariant under this local symmetry were found. Writing them down required even more efforts than those needed for the global supersymmetry case. Nevertheless this was achieved and a superfield notation was discovered as well. The Lagrangians are rather lengthy and we do not display them here. The system had additional interesting features.

- There is a Higgs like feature for such systems. The massless gravitino became massive when spontaneous breaking of supersymmetry occurred. The would-be goldstino became part of the massive gravitino. This may resolve the issue of the missing Goldstino.
- SUSY may persist in the presence of a negatively valued cosmological constant. Thus the spontaneous breaking of supersymmetry may be fine tuned so as to obtain a zero value or very small value for the cosmological constant. This is done by balancing the positive vacuum energy resulting from breaking supersymmetry against the value of the negative cosmological constant.
- The presence of gravity renders the Lagrangians to be superficially non-renormalizable. In fact in these theories the ultra violet divergences are much less severe than what would be expected by power counting. In the presence of local SUSY the supersymmetry can be enhanced up to $N = 8$ in four dimensions. Such a theory is intimately related to a ten dimensional theory with $N = 1$ supersymmetry in ten dimensions. Some scientists have the hope that this very special theory is in fact finite. Time will tell.
- Once global SUSY is embedded in supergravity one can imagine also terms which softly break supersymmetry and do not result from spontaneous breaking, i.e. one can add relevant terms to a Lagrangian describing for low energies a systems of particles containing superparticles with non-supersymmetric masses and interactions.

8.2.8 *The Hierarchy Problem*

SUSY was uncovered on the route of circumventing the no go theorem concerning the unification of a internal and space time symmetries into a unified group as well as attempting to find a symmetry which relates the different couplings in a rather general Lagrangian involving both bosons and fermions. It was taken up again when it was decided to declare the very large value of the ratio of the planck scale and the weak interaction scale as a problem. On a more technical level the problem was stated as follows. Consider a theory with an interacting scalar particle. Notice that to this date no fundamental spin zero elementary particles were observed, the discovery of an elementary spin zero Higgs particle would change this unfamiliar situation, a situation in which the simplest realization of a symmetry is not manifested in nature on an elementary level. No matter what the original mass of the scalar is, the interactions shift the mass. The mass shift is divergent as is the case for renormalizable theories. However according to the renormalization group ideas one always encodes the present knowledge valid up to an energy scale Λ in an effective low energy theory. In that case the mass shift is proportional to the cutoff Λ . As long as there is no physical scale near that of the weak interaction's scale of 1 TeV one needs to fine tune the input initial mass to obtain a Higgs mass of the weak interaction scale. This is to be contrasted with the case of a fermion mass. In that case the fermion mass is shifted by the interactions by an amount which is

proportional only to the logarithm of the cut off and is proportional to the initial mass of the fermion. The shift vanishes when the initial fermion mass vanishes. The reason behind the vanishing of the mass of the fermion is that for a massless fermion the system obtains a new symmetry, chiral symmetry. This symmetry should be restored in the zero mass limit. As long as that symmetry is not broken the mass remains zero. If the symmetry is dynamically broken the mass shift can be very small. In the case of supersymmetrical systems it is the fermionic nature which prevails and thus the bosonic mass shifts are small, in a supersymmetric theory one may imagine that the hierarchy problem is solved. (The softer divergences of the supersymmetric systems are of similar origin as those responsible for the no renormalization theorems described before) The problem *ab initio* is a problem which involves in some manner theoretical taste and in any case it seems from the LEP data that if one insists on a hierarchy problem it is already present. SUSY has not come to the rescue in time and if it is a symmetry of nature, solving the hierarchy problem may well not be its main purpose.

An earlier attempt to solve the hierarchy problem involved the introduction of a new gauge symmetry similar in many ways to color called technicolor. This is a specific realization of the idea that the Higgs particle is not a fundamental particle. Indeed both in describing aspects of superconductivity and superfluidity the Higgs scalar is only an effective degree of freedom. As of 2007 these line of ideas were not consistent with some of the experimental data. An often quoted problem is that such an interaction induces flavor changing neutral interactions at a too high rate.

8.2.9 *Effective Theories*

One may wonder why the Lagrangians describing the basic interactions are expressed in terms of a finite number of terms rather than by an infinite one. As long as the laws of physics allow the decoupling of far away scales this can be explained. The theory is written down in full generality in the presence of a short distance/high energy cutoff. The cutoff Λ may reflect the scale below which one has no knowledge on the interactions. The terms are constrained only by possible symmetries, their number is *a priori* infinite. The physics beneath any lower energy scale, Λ' is obtained by integrating out all the degrees of freedom which are heavier than Λ' . A new set of terms replaces the original set, it contains generically less terms. This process can be repeated till a theory approaches a critical surface. The theory on the critical surface is scale invariant. The resulting theory near the critical surface is well described by operators whose scaling dimensions, in d space-time dimensions, can be determined near the surface and happen to be smaller and at most not much larger than d . In many cases there is only a finite number of such operators, i.e., only a finite amount of terms in the Lagrangian describe the physics near the critical surface. This not only explains the concise form of the Lagrangian but provides one with a systematic method to classify the allowed terms to appear, the power of the method is further enhanced when symmetries are present, as those

constrain further the allowed terms. This can also sometimes be turned around. The collection of marginal and relevant operators may exhibit a symmetry of its own. This offers a proof that the symmetry should be present at low energies but may well disappear at higher energies if the irrelevant operators do not respect it. The resulting Lagrangian is called the effective low energy Lagrangian. It should contain all the light particles of the system and all the symmetries, may they be realized linearly or nonlinearly, (when the symmetry at question is spontaneously broken). Integrating out heavy particles results in a local Lagrangian consisting of a finite number of terms. Integrating out light degrees of freedom is likely to lead to non-local effects. The terms whose quantum scaling dimensions are smaller than d are called relevant terms. They become very large as the system is probed at lower energies and disappear at high energies, their number is usually finite and in many cases small. Examples of such terms are mass terms for both bosons and fermions in general and the non-Abelian Maxwell term for asymptotically free systems such as QCD. The terms whose scaling dimensions are exactly d are called marginal operators, they include the full Lagrangian for exactly scale invariant systems. Their number is also generically finite. They are equally important at all scales. Terms whose dimension is larger than d are called irrelevant operators. There is an infinite number of them. Each term on its own becomes insignificant in the low energy region and renders the theory non-renormalizable for high energies. (One can imagine examples where an infinite number of such terms collaborate to become relevant but that would most likely mean that the expansion around the critical surface should be modified). Those terms which have scaling dimensions not very much larger than d (such as five and six in four dimensions), can be useful hints for the scale at which the physics needs to be modified. For example the original four Fermi interactions are dimension six operators in four space-time dimensions. Their coefficient, the so called Fermi coupling, has dimension (-2) and hints at the nearly TeV scale of the weak interactions. Indeed at higher energies this irrelevant term is replaced by the classically marginal gauge interactions of the standard model. The replacement of an irrelevant operator by a relevant (or marginal) one at high energy leads to a well defined theory and it called UV completion. The UV completion is not unique, but its simplest version seems to do the job in the weak interactions case. This is clearly to be done when, like for the weak interactions in the 1950s, the leading term in the effective Lagrangian is irrelevant. When the irrelevant term appears in addition to marginal and relevant ones it may or may not indicate new physics.

8.2.10 *MSSM Lagrangian*

All this said one can write down the simplest low energy theory that by definition contains only marginal and relevant operators. For the case of systems with broken SUSY which still do not have a hierarchy problem one may allow in addition only those of the above operators which retain the at most logarithmic divergence structure of the theory. This requires that the classical marginal operators are all

Table 8.1 Internal quantum numbers of the Higgs superfields and one generation of matter superfields comprising the MSSM model.

