Chapter 12

Outlook for the Future

C. Joshi, A. Caldwell, P. Muggli, S. D. Holmes, and V. D. Shiltsev

12.1 Plasma Accelerators

C. Joshi · A. Caldwell · P. Muggli

12.1.1 Introduction

The charge separation between electrons and ions that exists within an electron plasma density wave can create large electric fields. In 1979 Tajima and Dawson first recognized that the longitudinal component of the field of a so-called “relativistic” wave (one propagating with a phase velocity close to c), could be used to accelerate charged particles to high energies in a short distance [1]. The accelerating gradient of such a plasma wave, E_0 , can be approximated—assuming a total separation of

Coordinated by C. Joshi, A. Caldwell

C. Joshi
Los Angeles, CA, USA
e-mail: joshi@ee.ucla.edu

A. Caldwell (✉) · P. Muggli
Max-Planck-Institut, Munich, Germany
e-mail: caldwell@mpp.mpg.de; muggli@mpp.mpg.de

S. D. Holmes · V. D. Shiltsev
Fermi National Accelerator Laboratory, Batavia, IL, USA
e-mail: shiltsev@fnal.gov

electrons and ions in such a wave with wavelength $\lambda_p = 2\pi c/\omega_p$ —as

$$E_0 \approx \frac{m_e c \omega_p}{e},$$

$$E_0 \text{ (eV/m)} \approx 96.2 \sqrt{n_0 \text{ (cm}^{-3}\text{)}}. \quad (12.1)$$

Here λ_p is the plasma wavelength, $\omega_p = \sqrt{4\pi e^2 n_0 / m_e}$ is the plasma frequency and n_0 is the background plasma density, c is the speed of light, m_e and e are the mass and charge of an electron, respectively. The majority of plasma accelerators have been operated at plasma densities $10^{14} \text{ cm}^{-3} < n_0 < 10^{20} \text{ cm}^{-3}$ giving $10^9 < E_0 \text{ (eV/m)} < 10^{12}$. Such ultrahigh accelerating gradients are the principal attraction of plasma accelerators.

In a laser-plasma accelerator (LPA), an accelerating structure is created as an intense laser pulse propagates through the plasma driving an electron plasma wave also known as a *wake*. Specifically it is the ponderomotive force, F_p , of the intense laser that displaces the electrons from the heavier ions creating the charge separation and accelerating *wakefield*. The ponderomotive force is related to E_l or A_l , the electric field and vector potential of the laser respectively via $F_p \propto \nabla E_l^2 \propto \nabla A_l^2$. Often the normalized vector potential of the laser driver, a_0 , is used to characterize the strength or magnitude of the laser driver and subsequent wake that is driven. The normalized vector potential of the laser is given by

$$a_0 = \frac{e A_l}{m_e c^2} = \frac{e E_l}{\omega_0 m_e c} \simeq 8.6 \times 10^{-10} \lambda_0 \text{ (}\mu\text{m)} \sqrt{I_0 \text{ (W/cm}^2\text{)}}, \quad (12.2)$$

where I_0 is the focused intensity of the laser pulse, and λ_0 and ω_0 are the vacuum laser wavelength and frequency respectively. In laser wakefield accelerators, laser intensities of $10^{17} < I_0 \text{ (W/cm}^2\text{)} < 10^{20}$ are commonly used. For $a_0 \approx 1$ and laser pulse duration on the order of half-a-plasma wavelength (typically 50–100 fs), wakes with accelerating fields of approximately 100 GeV/m are produced in a $10^{18}/\text{cm}^3$ density plasma. This gradient is more than three orders of magnitude larger than the accelerating gradient in a conventional RF driven accelerator.

It was recognized in the 1980s that wakes in plasmas could also be driven by a relativistic beam of electrons [2]. Given an intense enough beam of electrons, the plasma is both created [3] and excited by the passage of the bunch. Recently, driving the wake with a proton bunch has also been suggested [4].

In a beam-driven plasma wakefield accelerator (PWA), it is the space charge force of a highly relativistic beam of particles which excites the wakefield. For such a bunch, the electric field seen by the plasma electrons is in the transverse direction, and the plasma electrons initially move away from the beam axis, while the more massive ions remain effectively frozen. These transversely expelled electrons are attracted back towards the beam axis by the space charge force of the ions, creating a cavity with very strong electric fields. An appropriately timed witness bunch can be placed in a region of very strong longitudinal component of the electric field of this cavity and accelerated. See Fig. 12.1. The cavity also provides a radial force that

Fig. 12.1 Schematic of the accelerating structures produced in a plasma in the case of (a) electron beam-driven plasma wakefield accelerator and (b) laser wakefield accelerator (LWFA), both operating in the blow-out regime. In this regime all the plasma electrons are expelled by the electron beam or the laser pulse thereby creating a “cavity” containing only plasma ions. The extremely nonlinear longitudinal electric field generated in both cases is shown as a black curve in (a). In the LWFA case the laser spot size is matched to the plasma to produce a near spherical ion cavity with radius R_b equal to the laser spot size. The blow-out regime is attractive because the transverse focusing field increases linearly and the longitudinal accelerating field is constant with radius from the axis

Table 12.1 This table summarizes the scaling laws for the three distinct LWFA regimes characterized by the laser a_0 , normalized spot-size $k_p w_0$ and longitudinal electric field amplitude $\epsilon_{LW} (m_e c \omega_p / e)$