Field	SU(3) _c	SU(2) _L	U(1) _Y
$\hat{L} = \begin{pmatrix} \hat{\nu}_{eL} \\ \hat{e}_L \end{pmatrix}$	1	2	−1
$\hat{E}^c$	1	1	2
$\hat{Q} = \begin{pmatrix} \hat{u}_L \\ \hat{d}_L \end{pmatrix}$	3	2	$\frac{1}{3}$
$\hat{U}^c$	3 _*	1	− $\frac{4}{3}$
$\hat{D}^c$	3 _*	1	$\frac{2}{3}$
$\hat{H}_u = \begin{pmatrix} \hat{h}_u^+ \\ \hat{h}_u^0 \end{pmatrix}$	1	2	1
$\hat{H}_d = \begin{pmatrix} \hat{h}_d^- \\ \hat{h}_d^0 \end{pmatrix}$	1	2 _*	−1

supersymmetric as well as some of the classical dimension three operators. The dimension two relevant operators, namely the bosonic mass terms may break SUSY as long as the classically marginal terms are supersymmetric. The, so far, simplest model imposes minimality. It contains the Standard Model particles and interactions and their minimal extensions. Each Standard model particle is accompanied by a superpartner. Only one new supersymmetric multiplet is added in which neither the bosons nor the fermions are part of the Standard Model, this is a second Higgs field. The interactions are minimally extended to be N = 1 supersymmetric. The system resulting for this construction is called the Minimal Super Symmetric Model (MSSM). The number of resulting terms may be minimal but it can hardly be considered as small, in fact it has 178 parameters. The superpotential in the MSSM contains the following terms:

$$\hat{f} = \mu \hat{H}_u^a \hat{H}_{da} + \sum_{i,j=1,3} \left[(f_u)_{ij} \epsilon_{ab} \hat{Q}_i^a \hat{H}_u^b \hat{U}_j^c + (f_d)_{ij} \hat{Q}_i^a \hat{H}_{da} \hat{D}_j^c + (f_e)_{ij} \hat{L}_i^a \hat{H}_{da} \hat{E}_j^c \right]. \quad (8.120)$$

The indices of SU(2)_L doublets are denoted above as *a* and *b*. The $(f_k)_{ij}$ are the appropriate Yukawa couplings. The Quarks, Leptons and Higgs particles are all chosen to be chiral fields and the notation, for one generation, is made explicit in Table 8.1.

The terms above were constructed to be invariant under the standard model symmetries, they turn out to conserve also both baryon (*B*) and lepton (*L*) numbers. However, the terms listed below are marginal and relevant terms which also conserve the standard model symmetries but do not conserve either *B* or *L*. The generic amount of violation induced by these terms is not consistent with the experimental data. Thus in MSSM one requires both *B* and *L* conservation. By the no renormalization theorems the symmetry will be respected to all orders in perturbation theory. Such symmetries which are preserved quantum mechanically in perturbation theory, once imposed classically, are denoted as natural in a “technical”

sense.

$$\hat{f}_L = \sum_{ijk} \left[\lambda_{ijk} \epsilon_{ab} \hat{L}_i^a \hat{L}_j^b \hat{E}_k^c + \lambda'_{ijk} \epsilon_{ab} \hat{L}_i^a \hat{Q}_j^b \hat{D}_k^c \right] + \sum_i \mu'_i \epsilon_{ab} \hat{L}_i^a \hat{H}_u^b. \quad (8.121)$$

$$\hat{f}_B = \sum_{ijk} \lambda''_{ijk} \hat{U}_i^c \hat{D}_j^c \hat{D}_k^c. \quad (8.122)$$

The B and L symmetries are in any case not respected by non-perturbative effects. The above mentioned terms can also be forbidden by imposing a different global symmetry called R -parity which was already mentioned in the context of SUSY breaking. It is defined here as:

$$R = (-)^{3(B-L)+2s}, \quad (8.123)$$

s denoting the spin of the particle. The standard model particles are even under the R -parity, while their superpartners are odd under it. In MSSM the bosonic partners of the standard model matter fermions carry non-zero L and B quantum numbers. The (non)conservation of B and L symmetries are not in general correlated. They happen to coincide on the terms disallowed above. For example, the terms below both respect R -parity but the first violates L number conservation and the second does not respect B number conservation.

$$\epsilon_{ab} \hat{L}^a \hat{H}_u^b \epsilon_{cd} \hat{L}^c \hat{H}_u^d \quad (8.124)$$

and

$$\hat{U}_c \hat{U}_c \hat{D}_c \hat{E}_c \quad (8.125)$$

For the record we write down all the dimension three and dimension two operators which softly break SUSY in the MSSM model. i and j run over the generations and are summed over as are the doublet $SU(2)$ indices a and b .

$$\begin{aligned} \mathcal{L}_{soft} = & - \left[\tilde{Q}_i^\dagger m_{Q_{ij}}^2 \tilde{Q}_j + \tilde{d}_{R_i}^\dagger m_{D_{ij}}^2 \tilde{d}_{R_j} + \tilde{u}_{R_i}^\dagger m_{U_{ij}}^2 \tilde{u}_{R_j} \right. \\ & + \tilde{L}_i^\dagger m_{L_{ij}}^2 \tilde{L}_j + \tilde{e}_{R_i}^\dagger m_{E_{ij}}^2 \tilde{e}_{R_j} + m_{H_u}^2 |H_u|^2 + m_{H_d}^2 |H_d|^2 \Big] \\ & - \frac{1}{2} \left[M_1 \bar{\lambda}_0 \lambda_0 + M_2 \bar{\lambda}_A \lambda_A + M_3 \bar{\tilde{g}}_B \tilde{g}_B \right] \\ & - \frac{i}{2} \left[M'_1 \bar{\lambda}_0 \gamma_5 \lambda_0 + M'_2 \bar{\lambda}_A \gamma_5 \lambda_A + M'_3 \bar{\tilde{g}}_B \gamma_5 \tilde{g}_B \right] \\ & + \left[(a_u)_{ij} \epsilon_{ab} \tilde{Q}_i^a H_u^b \tilde{u}_{R_j}^\dagger + (a_d)_{ij} \tilde{Q}_i^a H_{da} \tilde{d}_{R_j}^\dagger + (a_e)_{ij} \tilde{L}_i^a H_{da} \tilde{e}_{R_j}^\dagger + h.c. \right] \\ & + \left[(c_u)_{ij} \epsilon_{ab} \tilde{Q}_i^a H_d^{*b} \tilde{u}_{R_j}^\dagger + (c_d)_{ij} \tilde{Q}_i^a H_{ua}^* \tilde{d}_{R_j}^\dagger + (c_e)_{ij} \tilde{L}_i^a H_{ua}^* \tilde{e}_{R_j}^\dagger + h.c. \right] \\ & + \left[b H_d^a H_{da} + h.c. \right]. \end{aligned} \quad (8.126)$$

The model, as written, has 178 independent parameters. Their number could be reduced if they had their origin in an underlying microscopic theory. In some present versions the matrices of parameters denoted above by c_{ij} are set to zero thus reducing by fiat the number of free parameters to 124. This is but the tip of an iceberg, one proceeds from versions of the MSSM and derives the mass spectrum of the supersymmetric particles while ensuring to preserve the known properties of the standard model particles as well as a variety of experimental bounds. The bounds range from cosmological ones to bounds on rare decays. We hope that in the not too distant future the fog will disperse and one will be able to write a rather concise item in a physics encyclopedia which would select the relevant physical components of this effort. As mentioned any model for SUSY breaking should respect present experimental constraints. Many models of spontaneous SUSY breaking fail to do this as a result it was suggested to add to the yet to be seen superpartners of the standard model particles also a hidden sector. In such models SUSY is to be broken in the hidden sector and its effects are supposed to be mediated to the “seen” sector. The agents or messengers which couple the two sectors vary. In some models the coupling is by the gravitational force at tree level, in others the coupling occurs first only at the one loop level and is called anomaly (Weyl anomaly) mediated. There are models in which the coupling is done through gauge interactions, they have a lower energy scale than the gravity mediated interactions. Each of these models has some advantages as well as disadvantages and are at this stage an active area of research.

We will end the section with a short glossary of terms currently used in describing MSSM features which were not mentioned above.

Short Glossary:

- **Chargino**—Charged supersymmetric partner of a charged standard model particle.
- **Gaugino**—Spin one half supersymmetric partner of a standard model gauge particle.
- **LSP**—Lightest supersymmetric partner of a standard model particle. It is long lived in many models.
- **μ term**—Term coupling the two different Higgs chiral supermultiplets.
- **Neutralino**—Neutral supersymmetric partner of a neutral standard model particle.
- **squark**—Spin zero supersymmetric partner of a standard model quark.
- **slepton**—Spin zero supersymmetric partner of a standard model lepton.
- **$\tan(\beta)$** —Ratio of the expectation values of the two Higgs fields.

8.3 Unification

8.3.1 Gauge Group Unification [2]

The standard model is unified along the lines that all of its components, the colour, weak and electromagnetic interactions are described by gauge theories. They are $SU(3) * SU(2) * U(1)$. The gluons and the photons, the electrons and the quarks belong to different representations of the product gauge group, it is natural to attempt to have all of the elementary particles and interactions as a single representation of a single gauge group. Intermediate algebraic solutions to this problem have been found. For example the gauge group $SU(5)$ does very economically unify the known gauge groups. Moreover it predicts that quarks and leptons can transmute into each other, in particular violating Baryon number conservation. The proton could decay in such models for example into a positron and a neutral pion. The original estimates of the half life time of the proton placed it possibly around 10^{31} years. That prediction was within experimental reach and initiated the construction of very ingenious experiments. Proton decay at that rate was not found, the lower bound on the proton life time was improved to be 10^{35} , thus invalidating the simplest version of the grand unified group $SU(5)$. In that version the particles did not all belong to the same representation as one may have wished based on esthetics. There have been many other attempts since to find an appropriate unifying group these included $SO(10)$ (which incorporated naturally a right handed $SU(2)$ singlet neutrino) and exceptional groups such as $E(6)$. This was done with and without SUSY. Using the renormalization group it was found that in the presence of SUSY the interactions may indeed unify in magnitude at a high energy not much below the Planck scale of 10^{19} GeV. At such scales it becomes difficult to ignore quantum gravity. In any case we will not review this vast subject further here.

8.3.2 Extra Dimensions and Unification [3]

The first ideas of unification by increasing the number of dimensions were suggested in classical field theory by Kaluza and Klein (KK) in the period of the 1920s. At the time there were two well known interactions, gravity and electromagnetism. KK suggested that space time is five dimensional rather than four dimensional and that there is but one fundamental interaction-gravity. In order to be consistent with observations it was suggested that the five dimensional space time is composed out of a fifth dimension which is a spatial circle of inverse radius m , and the usual four dimensional Minkowski component. The radius $1/m$ should be small enough to have not been observed yet. The resulting low energy (i.e. energies much lower than m) five dimensional Lagrangian decomposes into several four dimensional Lagrangians. They describe, four dimensional gravity, four dimensional Maxwell electrodynamics and the coupling of a neutral spin zero additional particle. The

symmetry group of the low energy Lagrangian consists of four dimensional general coordinate invariance as well as a U(1) four dimensional gauge symmetry. This result is obtained in the following manner [3].

The basic five dimensional gravitational action is thus given by:

$$\hat{S} = \frac{1}{2\hat{\kappa}^2} \int d^5\hat{x} \sqrt{-\hat{g}} \hat{R} \quad (8.127)$$

With $\hat{\kappa}^2$ being the five dimensional Newton constant. The action $\hat{S}$ is invariant under the fivedimensional general coordinate transformations

$$\delta \hat{g}_{\hat{\mu}\hat{\nu}} = \partial_{\hat{\mu}} \hat{\xi}^{\hat{\rho}} \hat{g}_{\hat{\rho}\hat{\nu}} + \partial_{\hat{\nu}} \hat{\xi}^{\hat{\rho}} \hat{g}_{\hat{\rho}\hat{\mu}} + \hat{\xi}^{\hat{\rho}} \partial_{\hat{\rho}} \hat{g}_{\hat{\mu}\hat{\nu}} \quad (8.128)$$

A useful 4 + 1 dimensional ansatz was:

$$\hat{g}_{\hat{\mu}\hat{\nu}} = e^{\phi/\sqrt{3}} \begin{pmatrix} g_{\mu\nu} + e^{-\sqrt{3}\phi} A_{\mu} A_{\nu} & e^{-\sqrt{3}\phi} A_{\mu} \\ e^{-\sqrt{3}\phi} A_{\nu} & e^{-\sqrt{3}\phi} \end{pmatrix}. \quad (8.129)$$

The fields depend on the five dimensional coordinate $\hat{x}^{\hat{\mu}}$, which were written as: $\hat{x}^{\hat{\mu}} = (x^{\mu}, y)$, $\mu = 0, 1, 2, 3$, and all unhatted quantities are four-dimensional. The fields $g_{\mu\nu}(x)$, $A_{\mu}(x)$ and $\phi(x)$ are the spin 2 graviton, the spin 1 photon and the spin 0 dilaton respectively. The fields $g_{\mu\nu}(x, y)$, $A_{\mu}(x, y)$ and $\phi(x, y)$ may be expanded in the form

$$\begin{aligned} g_{\mu\nu}(x, y) &= \sum_{n=-\infty}^{n=\infty} g_{\mu\nu n}(x) e^{inmy}, \\ A_{\mu}(x, y) &= \sum_{n=-\infty}^{n=\infty} A_{\mu n}(x) e^{inmy}, \\ \phi(x, y) &= \sum_{n=-\infty}^{n=\infty} \phi_n e^{inmy} \end{aligned} \quad (8.130)$$

Recall that m is the inverse radius in the fifth dimension.

In general the five dimensional theory can be described in terms of an infinite number of four dimensional fields. It also has an infinite number of four-dimensional symmetries appearing the Fourier expansion of the five dimensional general coordinate parameter $\hat{\xi}^{\hat{\mu}}(x, y)$

$$\begin{aligned} \hat{\xi}^{\mu}(x, y) &= \sum_{n=-\infty}^{n=\infty} \xi^{\mu}_n(x) e^{inmy}, \\ \hat{\xi}^4(x, y) &= \sum_{n=-\infty}^{n=\infty} \xi^4_n(x) e^{inmy}. \end{aligned} \quad (8.131)$$

However, at energy scales much smaller than m only the $n = 0$ modes in the above sums enter the low effective low energy action.

The $n = 0$ modes in (8.130) are just the four dimensional graviton, photon and dilaton. Substituting (8.129) and (8.130) in the action (8.127), integrating over y and retaining just the $n = 0$ terms one obtains (dropping the 0 subscripts)

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} \left[R - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{4} e^{-\sqrt{3}\phi} F_{\mu\nu} F^{\mu\nu} \right] \quad (8.132)$$

where $2\pi\kappa^2 = m\kappa^{\wedge 2}$ and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. From (8.128), this action is invariant under general.