Regime	a_0	$k_p w_0$	$\epsilon_{LW}(E_0)$	$k_p L_d$	$k_p L_{pd}$	λ_W	γ_ϕ	$\Delta W(mc^2)$
Linear	<1	2π	a_0^2	$\frac{\omega_0^2}{\omega_p^2}$	$\frac{\omega_0^2}{\omega_p^2} \frac{\omega_p \tau'}{a_0^2}$	$\frac{2\pi}{k_p}$	$\frac{\omega_0}{\omega_p}$	$a_0^2 \frac{\omega_0^2}{\omega_p^2}$
1D Nonlinear	>1	2π	a_0	$4a_0^2 \frac{\omega_0^2}{\omega_p^2}$	$\frac{1}{3} \frac{\omega_0^2}{\omega_p^2} \omega_p \tau'$	$\frac{4a_0}{k_p}$	$\sqrt{a_0} \frac{\omega_0}{\omega_p}$	$4a_0^2 \frac{\omega_0}{\omega_p^2}$
3D Nonlinear	>2	$2\sqrt{a_0}$	$\frac{1}{2} \sqrt{a_0}$	$\frac{4}{3} \frac{\omega_0^2}{\omega_p^2} \sqrt{a_0}$	$\frac{\omega_0^2}{\omega_p^2} \omega_p \tau'$	$\sqrt{a_0} \frac{2\pi}{k_p}$	$\frac{1}{\sqrt{3}} \frac{\omega_0}{\omega_p}$	$\frac{2}{3} \frac{\omega_0^2}{\omega_p^2} a_0$

Here $k_p L_d$ and $k_p L_{pd}$ are the normalized dephasing and pump depletion lengths respectively, λ_W is the plasma wavelength, γ_ϕ is the Lorentz factor associated with the phase velocity of the wake, and $\Delta W(mc^2)$ is the dephasing limited net energy gain in a single stage of a LWFA. τ' is the characteristic laser pulse duration given approximately by $\tau' = \frac{4}{3} \frac{1}{\omega_p} \sqrt{a_0}$. This table has been reproduced from [70]

keeps the witness bunch from expanding radially. A conceptual $e^- e^+$ linear collider based on plasma acceleration envisions multiple plasma wakefield stages powered by either a laser or a particle beam driver. Each stage adds typically 10 GeV to the accelerating beam. Using a multi-TeV class proton beam as a driver, it may be possible to accelerate an electron beam to energies of interest (TeV) in a single stage.

12.1.2 Physical Concepts

We now discuss some of the important physical concepts in plasma accelerators. These concepts will determine the necessary laser/beam and plasma conditions for the efficient acceleration of electrons. Unless specified otherwise the expressions given in this section are in the limit of small amplitude or sinusoidally varying (linear) wakes.

12.1.2.1 Phase Velocity v_ϕ

The phase velocity, v_ϕ , of the accelerating wake structure is equal to the group velocity of the driver. In the case of a LPA, v_ϕ of the wakefield is equal to $v_g = c \left(1 - \omega_p^2/\omega_0^2\right)^{1/2}$ of the laser pulse as it propagates through the plasma. Often it is also useful to define a relativistic Lorentz factor of the wake $\gamma_\phi = \left(1 - v_\phi^2/c^2\right)^{-1/2} \simeq \omega_0/\omega_p$. For a PWA driven by a highly relativistic electron bunch $v_\phi \approx c$.

12.1.2.2 Dephasing Length L_d

In a LPA, electrons that are accelerated by the wakefield can gain enough energy as to move faster than the phase velocity of the accelerating wake structure. The difference in velocity causes a relative slip in position between the electron beam and the accelerating phase of the wakefield. Eventually the electron beam will slip out of the accelerating phase and into the decelerating phase of the wakefield and begin to lose energy. The length over which the electron beam can gain energy from the accelerating phase of the wakefield is known as the dephasing length, L_d . Within the wake, the accelerating phase extends over approximately half a plasma wavelength and the dephasing length is approximated by assuming that the velocity of the relativistic electrons equals c . The dephasing length is then given by $L_d = c\lambda_p/\{2(c - v_\phi)\}$. This length limits the maximum energy gain that an electron experiences in a LPA.

In a PWA driven by an electron beam, where $v_\phi \approx c$ energy gain is not limited by dephasing. However, in a proton driven PWA the dephasing between the drive and witness bunch must be considered. The phase slippage between the drive and witness bunch has been estimated as [5]

$$\delta \approx \frac{\pi L}{\lambda_p} \left[\frac{1}{\gamma_f \gamma_i} \right], \quad (12.3)$$

where L is the distance traveled, and γ_i and γ_f are the initial and final Lorentz factors of the particles in the driving bunch. With an initial proton energy of 1 TeV and final energy of 0.5 TeV, it is nevertheless possible to have dephasing lengths of many hundreds of meters.

In either LPA or PWA cases, it is conceivable to control the plasma wavelength by adjusting the density of the plasma, thus in part or fully compensating for the phase slippage.

12.1.2.3 Pump Depletion Length L_{pd}

In a LPA, the laser pulse is depleted of its energy as it excites the wake. Additionally, laser light that is not coupled into driving the wakefield is lost due to diffraction/refraction within the plasma. The depletion of energy from the laser pulse limits the distance over which the laser remains intense enough to drive a wakefield. The characteristic distance over which the laser pulse is depleted of its energy is defined as the pump depletion length L_{pd} . In order to gain further energy, two or more laser-plasma accelerator stages must be combined in series. This is known as staging. In a PWA if $\gamma_f \ll \gamma_i$, then the pump depletion length is approximately $\gamma_i mc^2 / E_-$, where E_- is the decelerating field of the wake.