coordinate transformations with parameter ξ^μ_0 , i.e. (again dropping the 0 subscripts)

$$\begin{aligned} \delta g_{\mu\nu} &= \partial_\mu \xi^\rho g_{\rho\nu} + \partial_\nu \xi^\rho g_{\mu\rho} + \xi^\rho \partial_\rho g_{\mu\nu} \\ \delta A_\mu &= \partial_\mu \xi^\rho A_\rho + \xi^\rho \partial_\rho A_\mu \\ \delta \phi &= \xi^\rho \partial_\rho \phi, \end{aligned} \quad (8.133)$$

local gauge transformations with parameter ξ^4_0

$$\delta A_\mu = \partial_\mu \xi^4 \quad (8.134)$$

and global scale transformations with parameter λ

$$\delta A_\mu = \lambda A_\mu, \delta \phi = -2\lambda/\sqrt{3} \quad (8.135)$$

The symmetry of a vacuum, determined by the VEVs

$$\langle g_{\mu\nu} \rangle = \eta_{\mu\nu}, \langle A_\mu \rangle = 0, \langle \phi \rangle = \varphi_0 \quad (8.136)$$

is the four-dimensional Poincare group $\times R$. These expectation values have not been determined dynamically but let us settle here for them. At this level of the analysis the masslessness of the graviton is due to unbroken four dimensional general covariance, the masslessness of the photon is consistent with the four dimensional gauge invariance and the dilaton seems massless because it is the Goldstone boson associated with the spontaneous breakdown of the global scale invariance. In fact taking into account the actual periodicity in y which is not manifested for only the $n = 0$ low energy modes the symmetry acting on the scalar field is $U(1)$ rather than R and is thus not a true scale symmetry. The field φ_0 is a pseudo-Goldstone boson and does not really deserve to be called a dilaton. Further analysis uncovers an infinite tower of charged, massive spin 2 particles with charges e_n and masses m_n given by

$$e_n = n\sqrt{2}km, m_n = |n| m \quad (8.137)$$

Thus the KK ideas provide an explanation of the quantization of electric charge. (Note also that charge conjugation is the parity transformation $y \rightarrow -y$ in the fifth dimension.) If one indeed identifies the fundamental unit of charge $e = \sqrt{2}km$ with the charge on the electron, then one is forced to take m to be very large: the Planck mass 10^{19} GeV. Such extra scalar particles seem to be present abundantly in string theories as well. The experimental question was mainly to set bounds on the value of the radius $R(= 1/m)$ of the fifth dimension. The limits are obtained from precision electroweak experiments and require $m > 7\text{TeV}$. This is for models with one extra dimension, in which the gauge bosons propagate in the bulk but the fermions and Higgs are confined to four dimensions. More on this shortly. There are also bounds from astrophysics they are not model independent bounds. They are important for large dimensions in which only the graviton propagates, and they depend on number of large dimensions. The strongest bounds arise from supernova emissions, giving $1/m < 10^{-4}$ mm for the case of two large extra dimensions. Although researchers such as Einstein and Pauli had studied such theories for dozens of years this path has been abandoned. Extra dimensions resurfaced when string theory was formulated. It was found out that strings are much fussier than particles, (super) strings can propagate quantum mechanically only in a limited number of dimensions. In fact, not only was the number of allowed possible dimensions dramatically reduced, the allowed values of the number of space-time dimensions did not include the value 4. These dimensions are usually required to be 10 or 26 (The origin of this basic number is, at this stage, disappointingly technical.) In fact while the numbers 10 and 26 result from the theory they need not always be related directly to extra dimensions. When the extra dimensions emerged in string theory it was suggested in the spirit of KK that any extra dimensions are very small. As interest was diverted from string theory back to field theory the ideas of KK were revised allowing one to take into account the extra interactions discovered since the original work of KK. It turned out that if one which to trade all the known standard model gauge interactions, i.e. the all $SU(3) * SU(2) * U(1)$ interactions, for a theory of gravity alone one was led to consider eleven dimensional gravity. Such theories raised interest also for other reasons. The return of string theory brought back with it an intense study of extra dimensions. In particular extra dimensions in the form of compact Calabi-Yau manifolds which allowed to have four dimensional effective theories with $N = 1$ SUSY. The topology of the extra dimensions determined in some cases the number of zero modes on them, that is the spectrum of massless particles. The number of generations of particles for example could be correlated thus to the topology of those extra dimensions offering a solution to the origin of the repetitive structure of the elementary particles. A vast amount of research on the possible extra dimensions is ongoing. There have also been attempts to understand dynamically the origin of the difference between the four large extra dimensions and the rest. Some efforts were directed to explain how a spontaneous breaking of space time symmetries could lead to such asymmetry in the properties of the different dimensions. Solid state systems such as liquid crystals also exhibit such differences. Other efforts focused on determining why an expanding universe would expand asymmetrically after a while only in four directions. Another development occurred

following the realization that string theory in particular allows what are called brane configurations. Branes are solitons, extended objects embedded stably in a space time of larger dimension. Vortices and magnetic monopoles were mentioned as such lower dimensional objects. There is a consistent possibility that for example a four dimensional universe can be embedded in higher dimensions. The known gauge interactions living only on the brane while gravity extends to the full space. If our universe has this structure many things can be explained and in particular this has given rise to the possibility the extra dimensions could be much larger than previously expected. Actually they could extend up to the submicron region. This could lead to measurable deviations from Newton's gravity at those distance scales. Astrophysics gives upper bounds on how large such extra dimensions may be but in any case this is a very significant relaxation of a bound, in fact I am not aware of any bound on such a fundamental quantity in physics that has been altered to such an extent by a theoretical idea. All this said one should stress the obvious, also the bounds have been relaxed the true value of the extra dimensions may still be very small, perhaps even Planckian. Some suggestions of larger extra dimensions could well be tested at the LHC.

The fact that unification may occur not far from the planck scale brings quantum gravity to the front row and with it a theory which attempts to be able to indeed tame quantum gravity that is string theory.

8.4 String Theory [4]

8.4.1 *No NOH Principle*

String Theory is at this time far from being a complete theoretical framework, not to mention a phenomenological theory. That said, the theory of extended objects has evolved significantly and has shaped and was shaped by aspects of modern mathematics. In my opinion is it appropriate to highlight the qualitative aspects of string theory and the ideas behind it. This choice is not made for lack of formulas or precision in string theory. An essential catalyst in the process of the formation of string theory was urgency, the urgency created out of the near despair to understand the amazing novel features of the hadronic interactions as they unfolded in the 1960's. String theory was revitalized in the latter part of the twentieth century by the urgency to understand together all of the four known basic interactions. Here I shall briefly review some of the motivation to study a theory of extended objects and discuss the challenges string theory faces, the successes it has had, and the magic spell it casts. We start by reviewing a hardware issue, the hard wiring of some scientific minds. Researchers using string theory are faithful followers of an ancient practice, that of ignoring the NOH principle. The NOH principle is very generic, it states that Nobody Owes Humanity neither a concise one page description of all the basic forces that govern nature nor a concise description of all the constituents of

matter. Actually no reductionist type description is owed. Time and again physicists driven by their hard wiring processed the experimental data utilizing theoretical frameworks and were able to come up with explicit formulae; formulae that were able to express on less than one page the essence of a basic force. A key assumption used and vindicated was that the basic constituents of nature are point-like. The electromagnetic forces, the weak interactions and even the powerful color forces were rounded up one by one, straddled by the rules of quantum mechanics and then exhibited on a fraction of a page. Using these formulae, an accomplished student abandoned on an isolated island can predict with an amazing accuracy the outcome of some very important experiments involving the basic forces. The level of accuracy ranges from a few percent in the case of some aspects of the color forces to better than 10^{-10} for some features of the electromagnetic interactions.

8.4.2 Why Change a Winning Team: Extended Constituents Are Called Upon to Replace Point-like Ones

Point particles have done a tremendous job in shouldering the standard model of particle interactions. Why replace them by extended objects? In the section on SUSY we have outlined several of the reasons for that. The standard model does reflect the enormous progress made in the understanding of particle physics but it is not perfect, at least as seen by a critical theorist through his TPs. The model is afflicted with tens of parameters as well as what are termed naturality problems. The values of some physical quantities, such as the mass of the Higgs particle, are required to be much lighter than the theory would naturally suggest. Ignoring the NOH principle, scientists find themselves once again on the path of searching for a more concise description. One effort was directed towards unifying the three interactions, thus following the conceptual unification of the electromagnetic and weak interactions, described each by a gauge theory. That direction led to possible unifications at scales not so distant from where quantum gravity effects are definitely supposed to become important. In fact, gravity, the first of the basic forces that was expressed by a mathematical formula, is the remaining unbridled known force which is considered currently as basic. Here the quantum breaking methods based on the concept of a basic point like structure have failed. Sometimes what is considered failure is also a measure of intellectual restlessness, still it made sense to search for alternatives. Actually even with no apparent failure lurking upon basic physics, a study of a theory of fundamentally extended objects is called for. After all, why should the basic constituents be just point like? String theory is a natural extension of the idea that the basic constituents are point-like, it is an investigation into the possibility that basic constituents are ab initio extended objects. The simplest such extended objects being one-dimensional, that is strings. In a sense one may regard also point particles as extended objects dressed by their interactions. The direct consequences in this case are however totally different. It turned out that

such a generalization did eventually reproduce many properties of point particle interactions and in addition enabled one to formulate what seems as a theory of gravity. This it should do, following the tradition of subjecting any new theory to the test of the correspondence principle. The theory is well defined up to several orders and perhaps to all orders in perturbation theory. The perturbations are small at the vicinity of a very small distance scale called the string scale, as small as that distance is, it is sometimes expected to be larger than the Planck length and thus is associated with lower energies than the Planck energy (it is related to the plank scale by the string interaction coupling). Interactions among strings are, to a large extent, softer and fuzzier than those among point particles. Recall that a large number of successes of particle physics consisted of explaining and predicting the physics of the various interactions at scales at which they were weakly coupled, otherwise, a large dose of symmetry was essential. This is perhaps the most significant achievement of string theory. It is also its major source of frustration, if the string scale is indeed of the order of 10^{-32} cm or even slightly larger, it is not known today how to verify, experimentally, any prediction resulting from perturbative string theory. It is very difficult to imagine, for example, how to set up an experiment measuring the differential cross section of graviton-graviton scattering at the string scale. We will return to this extremely important issue later on and continue to follow for a while the path of the theoreticians.

8.4.3 *New Questions*

The impact of scientific progress can be tested by the type of questions it makes us aware of, as well as by the answers it eventually provides for them. By studying string theory new questions do arise. Values taken for granted are subjected to query and downgraded to being parameters. String theory suggests questioning the values of some such physical quantities and in some cases offers scientific answers to the question. A prime example is that of the number of space-time dimensions. In a point like constituent theory there is a very large degree of theoretical freedom. As far as we know spin zero point particles, for example, could have propagated in any number of dimensions, their interactions would have been different depending on the dimension of space-time but nevertheless they would have been allowed. A theory of spin one half particles can turn out to be in consistent on certain type of manifolds but they do not restrict the dimensionality of the space in which they propagate. Strings are much fussier than particles, (super) strings can propagate quantum mechanically only in a limited number of dimensions. In fact, not only are the number of allowed possible dimensions dramatically reduced, the allowed values of the number of space-time dimensions do not include the value 4. These dimensions are usually required to be 10 or 26 depending on the amount of supersymmetry the string is endowed with. The origin of this basic number is, at this stage, disappointingly technical and one should note that though, strictly speaking, the number 26 does indeed appear in string theory, it is not

always possible to associate this number with the number of dimensions. In fact the numbers appearing are 15 and 26 and they reflect from one point of view a conformal anomaly in the theory describing first quantized string theory. In first quantized field theory of particles can be expressed as a sum over one dimensional field theories, i.e. quantum mechanics, of a particle moving in space time and interacting along its space time trajectory. The action of the quantum mechanical system is geometrical and describes in its simplest form the length of the particle's trajectory. The result should not depend on the parameterization of trajectory and this should be general coordinate invariant. In a more fancy manner the action describes a particle coupled to gravity evolving in one space time dimension. The generalization to a description of the motion of a one dimensional object, a string, follows. As the string is a one dimensional object, its motion in space time spans a two dimensional manifold. The theory describing its motion is that of a particle, coupled to gravity, moving in two space-time dimensions. The two dimensions are called the world sheet and the theory describing them using first quantization is called the World Sheet theory. The usual counting of quantum degrees of freedom leads to the conclusion that gravity has minus one degrees of freedom in both one and two dimensions. In other words the system is over constrained and will require a higher degree of symmetry to be consistent. In addition the two dimensional gravitational system has an anomaly which restricts the algebraic structure of the conformal system describes by the string. A motion of a (super) string in (10) 26 flat dimensions removes the anomaly. There are many other ways to remove it and perhaps many more to be discovered. Sometimes the background on which the string can propagate has no obvious geometrical description, just an algebraic one. The space in which the strings moves is called the Target Space. The interactions of strings are described by them parting or joining, this is described (in a Euclidean formulation) by a compact two dimensional surface with non-trivial topology. These manifolds are called Riemann manifolds and are distinct by their topology. The freely propagating string is described by a world sheet 2 sphere, the first interactions occur when the world sheet is a torus. The theory is defined perturbatively by summing with a certain weight over all such two dimensional Riemann surfaces. The non-perturbative structure is non-trivial but less understood at this stage. But setting that aside, one would like to detect extra dimensions experimentally. As the idea of having extra dimensions was raised originally in field theory, upper bounds on the length of such dimensions were already available, more recently under the security net of string theory these bounds have been revised (they have been relaxed by more than 10 orders of magnitude and brought up to the sub micron regime), in fact I am not aware of any bound on a fundamental quantity in physics that has been altered to such an extent by a theoretical idea. This fact about strings illustrates the tension inherent in attempts to search for experimental verification of extremely basic but very weakly coupled phenomena. Sitting in our chairs we sometimes forget how frail gravity is. It is all the planet earth that gravitationally pulls us towards its center, yet all this planetary effort is easily counterbalanced by the electromagnetic forces applied by the few tiles beneath our chair. Although string theory sets constraints on the possible number of dimensions, that is not to

say that point particles allow everything. We mentioned that spin one half particles can't be defined on all manifolds but the number of known restrictions is much smaller. The interactions could be of an infinite (sometimes classifiable) variety, the color group, which happens to be $SU(3)$, could have been for all the theory cares, $SU(641)$, the electromagnetic force could have been absent or there could have been 17 photons. Not everything is allowed, but an infinite amount of variations could have been realized instead of what one actually sees in nature today. The same goes for many (but not all) of the properties of the space-time arena in which all the interactions are occurring, in particular effective low energy theories (and one seems to be constrained to use only such theories for a long time to come) contain many more free parameters such as masses and some of the couplings. In a string theory one may have expected that all or most of these parameters are fixed in some manner. Actually is it more complicated, for some string backgrounds, some parameters are fixed, while other parameters are not. In addition there seemed to be several string theories leading to even more possibilities.

8.4.4 *The One and Only?*

As we have discussed the desire to find the one and only theory encompassing all fundamental physical phenomena seems hard wired in many scientific minds. Is string theory an example of such a theory? Before addressing this question let us reflect upon the limitations of the methods used. What would in fact satisfy the purest (or extremist) of reductionists? A one-symbol equation? Be her desires, what they may be, one should recall that one is using mathematics as our tether into the unknown. This tool, which has served science so well, has its own severe limitations, even ignoring the issue of using differential equations as tool to probe short distance, a generic problem in mathematics can't be solved, maybe the NOH principle will eventually have its day, one will run out of interesting questions which have answers, maybe that day is today! (Although some mathematicians suggest to use physicists as hound dogs that will sniff one's way towards interesting and solvable problems). Having this in mind the key question is posed in hope it does have an answer: find the unique theory describing the fundamental forces. There have been ups and downs for the proponents of the one and only string theory. The understanding of the situation has passed several evolutionary stages. From the start it was recognized that one may need to settle for more than only one string theory. It was thought that a distinction could be drawn between a theory of only closed strings and a theory containing both open and closed strings. A theory with only open strings was realized to be perturbatively inconsistent. Even those theories needed not only a fixed dimension but in addition an extra symmetry, supersymmetry, to keep them consistent. It was very quickly realized that actually there are an infinite varieties of backgrounds in which a string could move. For example, consider 26 dimensional spaces in which one dimension is actually a circle of radius R . It turned out that all possible values for the radius, R , of the circle are allowed. It was then realized that