12.1.2.4 Injection and Trapping

Once a plasma accelerator has been created, electrons can be *injected* into the accelerator to gain energy. In a LPA, the background plasma typically provides the source for injected electrons. In order to gain a significant amount of energy these injected plasma electrons must be *trapped* by the wakefield. An electron is considered to be trapped once it has gained enough energy to have a longitudinal velocity equal to the phase velocity of the wakefield. After an electron is trapped it can remain in the accelerating phase of the wakefield and continue to gain energy until it travels a dephasing length. Next different methods of electron injection are introduced.

In a PWA, a relativistic witness beam is injected at the appropriate phase with respect to the drive beam in order to experience the accelerating phase of the wakefield. Since both the witness beam and the wake propagate at c , there is no relative phase slippage.

Self-Injection

In a thermal plasma, (such as those typically created using lasers with joules of energy and pulse durations on the ps-ns time scale), some electrons in the tail of the electron distribution function may have sufficiently high momentum so as to reside on a “trapped-orbit”. This occurs when the electron velocity is equal to v_ϕ in the plasma wake potential.

In a cold plasma, plasma electrons from outside the wake can cross into or be self-injected into the accelerating phase of the wake. If the wake amplitude is large enough, these injected electrons can quickly gain enough momentum from the accelerating field to move with the wake and become trapped [6, 7]. Self-injection process can be facilitated in a density downramp [8] or at a sudden transition between high and low density plasma regimes [9].

Downramp Injection

If the drive laser pulse or the particle bunch traverses across a plasma downramp [9] some of the plasma electrons forming the sheath around the plasma ions of a 3D nonlinear wake (discussed later) can be injected into the wake. This happens because the wavelength of the wake increase as the drive bunch goes from higher to lower density causing some electrons to cross the sheath and gain sufficient longitudinal velocity to move synchronously with the wake as they reach the back of the wake. These electrons can also have a very small transverse emittance because their transverse velocity is reduced to near zero by the strong defocusing field they feel as they converge on the axis of the wake.

Colliding-Pulse Injection

In a LPA, if a second laser pulse is collided with the laser pulse that induces the wake, then the ponderomotive force associated with the beating of the two pulses can impart enough longitudinal momentum to the initially untrapped electrons so that they are injected into the separatrix of the wakefield to be trapped [10, 11].

Ionization Injection

Electrons can be injected into a fully formed wake via ionization-trapping [12, 13]. A binary mixture of atoms is ionized by the field of the laser or space charge of the driving bunch. The minority atoms in this mixture typically have a step in the ionization potential such that the inner electrons are ionized close to the wake axis, near the peak field of the laser pulse or beam driver. These electrons experience a much greater wake potential so as to be trapped by the wake.

External Injection

In analogy to a conventional traveling wave accelerating structure, a witness beam of electrons or other charged particles may be pre-accelerated and injected into a plasma accelerating structure to increase their energy [14]. Within the wake the magnitude of accelerating field varies with space. Therefore, to minimize the difference in the accelerating gradient that is experienced, the injected electron beam should ideally have a longitudinal length that is much less than the plasma wavelength of the accelerator. For a plasma density of 10^{18} cm^{-3} , $\lambda_p \sim 30 \text{ } \mu\text{m}$ ($\sim 100 \text{ fs}$). Additionally, the external electron beam must be synchronized to the accelerating structure such that it is injected into the correct accelerating and focusing phases of the wakefield.

12.1.2.5 Net Energy Gain ΔW

For a LPA, The net energy gain of an electron ΔW , can be estimated as

$$\Delta W = e E_{LW} L_{acc} = \left(\frac{E_{LW}}{E_0} \right) \left(\frac{\omega_p}{c} L_{acc} \right) m c^2. \quad (12.4)$$

Here E_{LW} is the local longitudinal accelerating field seen by the electron of the wake, and L_{acc} is the distance over which the electron interacts with the accelerating field.

12.1.2.6 Beam Loading

The accelerated particles can modify the wakefield in a process known as beam loading [15, 16]. It can be understood as destructive interference between the fields of the wake and fields of the accelerated particles. Beam loading places limitations on the number of particles, the peak current, the energy spread and duration of the accelerated particles. The plasma-accelerator can be optimized for one or more of these parameters. The maximum number N_0 of electrons that can be loaded into the accelerating phase of the wake is [15]

$$N_0 = 5 \times 10^5 (E_1/E_0) n_0^{1/2} \left[\text{cm}^{-3} \right] S \left[\text{cm}^2 \right], \quad (12.5)$$

where $S \gg \pi/k_p^2$ is the cross sectional area of the wake. As $N \rightarrow N_0$ the energy spread $\Delta\gamma/\gamma \rightarrow 1$. From energy conservation arguments the beam loading efficiency scales as $(N/N_0)(2 - N/N_0)$.

12.1.2.7 Drive Pulse Evolution

In a LPA, plasma can act as a lens to focus the spot size of the laser in a process known as relativistic self-focusing [17]. Relativistic self-focusing of the laser pulse occurs when the power of the laser pulse exceeds P_c , the critical power for self-focusing given by

$$P_c(\text{GW}) \simeq 17.4 \frac{\omega_0^2}{\omega_p^2}. \quad (12.6)$$

The laser pulse will continue to focus until all the electrons are expelled from within the spot size of the laser and there is no longer a gradient in the index of refraction in the radial direction. The ratio of the laser power, P , to P_c gives a good indication of how strongly the laser pulse will be focused by the plasma.

The laser pulse can also be compressed longitudinally in time by the density gradient of a wake via photon acceleration and deceleration [18, 19]. If the laser pulse width is on the order of λ_p the entire pulse will be compressed and this can lead to an increase the laser intensity and a_0 [20]. However, if the laser pulse width is much greater than a plasma wavelength, the laser pulse will be compressed within each wake driving a laser-plasma accelerator in the self-modulated regime.

In a PWA, the particles in a symmetric drive pulse lose their energy to the wake at a different rate. For ultra-relativistic electrons, the pulse shape of the drive beam does not evolve longitudinally since there is insignificant relative motion between the particles. But for a proton drive beam the spatial spread of particles due to momentum spread will induce a lengthening of the bunch. The effect can be evaluated for vacuum propagation as

$$d \approx \frac{L}{2\Delta\gamma^2} \approx \left(\frac{\sigma_p}{p}\right) \frac{M_p^2 c^4}{p^2 c^2} L, \quad (12.7)$$

where M_p is the proton mass, and p is the proton momentum. Given a 1 TeV proton beam, a 10% momentum spread leads to a growth of $\sim 0.1 \mu\text{m/m}$. Large relative momentum spreads will still allow for long plasma acceleration stages provided the drive beam is relativistic.

12.1.2.8 Guiding

The length of the accelerating structure can limit the energy gain of the particles. For a LPA the length of the accelerator (assuming Gaussian optics) is approximately equal to π times the Rayleigh length of the focused laser pulse, $z_R = \pi w_0^2 / \lambda_0$, where w_0 is the spot size of the laser. Often, in order to reach the intensities necessary to drive large amplitude wakefields, the laser must be focused to a spot sizes on the order of $\sim 10 \mu\text{m}$. This limits the accelerating structure length to $\sim 1 \text{ mm}$ and subsequently severely limits the energy gain of electrons if the L_d is longer than this.

To overcome this limitation, a channel can be created that has an appropriate radial plasma density profile such that the diffraction of the laser pulse is minimized. This allows the spot size of the laser pulse to remain small, or 'guided', over the length of the channel that is much greater than a Rayleigh length. For a channel with a radial parabolic electron density profile, a change in electron density of $\Delta n = 1/(\pi r_e w_0^2)$ is required to optical guide the laser pulse, where here $r_e = e^2/m_e c^2$ is the classical electron radius. Two main methods have been used to guide short laser pulses.

In the first method, an electrical discharge is struck across a tube, or capillary containing hydrogen gas creating a plasma. The electron temperature close to the capillary wall is lower than that near the center of the discharge. Therefore over a short amount of time ($\sim 100 \text{ ns}$), hotter electrons located on axis of the discharge will diffuse radially, creating an appropriate density channel that can guide a laser pulse [21].

A second method, known as self-guiding, relies on matching the ponderomotive force of the laser, which expels the plasma electrons and drives the wake, to the space-charge attraction force that the plasma ions exert on the expelled electrons. When this is achieved, the radial plasma density profile of the wake that is created has the appropriate shape and depth to minimize the diffraction of the laser pulse [22, 23].

Both of these techniques have been used to extend the length of a laser-plasma accelerator from hundreds of microns to centimeter scale lengths [24–26].

In a PWA, if the density-length product of the plasma is large enough, the electron drive pulse can be guided by the transverse focusing force provided by the plasma ions. If the beam density $n_b > n_o$, the beam electrons can blow-out all the plasma electrons creating an ion channel. In this blow-out regime, the beam envelope is described by the differential equation [27]:

$$\sigma_r'' + \left[K^2 - \epsilon_{Nr}^2 / \gamma^2 \sigma_r^4(z) \right] \sigma_r(z) = 1, \quad (12.8)$$

where $K = \omega_p / (2\gamma)^{1/2} c$ is the restoring force provided by the plasma ions or equivalently the betatron wavenumber $k_\beta = \omega_\beta / c$ where ω_β is the betatron frequency. The beam is said to be matched to the plasma if $\beta_{beam} = 1/K = \beta_{plasma}$. In this case the beam propagates through the plasma with a constant radius [28]. The matched beam radius r_b is found by letting $\sigma_r''(z) = 0$ in this equation giving $r_b = (\epsilon_N / \gamma k_\beta)^{1/2}$. The beam now propagates through the plasma until it is either pump depleted or its front is slowly eroded away by finite emittance of the particles.

12.1.2.9 Head Erosion

In a LPA, the front portion of the laser pulse will diffract at a rate of some fraction of c/ω_p per z_R unless the pulse is guided in a pre-formed plasma channel [23, 29]. This head erosion will eventually limit the distance over which a wake can be excited. Similarly, in a PWA, the head of the drive bunch propagates in a region of no wakefield, so that the beam emittance will eventually erode the drive bunch [30]. A lower emittance drive beam can reduce head erosion as an obstacle limiting the energy gain, thus extracting more energy from the bunch. Proton beams have larger emittances than electron beams, therefore longer beams, lower plasma densities and thus long plasma cells are envisaged for efficient energy extraction. Therefore strong magnetic focusing of the proton drive bunch along the length of the plasma channel is likely necessary.

12.1.2.10 Instabilities, Scattering, and Radiation Loss

Generally short laser/bunch drivers are relatively immune to laser-plasma instabilities because both the driver and the wake are continuously entering undisturbed

plasma at $\sim c$. Nevertheless, there are two instabilities that can grow the emittance of the beam being accelerated. These are laser and electron beam hosing instability [31] and plasma ion motion [32].

In the hosing instability any transverse offset of the electron beam from the axis of propagation of the wake or any head-to-tail tilt can grow. The growth rate depends on the initial offset of a particular slice, how far the slice is from the front of the bunch and how far the bunch has propagated into the plasma. In the nonlinear 3D regime the instability involves a coupling of the centroid of the wake cavity and the bunch. In plasma accelerators the ions are usually assumed to be immobile because they are far more massive than the electrons. However if the accelerating bunch density exceeds the plasma density by a factor (n_b/n_e), the focusing force exerted by the electrons on the plasma ions becomes so large that the plasma ions implode inward toward the axis. This can locally alter the linear focusing force provided by the plasma ions and affect the emittance of the accelerating electrons.