Fig. 8.7 The potential, familiar from the standard model, has a shape of a sombrero. In the figure, its minima lie on a circle, each point along the circle, can be a basis for a ground state. The physics around each is equivalent. In string theory and in supersymmetric systems flat potential arise

Fig. 8.8 The potential has two orthogonal flat directions. The landscape of the valleys is generically more elaborate. Each point along the flat directions can serve as a basis of a ground state. In many cases. The physics around each ground state is different

there is a large variety of 22 dimensional compact manifolds in the case of bosonic strings (6 dimensional compact dimensions in the case of the supersymmetry), which could accompany the four dimensional Minkowski space-time one is familiar with. Each such compact manifold was called a string compactification. Next it was suggested that actually all possible compactifications are nothing but different solutions/ground states of a single string theory. Each of the solutions differing from each other by the detailed values of the physical parameters it leads to. But the ground states have also many common features, such as generically the same low energy gauge group. To appreciate this consider the difference between the potential of the Higgs particles which may describe the mass generation of the carriers of the weak interactions and the effective potential leading to the ground states in string theory. For the electro-weak interaction case the potential has the form of a Mexican hat, the ground state may be described by any point in the valley (Fig. 8.7) but all choices are equivalent, they describe exactly the same physics.

In the case at hand the valley of minima is flat and extends all the way to infinity. Any point along the valley can be chosen as a ground state as they all have the same energy and yet each describes different physics. The manifold describing the infinity of these degenerate ground states is called the moduli space when they are continuously connected (Fig. 8.8).

These connected regions of valleys trace out very elaborate geography and describe different possible solutions of string theory. All known stable ones share in common the property of describing a supersymmetric theory. In such theories each bosonic particle has a fermionic particle companion. It then turned out that there actually are several different types of string theories, each having its own infinite set of degenerate ground states. What made the string theories different were details, most of which are rather technical, which seemed to affect the particle content and gauge symmetries of the low energy physics. There was a stage, in the eighties of the last century, at which some researchers imagined they were on the verge of being able to perform reliable perturbative calculations of a realistic theory containing all the aspects of the standard model. That turned out not to be the case. However, in the process, the geography of many interesting ground states was surveyed. Next, a large amount of circumstantial evidence started to accumulate hinting that all these theories are after all not that distinct and may actually be mapped into each other by a web of surprising dualities. This led once again to the conjecture that there is but one string theory, this time, the theory that was supposed to encompass all string theories and bind them, was given a special name, M theory. Even in this framework all string theories brought as dowries their infinite moduli space. M theory still had an infinite number of connected different degenerate supersymmetric ground states, they were elaborated and greatly enriched by what are called brane configurations. Branes are a special type of solitons that appear in string theory. They come in various forms and span different dimensions. What is common to several of them, is that open strings may end on them.

Thus what was thought to be a theory of only closed strings contains open strings excitations in each of its sectors that contain branes. Moreover, every allowed brane configuration leads to its own low energy physics. In fact there are suggestions that our four dimensional experience results from us living on one or several branes all embedded in a larger brane configuration placed in a higher dimensional space-time. How does that fit with the expectations to have the One and Only string theory? Well, in this framework there is one theory but an infinite amount of possible ground states. Are we owed only once such state? There are those who hope that the vacua degeneracy will be lifted by some yet to be discovered mechanism leaving the one unique ground state. Others are loosing their patience and claiming that the NOH principle takes over at this stage, the theory will retain its large number of ground states.

Moreover a crucial ingredient of string theory at present is super symmetry which was discussed above, a beautiful symmetry not yet detected in nature, and even if detected, it seems at this stage to play only a role of a broken symmetry. It is not easy to obtain even remotely realistic models of string theory in which supersymmetry is broken (that is the symmetry between fermions and bosons is broken), a nearly vanishing cosmological constant and light particles whose mass values are amenable to a calculation within a reliable approximation. Candidates for such ground states have been suggested recently, these particular candidates consist of very many isolated solutions living off the shore of the supersymmetric connected valleys. Many of these solutions actually have different values for the

vacuum energy and perhaps may thus decay from one to the other. In particular bubbles may form within one configuration, bubbles that contain in them a lower energy configuration. Some researchers would like to see the universe itself, or the bubbles that form in it, as eternal tourists who will end up visiting many of these metastable states. Some suspect that many of these metastable states are stable. Some would encourage brave sailors to take to sea again, as they did in the 1980s, and charter this unknown geography, some are trying to make this endeavor quantitative by counting vacua which have a given property, such as a given cosmological constant. This subject is still at its infancy, many hopes and opinions are currently expressed, it would be nice if this issue will turn out to fall within the realm of solvable problems.

The challenges to string theory on the experimental side have been to reproduce the standard model. This has not been done to this day. Many new ideas and insights have been added. Bounds have been shifted several times, one has felt being very near to obtaining the desired model, each new approach seemed to bring one nearer to that goal, but obtaining one completely worked-out example, which is the standard model, remains a challenge for string theorists. Another important issue is that of the value of the cosmological constant. It is usually stated that the small value of the cosmological constant is a major problem. These arguments are centered around some version of a low energy effective field theory. That theory could be valid below a few tens of electron volt thus describing only aspects of electromagnetism or it could be at a TeV scale describing both the electroweak interactions. The arguments goes on to say that the vacuum energy of such an effective theory should be, on dimensional arguments grounds, proportional to the scale of the cutoff, (the cutoff is set at that energy beyond which the ingredients of the physics start to change) that is eVs or TeVs in the above examples. In either case, the value of this vacuum energy, which is the source of the cosmological constant, is larger by an astonishing number than the known bounds on the value of the cosmological constant. The argument is not correct as stated, it is not only the cutoff that contributes to the cosmological constant, all scales do actually contribute to the vacuum energy and if a cutoff should be invoked, it is the highest one, such as the Planck scale, that should be chosen. Moreover, this argument does not take into account the possibility that the fundamental theory has a certain symmetry which could be the guardian of vanishingly small value of the cosmological constant. Such a symmetry exists, it is called scale invariance, and it has shown to be a guardian in several settings. Scale invariance is associated with the absence of a scale in the theory. There are many classical systems which have this symmetry and there are also quantum systems which retain it. In a finite, scale invariant theory not only does the vacuum energy get contributions from all scales, they actually add up to give rise to a vanishing vacuum energy. This remains the case even when the symmetry is spontaneously broken. Being a consequence of a symmetry, the vacuum energy contribution to the cosmological constant vanishes also is the appropriate effective theory. What does string theory have to say about that? String theory could well be a finite theory, it is not scale invariant, as it contains a string scale, if that scale would be spontaneously generated perhaps one would be nearer to

understanding the value of the cosmological constant. If that scale is indeed not elementary, one would be searching for the stuff the strings themselves are made off. Such searches originating from other motivations are underway, some go by the name of Matrix Model. Particle Physics as we know it is described by the standard model evolving in an expanding universe. The methods used to study string theory are best developed for supersymmetric strings, and for strings propagating on time independent background, nature is explicitly neither. What then are the successes of string theory?