Coulomb scattering in the plasma can increase the emittance of the witness bunch. In plasma accelerators, typical values of n_o will be in the range of $10^{14} - 10^{17} \text{ cm}^{-3}$, and both the radiation length and the mean-free-path are orders of magnitude larger than the expected plasma length on the order of 100 m. A 1 TeV proton beam in Li vapor of density $1 \times 10^{15} \text{ atoms/cm}^3$ gives a transverse growth rate of the proton beam of less than $0.01 \text{ } \mu\text{m/m}$ due to multiple scattering, which is small compared to the size of the drive bunch. Multiple scattering for high energy electrons will also be small.

In addition to these instabilities, as particles are accelerated they will oscillate transversely emitting betatron radiation. The energy lost to radiation per unit is given by [33, 34]:

$$\frac{dW_{loss}}{dz} = \frac{1}{3} r_e \gamma_b^2 \omega_\beta^2 K_w^2. \quad (12.9)$$

Here $K_w^2 = \gamma_b \omega_\beta r_o / c$ is the effective wiggler strength of the ion column. At extremely high energies, the betatron radiation loss rate will equal the rate particles gain energy. This ultimately limits the maximum energy gain in a plasma accelerator.

12.1.3 Beam Driven Plasma Wakefield Accelerators

12.1.3.1 Electron Beam Driven PWA

The basic concept of the plasma wakefield accelerator involves the passage of an ultra-relativistic charged particle bunch through a stationary plasma. The plasma may be formed by ionizing a gas with a laser or through field-ionization by the Coulomb field of the relativistic bunch itself. This second method allows the production of meter-long, dense ($10^{16} - 10^{17} \text{ cm}^{-3}$) plasmas suitable for the PWA

and greatly simplifies the experimental set-up. In single bunch experiments [35] carried out with ultra-short electron bunches, the head of the bunch creates the plasma and drives the wake. The wake produces a high-gradient longitudinal field that in turn accelerates particles in the back of the bunch. The system effectively operates as a transformer, where the energy from the particles in the bulk of the bunch is transferred to those in the back, via the plasma wake. The physics is unchanged if there are two bunches rather than a single bunch; energy from the leading drive bunch is transferred to a trailing witness bunch. The maximum energy which can be given to a particle in the witness bunch in a PWA is limited by the transformer ratio, defined as

$$R = \frac{\Delta W_{max}^{witness}}{\Delta W_{max}^{drive}} \leq 2 - \frac{N_{witness}}{N_{drive}} , \quad (12.10)$$

which is at most two for longitudinally symmetric drive bunches and an unloaded wake ($N_{witness} \ll N_{drive}$) [36]. Here ΔW_{max} is the change in energy of the particles in the drive/witness beam and N is the number of particles in the witness/drive beam. This upper limit can in principle be overcome by nonsymmetric bunches [37]. Another option currently under study is to use an appropriately phased train of bunches so that the wakefield is increased with each bunch [38]. According to linear plasma theory [39], maximum accelerating field of the wake is given by [40]:

$$eE_{linear} \simeq 100 \frac{N}{2 \times 10^{10}} \frac{20}{\sigma_z (\mu\text{m})} \ln \sqrt{\frac{2.5 \times 10^{17}}{n_o (\text{cm}^{-3})} \frac{10}{\sigma_r (\mu\text{m})}} \text{GeV/m}, \quad (12.11)$$

where N is the number of particles in the electron bunch and σ_z is the bunch length, and σ_r is the spot size. Linear theory is valid when the normalized charge per unit length of the beam, $\Lambda = \frac{n_b}{n_o} (k_p \sigma_z) \simeq 2.5 \left(\frac{N}{2 \times 10^{10}} \right) \left(\frac{20}{\sigma_z (\mu\text{m})} \right)$ is less than 1. Equation (12.11) indicates that generating large gradient wakefields requires short, high density electron bunches. For high Λ the nonlinear equivalent of Eq. (12.11) should be used [41].

In a PWA operating in the so-called blowout regime, a short but high-current electron bunch, with density n_b larger than the plasma density n_p , expels all the plasma electrons from a region surrounding the beam as shown in Fig. 12.1. The expelled plasma electrons rush back in because of the restoring force of the relatively immobile plasma ions and thus generate a large plasma wakefield. This wakefield has a phase velocity equal to the beam velocity ($\approx c$ for ultra-relativistic beams). Since the electrons in the bunch are ultra-relativistic, there is no relative phase slippage between the electrons and the wake over meter-scale plasmas.

When the wakes are in the blowout regime, shaped bunches can be used to transfer a substantial amount of energy from the drive bunch to the wake and optimize the energy extraction efficiency from the wake to the trailing bunch while not further increasing its energy spread. It can be shown that particles in a trapezoid shape drive bunch—with beam current increasing from the front to the back—lose

energy at a near constant rate. A trailing bunch with a reverse shape can load the wake in such a way that nearly all the particles gain energy at the same rate. Significant beam loading will occur [42] when the loaded charge

$$Q(nC) > [(0.047mcp)/eEs](1016/np(cm-3))^{1/2}(kpRb)^4 \quad (12.12)$$

Here, we take the normalized electric field $eEs/mcp = kpRb/2$ as the electric field seen by the accelerating electrons, $kp-1$ is the plasma skin depth, and Rb is the blowout radius of the wake.

12.1.3.2 Short Proton and Positron Beam Driven PWA

In contrast to plasma wakes driven by bunches of electrons, only limited investigations of the plasma wave excitation by a positively charged driver exist [42–46]. For a positively charged driver (such as a proton bunch or a positron bunch), plasma electrons are attracted towards the axis of the driver, overshoot and setup the accelerating wake structure [47, 48]. For positively charged drivers with $n_b/n_o \ll 1$ the electric field distribution of the wake is the same as that for the negative driver but shifted in phase. However, for a positively charged driver with $n_b/n_o \gtrsim 1$ the wake behavior is significantly different than for a negatively charged driver with the same beam density. Due to the radial symmetry, for the wakefield driven by a strong positively charged driver ($n_b/n_o \geq 1$), there is an electron density enhancement on-axis and effective increase of the local plasma frequency. As a result, the proton or positron driver beams must be even shorter in order to excite the plasma wake resonantly. However, given that protons can be accelerated to the TeV regime in conventional accelerators it is conceivable to accelerate electron bunches in the wake of the proton bunch up to several TeV (e.g., in the wake of a Large Hadron Collider (LHC) proton beam) in a single plasma stage.

12.1.3.3 Long Proton Beam Driven PWA

Proton bunches available today are in principle very interesting to drive wakefields because, besides being relativistic, they also have a large population and thus carry large amounts of energy (10s to 100s of kJ). A proton bunch can therefore drive wakefields over a very long single plasma and lead to very large energy gain of a witness bunch. It was shown through simulations [4] that a 10 GeV electron bunch injected into wakefields driven by a 100 μm -long, 10 kA, 1 TeV proton bunch can reach an energy of ~ 0.5 TeV in a single plasma, only ~ 300 m-long. This corresponds to an average accelerating gradient larger than 1 GeV/m in a plasma with density of $6 \times 10^{14} \text{ cm}^{-3}$. We note here that in these simulations the proton bunch length σ_z is shorter and the width σ_r is narrower than the wakefields period λ_p , a condition necessary to effectively drive wakefields. These two conditions are usually expressed as $k_p\sigma_z \leq 1$ and $k_p\sigma_r \leq 1$.

Proton bunches produced for example by the CERN Super Proton Synchrotron (SPS) or Large Hadron Collider (LHC) are long: $\sigma_z \sim 6\text{--}12\text{ cm}$.

Compression of proton bunches produced by these synchrotrons is in principle possible [49]. However, it would first require means to impart a %-level correlated energy chirp along the already high-energy bunch. Second, the magnetic compressor (chicane) would have to be rather long (km), again because of the high energy of the particles. Therefore, producing short proton bunches such as the one used in the simulations of [4] remains a challenge.

Matching the plasma density to the long SPS proton bunch ($k_p \sigma_z \cong 1$) would mean using a low density plasma ($\sim 10^{11} \text{ cm}^{-3}$) and driving low amplitude wakefields ($\sim 10\text{s}$ of MV/m range from E_0 in Eq. 12.11).

However, proton bunches can be focused to small transverse sizes (e.g., $\sigma_r = 200\text{ }\mu\text{m}$). Choosing the plasma density to reach $k_p \sigma_r \cong 1$, leads to a much larger plasma density ($7 \times 10^{14} \text{ cm}^{-3}$) and also much larger possible accelerating field ($E_0 \sim 2.5\text{ GV/m}$, Eq. 12.11). This choice of $k_p \sigma_r \cong 1$ means $k_p \sigma_z \gg 1$ since $\sigma_z \gg \sigma_r$. Such bunches are subject to a symmetric transverse self-modulation process [50]. The self-modulation process transforms the long continuous bunch into a train of short bunches separated by the wakefields period. The short bunches are formed by the action of the periodically focusing/defocusing transverse wakefields. The train can then resonantly drive wakefields to amplitudes on the order of the 2.5 GV/m predicted by Eq. 12.11. Furthermore, the process can be seeded, so that it becomes a well-controlled, seeded self-modulation (SSM) process [51]. In this case the beam-plasma system is turned into an amplifier of seed wakefields, with well determined final fields amplitude *and phase*. The process is also weakly sensitive to variations of the input bunch parameters ($\pm 5\%$) [52]. The SSM process makes it possible to deterministically inject an electron bunch where wakefields reach their peak amplitude and into the range of accelerating and focusing phase of the wakefields (corresponding to region $\sim \lambda_p/4$ long), a large number of periods ($\sim \sigma_z/\lambda_p$) behind the seed point [53]. Simulation results show that with this scheme, electrons can be accelerated to multi-TeV energies in a plasma a few km-long [54].

12.1.4 Laser-Driven Plasma Accelerators

12.1.4.1 Plasma Beat Wave Accelerator (PBWA)

Two co-propagating long ($c\tau \gg \lambda_p$) but moderate intensity ($a_0 < 1$) laser beams with frequencies ω_1 and ω_2 can resonantly excite a relativistic plasma wave when $\omega_1 - \omega_2 \cong \omega_p$ [55]. Here τ is the pulse width of the laser pulse. For a square pulse, the amplitude of the plasma wave grows linearly in time and eventually saturates due to change in plasma frequency caused by relativistic mass increase of the plasma electrons. For laser pulses with $a_0 \ll 1$, the maximum wave amplitude is given by $E_1/E_0 = (4a_{01}a_{02}/3)^{1/3}$ and occurs when $\omega_1 - \omega_2 = \omega_p(1 - (a_{01}a_{02})^{2/3}/8)$. The relativistic plasma wave is prone to modulational instability [56], and mode coupling

[57]. For linear wakes, electrons need to be externally injected for acceleration [58, 59].