8.4.5 Successes: Black Holes, Holography and All That . . .

A major success described above was to obtain a theory of gravity well defined to several orders in perturbation theory, in addition, no obvious danger signals were detected as far as the higher order in perturbation theory are concerned, non perturbative effects are yet to be definitely understood. This issue involves the short distance structure of string theory. String theory is supposed to be a consistent completion of General Relativity (GR). General relativity suffers from several problems at low energies. String theory, which according to a correspondence principle, is supposed to reproduce GR in some long distance limit, will thus have inherited in this merger, all the debts and problems of GR. If it does solve them it will need to do it with a “twist” offering a different point of view. That perspective could have been adopted in GR but was not. This has occurred in several circumstances, one outstanding problem in GR is to deal with the singularities that classical gravity is known to have. These include black holes, big bangs and big crunches as well as other types of singularities. String theory can offer a new perspective on several of these issues. Let us start with black holes. The first mystery of black holes is that they seem to possess thermodynamical properties such as temperature and entropy. This is true for charged black holes as well as for uncharged Schwarzschild ones. In the presence and under the protection of a very large degree of supersymmetry, string theory tools enabled to provide a detailed microscopical accounting of the entropy of some black holes. More precisely it was shown in these special cases that the number of states of a black hole is identical to the number of states of essentially a gauge theory. It was possible to count the microscopic number in the gauge theory case and the resulting number of states was exactly that predicted for black holes! These are mostly higher dimensional very cold black holes. This has not yet been fully accomplished for uncharged black holes. Black holes have several more non-conventional properties. The above mentioned entropy of a mass M black hole is much smaller, for many types of black holes, than that of a non gravitating system of particles which have the same energy, M . In fact one may suspect that such black holes dominate by far the high energy spectrum of any gravitating system. The Schwarzschild radius of these black holes actually increases with their mass. In general one associates large energies with short distances, in the case of black holes large energies are related also to large distances. A meticulous

separation of long and short distances is a corner stone of the renormalization group approach, this is not evident in the case of gravity. One possible implication of this behavior is that in gravitational systems the amount of information stored in a spatial volume is proportional to the boundary's surface area rather than to the bulk volume. String theory provides explicit and concrete realizations of this amazing behavior. A system of strings moving in particular ten dimensional space-times (of the anti de Sitter type) are equivalent to special highly symmetric supersymmetric non-Abelian gauge theories living in four dimensions and experiencing no gravitational forces. Moreover modifying the backgrounds on which the strings propagate by adding defects to them, one finds that strings propagating on certain backgrounds are equivalent to gauge theories in which color is confined. String theory methods are used to perform difficult calculations in hadronic physics. String theory, which was born out of hadronic physics, has returned to pay back its original debt! In addition, remarkable structures have been uncovered in supersymmetric gauge theories using string theory methods. Attempts have been made to confront strings with other singularities. Why should strings respond to singularities in any other way than particles do? A Talmudic story helps explain the manner in which extended objects modify problems associated with point particles. During a seminar, the following question came up: Assume a pigeon (a very valuable animal at the time) is found somewhere, to whom does it belong? The rule is, finders-keepers, as long as the pigeon is found further away, than some determined distance, from the entrance to an owned pigeon-hole. A student raised his hand and asked: "The pigeon is, after all, an extended object, what if one of its legs is nearer than the prescribed distance to the pigeon-hole but the other leg is further away?" The student was ejected from his place of study for raising the question. Strings definitely turn out to soften some harsh singularities which are time independent (called time like singularities). Strings are also found to punch through singularities which form at particular times such as a big crunch. The analysis of such systems is still at very early stages, still I would like to note a very interesting feature in common up the present to all attempts to study using string theory methods universes that are spatially closed. In these cases the compact universe is found to be immersed in one way or another in a non compact universe. The study of strings near these singularities could well be essential to appreciate their properties at short distances. A key ingredient driving some researchers is a new panoramic vista opening up to them. For some it is the precise calculations of experimentally measurable quantities, for others it is the Magic.

8.4.6 *Magic*

The universe as viewed with string probes is full of magic ambiguities and symmetries unheard of before. In fact most mathematical concepts used to describe the universe are veiled under symmetry. There are cases for which each of the following concepts becomes ambiguous: distances, the number of dimensions,

topology, singularity structure, the property of being commutative or not being commutative. Let us somewhat elaborate on the magic.

- **Distance**—The simplest example is that of a universe in which one of the dimensions is extended along identified with a circle of radius R . It turns out that an experimentalist using strings as his probes will not be able to determine if the universe is indeed best described but one of its dimensions being a circle of radius R (in the appropriate string scale) or by being a circle of radius $1/R$. For point particles moving a circle of radius R , the energy spectrum has large gaps, when R is small, and very small gaps, when the value of R is large. For strings, which are extended objects, the spectrum consists of an additional part, that of the strings wrapping around the circle. A point particle can't wrap around the circle. This part of the spectrum is narrowly gapped for small values of R and widely gapped for large values of R , this is because the energy required to wrap a string, which has its tension, around the circle, is proportional to the length of the wrapped string. For an experimentalist, who can use point particles as probes, the distinction is clear, not so for one using string probes. This is but the tip of the iceberg of an infinite set of ambiguities. Some geometries, judged to be different by point probes, are thus identified by extended probes.
- **Dimensions**—One could at least expect that the value describing the number of spacetime dimensions to be unique. It turns out not to be the case. Ambiguities in calling the "correct" number of space-time dimensions come in several varieties. In string theory there are examples of a string moving in certain ten dimensional space-times which measures exactly the same physics as that measured by a certain point particle gauge theory in four dimensions. Each system is made out of totally different elementary constituents, one system is defined to exist in ten dimensions, the other in four and yet both reproduce the same observables, the same physics. Another example consists of a strongly coupled string theory in ten dimensions whose low energy physics is reproduced by an eleven dimensional supergravity theory. This is true for large systems. For small, string scale, systems, more magic is manifested. A string in some cases can't distinguish if it is moving on a three dimensional sphere or a one dimensional circle. So much for non-ambiguous dimensions.
- **Topology**—Objects that can be deformed to each other without using excessive violence (such as tearing them) are considered to have the same topology. The surface of a perfect sphere is the same, topologically, as that of the surface of a squashed sphere, but different, topologically, than that of a bagel/torus. Well, once again that is always true only as far as point particles probes are concerned. In string theory there are examples in which the string probes can describe the same set of possible observations as reflecting motions on objects with different topologies. In addition in string theory there are cases where one may smoothly connect two objects of different topology in defiance of the definition given above for different topologies. Topology can thus be ambiguous.
- **Singularities**—A crucial problem of General Relativity is the existence of singularities. Bohr has shown how such singularities are resolved in electrodynamics

by quantum mechanics. In GR one is familiar with singularities which are stationary (time-like) such as the singularity of a charged black hole and those which form or disappear at a given instance instant such as the big bang and the big crunch (space-like). What happens in string theory to black holes big crunches or big bangs? A good question. Consider first the motion a particle on a circle of some radius and the motion of the string on a segment of some length. The circle is smooth, the segment has edges. For a point particle probe the first is smooth and the second is singular. In some cases, viewed by stringy probes, both are actually equivalent, i.e. both are smooth. The string resolves the singularity. Moreover there are black holes for which a string probe can't distinguish between the black hole's horizon and its singularity. There are many other cases in string theory where time-like singularities are resolved. The situation in the case of space-like singularities is more involved and is currently under study. Some singularities are in the eyes of the beholder.

- **Commutativity**—The laws of quantum mechanics can be enforced by declaring that coordinates do not commute with their conjugate momenta. It is however taken for granted that the different spatial coordinates do commute with each other and thus in particular can be measured simultaneously. The validity of this assumption is actually also subject to experimental verification. A geometry can be defined even when coordinates do not all commute with each other. This is called non-commutative geometry. There are cases when strings moving on a commutative manifold would report the same observations as strings moving on a non-commutative manifold. It seems many certainties can become ambiguities, when observed by string probes. Stated differently, string theory has an incredible amount of symmetry. This may indicate that the space-time one is familiar with, is in some sense suited only for a “low” energy description. At this stage each of these pieces of magic seems to originate from a different source. The hard-wired mind seeks a unified picture for all this tapestry. This review may reflect ambiguous feelings, challenges, success and magic each have their share. All in all, string theory has been an amazingly exciting field, vibrating with new results and ideas, as bits and pieces of the fabric of space time are uncovered and our idea of what they really are shifts.