12.1.4.2 Self-Modulated Laser-Wakefield accelerator (SM-LWFA)

An accelerating wakefield can be driven by a single long laser pulse ($c\tau > \lambda_p$) via the Raman forward-scattering (RFS) instability [60] if $\omega_0 > 2\omega_p$ and the laser power $P \cong P_c$ the critical power for relativistic self-focusing. In this process the laser pulse amplitude is modulated at ω_p by the creation of frequency sidebands at $\omega_0 \pm \omega_p$. The spatiotemporal gain for RFS instability [61] has been estimated as $G = e^g (2\pi g)^{-1/2}$, where g is given by

$$g = \frac{a_0}{\sqrt{2}} \left(1 + \frac{a_0^2}{2} \right) \left(\frac{\omega_p^2}{\omega_0^2} \right) \frac{\omega_0}{c} \sqrt{x\phi}, \quad (12.13)$$

where x is the length of the plasma, and ϕ/c is the time over which the plasma experiences a constant laser intensity. The normalized plasma wave amplitude is $\delta n/n_0 = \alpha_n G$, where α_n is the initial wave amplitude associated with the thermal noise of the plasma. When $\alpha_n G \approx 1$ the wave amplitude becomes large enough to self-trap and accelerate electrons [62, 63].

12.1.4.3 Laser Wakefield Accelerator (LWFA)

A laser wakefield accelerator (LWFA) is a laser-plasma accelerator which is created directly by the ponderomotive force of a single short ($c\tau \sim \lambda_p$), intense laser pulse. With the advent of Ti:sapphire laser systems, such ultra-short, ultra-intense laser pulses are readily available. An advantage of using such short laser pulses to drive an accelerating wake is that they are relatively immune to most laser-plasma instabilities. Additionally, these short, intense laser pulses can drive wakefields with large amplitudes, that are capable of self-trapping and accelerating electrons to high energies [64–67]. As a result of this, laser wakefield accelerators are the focus of the majority of current laser-plasma accelerator research. Next, details on a few specific regimes in LWFA research are presented.

12.1.5 LWFA Regimes

12.1.5.1 Linear Regime

The wake induced by a short laser pulse ($c\tau \sim \lambda_p$) is said to be in the linear regime if $\delta n/n_0 < 1$, where δn is the change in electron density associated with the wake. Laser drivers with modest intensities $a_0 < 1$ and $P/P_c \leq 1$, excite wakefields in the

linear regime. In the linear regime the electron density response and the longitudinal electric field of the wake vary sinusoidally. The transverse and longitudinal fields are $\pi/2$ out of phase with one another. Thus there is a quarter wavelength region where the fields are both accelerating and focusing for the particles.

12.1.5.2 Nonlinear Regime

A nonlinear wake is created when electron density perturbation of the wake is on the order of or greater than the background plasma density ($\delta n/n_o \gtrsim 1$) and the longitudinal accelerating field of the wake becomes larger than E_o . Such nonlinear wakes are typically created by using a laser driver with an $a_o \gtrsim 2$ or with a beam driver with $n_b/n_o \gg 1$ and a $k_p \sigma_r \ll 1$. A characteristic of a nonlinear wake is a “saw toothed” shape longitudinal electric field profile. Additionally, the plasma wavelength of a nonlinear wake increases as the a_o of the laser (wake amplitude) is increased. A nonlinear wake is said to be in the 1D regime if the normalized laser pulse spot is broad, i.e., $k_p w_o \gg 1$. However, when $k_p w_o \sim 1$, and $a_o > 1$ one approaches the 3D-nonlinear regime.

The electron density response and corresponding fields of nonlinear wakes driven by laser pulses in 3-D regime are difficult to study analytically. To gain a better understanding in this regime, particle in cell (PIC) codes have been used to model the LWFA in this regime. Early work in this regime focused on simulating and developing scalings to describe the dynamics of a singular nonlinear wake driven in the so called bubble regime by a laser pulse with an $a_o \geq 2\omega_o/\omega_p$ [68, 69]. More recently a nonlinear regime in which a laser pulse with an $2 \leq a_o \leq 2\omega_o/\omega_p$ is used to drive a periodic nonlinear wake, in the so called blowout regime, has been studied with simulations, a phenomenological theory and experimental investigation [23, 25, 26, 41, 70–73]. In both the bubble and blowout regimes, the ponderomotive force of the laser pushes out all the electrons from within the first period of the wake creating an ion bubble into which background plasma electrons are self-injected, trapped and accelerated.

An advantage of operating in the blowout regime is that there exists a matched self-guiding condition, in which the intensity, spot size and pulse width of the laser pulse are matched to the plasma density, such that the driven wake serves to minimize the diffraction of the laser pulse [70]. This allows a strong and stable wake to be sustained over tens of Rayleigh lengths and can increase the interaction length between trapped electrons and the accelerating field of the wake. The matched self-guiding condition is valid when the laser $a_o \gtrsim 2$ and $c\tau \sim \lambda_p/2$. When these conditions are met, the matching condition is given by

$$k_p w_o \simeq k_p R_b = 2\sqrt{a_o}, \quad (12.14)$$

where R_b is the blowout radius of the spherically shaped wake. Remarkably, even when the spot size and or pulse width are not precisely matched to the plasma density, the laser pulse evolves within the plasma towards the matched spot size

and pulse width, via relativistic self-focusing and longitudinal pulse compression [67, 72, 73].

Simulations indicate that in the blowout regime self-trapping of background electrons will occur for large amplitude wakefields when the normalized blowout radius $k_p R_b \approx 4 - 5$.