8.4.7 Human Effort and Closing Remarks

One may estimate that more than 10,000 human years have been dedicated up to now to the study of string theory. This is a global effort. It tells an amazing story of scientific research. In the mid eighties a regime/leadership change occurred. The scientific leaders which had nursed the standard model, have been replaced by a younger generation of string theorists. Some of those leaders complained that the field had been kidnapped from them. Mathematical methods of a new type were both applied and invented, many pieces of a vast puzzle were uncovered and a large number of consistency checks were used to put parts of the puzzle together

tentatively. As in any human endeavor alternative paths are suggested and critique is offered. I have incorporated in this note some of the serious challenges string theory faces. There are additional complaints, complaints that the new style leaves too many gaps in what is actually rigorously proven, complaints that a clean field theory description is needed and lacking. Research has been democratized and globalized to a quite unprecedented extent. This has mainly been achieved by making research results available simultaneously to most of the researchers by posting them on the web daily. There are some drawbacks inherent to that evolution, research attitudes have been largely standardized and the ranges of different points of view have been significantly focused/narrowed. The outside pressures to better quantify scientific production have found an easily accessible statistical database. In particular, that is leading to assigning a somewhat excessive weight for the citation counting in a variety of science policy decisions. The impact of that is yet to stabilize. All that said, one should notice that phenomenologists and experimentalists do turn to string theory and its spin offs in search of a stimulus for ideas on how to detect possible deviations from the standard model. The problems string theory faces are difficult, one approach is to accept this and plunge time and again into the dark regions of ignorance using string theory as an available guide, hoping that it will be more resourceful than its practitioners are. Another approach is to break away from the comfort of physics as we formulate it today and replace some of its working axioms by new ones, such as holography, the entropic principle or eventually perhaps something else. It seems that the question of the exact nature of the basic constituents of matter will remain with us for still quite awhile. My favorite picture is that it will turn out that asking if the basic constituents are point like, are one, two or three-dimensional branes is like asking whether matter is made of earth or air. A theory including the symmetries of gravity will have different phases, some best described by stringy excitations, some by point particles, some by the various branes, and perhaps the most symmetrical phase will be much simpler. Several of these will offer the conventional space time picture, other will offer a something new. String theory either as a source of inspiration, or as a very dynamic research effort attracting criticism leaves few people indifferent.

8.5 Hindsight from 2018

The years from 2007 to 2018 were yet another example of the well-known difficulty of predicting the future. Luckily what I wrote in 2007 contained, on this front, enough disclaimers to satisfy the most strict of lawyers.

During those years one could celebrate a gigantic engineering triumph. After succumbing to a very significant early setback the LHC outperformed its expectations. The experimental triumph was as impressive. The detectors performed well beyond what one could dream. They obtained results from picking out the Higgs particle out of haystack through rediscovering particles which were first detected at electron-positron machines to finding more hadrons in the heavier flavour sectors.

This success was not matched by validating any of the TPs (Theoretical Prejudices) which I had discussed in some detail. The discovery of the quite likely absence of new particles in the new energy vistas opened by the LHC had an impact. Of course one can still hope that positive new discoveries will be made and I for one do hope so; still it is more and more considered as a possibility that we will need to settle for a long time for accelerators that will validate the Standard Model and not uncover treasures beyond it. A school of thought is developing, since the activation of the LHC, that beyond the Standard Model lurks non other than the Standard Model itself. At least for a while.

This requires soul searching of the theoretical physics community. What could be the result of such an examination of the routes taken so far? Recall that NOH (Nobody Owes Humanity), it may be that using elegance and beauty as our guiding principles, and indeed SUSY and string theory are beautiful, has failed us. Perhaps our esthetics taste does not confirm with that of nature. Perhaps these TPs will manifest themselves at some higher energy and perhaps we are on a totally wrong track. Falsifying ideas is a most important component of the scientific method, but many would not agree that the TPs have been indeed falsified. New ideas should be encouraged and welcomed, there is nothing as good as a good new idea to revitalize our thinking. The results of such debates cannot be dictated. Each researcher will draw her/his own conclusions. Be that as it may, such an open debate is required. Hopefully there will be something to report on in the next edition of this book.

One aspect of “Beyond” physics that is considered more actively now involves testing the possibility that the Standard Model has scale or even conformal invariant features. Some of them I suggested long ago and pointed out in this volume, especially its relation to the puzzle of the value of the vacuum energy.

The fact that, depending on the exact value of the top quark mass, the Universe is living dangerously and may eventually self-destruct as it decays to some other place was also noted and investigated.

The dark matter elephant in the room remains very present.

The mathematical aspects of SUSY have been considerably sharpened during these years and have led to many new surprising exact results in more regions of strong coupling where no one ventured before. I was most attracted to a branch of research trying to attempt to use quantum information aspects as ingredients that build up semiclassical geometry and also quantum space time. These attempts are in their infancy but they have already covered quite some ground. Sometimes it has happened that a change of perspective and bringing in new tools was a way for a younger generation to take over the leadership of a field. The absence of experimental data to crown new leaders serves as a fertile ground for this mechanism.

Finally, back to experiment; for years black holes seemed as removed from experiment as the detection of stringy effects are 2017 has seen the hundred years search for the detection of gravitational waves reach a discovery moment. Humans observed particles collide at the LHC and black holes collide in the Universe—a great moment. Theorists are seriously considering the properties of those very black holes in terms of information processing. Seeds for a new TP have been sown.

Over this decade we have learned or should have learned the importance of patience, humility, and diversity of ideas. Not the type of lessons that theoretical physicists appreciate too much.

8.6 References for 8

We mention here only text books and reviews and only those which were used to prepare this entry. Most of the appropriate references for the works reviewed here can be found in them.

(1) For Super Symmetry:

Different narratives on the history of Super Symmetry by a large number of its pioneers has been collected in the book *The Supersymmetric World*, edited by G. Kane and M. Shifman. Published by World Scientific (2000). As most of them were still alive at the time this offers a unique perspective into how theoretical ideas develop.

- *Supersymmetry and Supergravity* by J. Wess and J. Bagger. Princeton Series in Physics (1992).
- *The Quantum Theory of Fields, Volume 3: Supersymmetry* by S. Weinberg. Cambridge University Press (2000).
- *Lectures on Supersymmetric Gauge Theories and Electric—Magnetic Duality* by K.A. Intriligator and N. Seiberg. Published also in *Proceedings of Cargese Summer School lectures*, Cargese.
- *Supersymmetric Gauge Theories* by D. Berman and E. Rabinovici. Published in: *Unity from Duality: Gravity, Gauge Theory and Strings*, Les Houches Session LXXVI. Editors: C. Bachas, A. Bilal, M. Douglas, N. Nekrasov and F. David. Publisher: Springer.
- *Weak Scale Super Symmetry: From Superfields to Scattering Events* by H. Baer and X. Tata. Cambridge University Press (2006).
- *Lectures on Supersymmetry Breaking* by K. Intriligator and N. Seiberg. Published in *Class.Quant. Grav.* (2007).

(2) For gauge theory:

This subject was barely touched upon here. A group theory reference is: *Group Theory for Unified Model Building* by R. Slansky. Published in *Phys. Rept.* **79** 1–128 (1981).

(3) For extra dimensions:

Kaluza-Klein Theory in Perspective. M.J. Duff (Newton Inst. Math. Sci., Cambridge). NI-94-015, CTP-TAMU-22-94, Oct. 1994. 38 pp. Talk given at The Oskar Klein Centenary Symposium, Stockholm, Sweden, 19–21 Sept. 1994. In **Stockholm 1994, The Oskar Klein Centenary** 22–35, hep-th/9410046.

(4) For strings:

For text books and many references see for example:

- M.B. Green, J.H. Schwarz, E. Witten: Superstring Theory. Cambridge Monographs On Mathematical Physics (1987).
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