12.1.5.3 Scaling Laws

The scaling laws for a LWFA in the regimes which were discussed above are now presented.

12.1.6 Status

12.1.6.1 LWFA and PWA

The development of laser-plasma accelerators has closely followed the progress of high-power, short pulse lasers [74]. Prior to 1990, laser-plasma accelerator experiments focused primarily on the PBWA [14] or SM-LWFA concepts [54, 56]. These demonstrated the existence of greater than many GeV/m gradients and acceleration of self-injected as well as externally injected electrons [14, 69]. The accelerated particles typically had an exponentially falling energy spectrum [57]. In the mid 1990s, the chirped pulse amplification (CPA) scheme was implemented using titanium-sapphire as a gain medium and enabled the development of terawatt-class, sub-100 fs laser systems. Such short pulses (cp) were ideal for exciting wakes in underdense plasmas. Through better control of plasma density and length, and laser pulse parameters, electron bunches with a significant charge and relatively narrow ($\lesssim 20\%$) energy spread were observed [58–60]. The electron beam energy spread and charge were controlled further by employing the colliding pulse injection technique [10, 11]. The CPA technique has extended the peak power reach of the Ti-sapphire lasers to multi-petawatt (PW) levels. For instance at Lawrence Berkeley National Laboratory (LBNL) a petawatt laser facility became operational as part of the Berkeley Lab Laser Accelerator (BELLA) project [74]. Using a plasma-channel to extend the acceleration distance, narrow energy electron bunches with energy up to 4.3 GeV have been observed [24] at LBNL. Similar peak power laser facilities have either come on line (for instance at GIST (Korea)) or expected to soon be operational in several institutes worldwide. Most of the LPA experiments are being conducted using the highly nonlinear 3D (sometimes called the blow-out or bubble) regime using either self-injected or ionization injected charge into the plasma wake. For example electrons were accelerated to beyond 1.4 GeV using ionization injection into a LPA operating in the nearly matched, self-guided blowout regime [70]. At present, several groups are exploring this regime to obtain electron beams for the generation of betatron and undulator radiation [71–73]. In past few years

much effort has been devoted to controlling the charge, minimizing the transverse emittance and energy spread of electrons created from laser wakefield accelerators [74]. New techniques for beam injection such as downramp injection [74] and transverse collide pulse injection [74] have been proposed for generating electron beams with sub 100 nm transverse emittances. Possible applications for such beams might be fifth generation light sources (ultra compact XUV and X-FELs) and high-energy electron therapy of cancerous tumors. As in the past, continued progress in this field will be tied to progress in making lasers more reliable, efficient and cheaper. Additionally, to extend the interaction between the electron beam and the LPA progress must be made in making reproducible, meter-scale plasma sources.

Experiments on beam-driven PWA have been going on at Argonne National Laboratory (ANL) [75], Fermilab [76], KEK [77], Brookhaven National Laboratory (BNL) [78] and at SLAC [79]. New facilities such as FLASHForward at GSI, Germany are being commissioned. With the exception of the SLAC experiments, all other work has hitherto used modest energy electron beam ($\lesssim 100$ MeV) to study both acceleration and focusing effect of plasmas on beams. Experiments at SLACs FFTB facility were initially carried out using 28 GeV electron and positron beams. These used 4 ps long beams containing more than 10 kA current to systematically study beam propagation, focusing, betatron radiation emission, and acceleration in meter-scale, laser-ionized plasmas [80, 81]. These were followed by experiments that used a sub-50 fs beam to both produce the plasma via tunnel ionization and excite the wakefield in much denser, meter-scale vapor columns. The front portion of the bunch was used to excite the field while the particles in the back of the same bunch were used to sample the accelerating wake. An example of data on changes in electrons energy in the above mentioned PWA experiment at SLAC is shown in Fig. 12.2. Most of the initially 28.5 GeV electrons in the pulse are seen to lose energy but electrons in the back of the same pulse are accelerated by the wake as its electric field changes sign. The energy gain is seen to increase with distance. When the electron beam energy was increased to 42 GeV some electrons were seen to gain more than 42 GeV in just 85 cm, implying that a gradient of 50 GeV/m was sustained throughout the plasma [82].

In 2006 the FFTB facility was replaced by a new experimental facility called FACET for Advanced Acceleration research [83]. Between 2019 and 2016 FACET provided electron and positron capability with a driver beam-witness beam structure so that high gradient acceleration of a significant number of charged particles with a narrow energy spread could be demonstrated. The drive and witness beams were only about 50 fs long and were separated by about 300 fs. In order to preserve its narrow energy spread the witness beam had to beam-load the wake so that particles in the witness beam flattened the electric field of the wake thereby experiencing a nearly uniform accelerating field. With this loading, the energy contained in the wake is efficiently transferred to the accelerating particles. In a landmark experiment carried out at FACET, it was shown that the efficiency of transferring drive bunch energy to the core of the accelerated bunch was up to 30% [84]. In a follow up campaign an energy gain of 9 GeV for a bunch containing 30 pC of charge with a 5% energy spread in a 1.2 m long plasma was observed [85]. Using the energy and

Fig. 12.2 Changes in electron energy in the PWA experiment at SLAC

the spot size changes experienced by different slices of the drive bunch itself, the PWA cavity in the nonlinear blowout regime was shown to have a longitudinal and transverse field structure that in principle will accelerate electrons without emittance growth, as long as the electrons (to be accelerated) are matched in and out of the plasma with minimal energy spread [86].

The plasma wake produced by an electron bunch cannot be used to accelerate a positron beam when the wake is in the nonlinear blow out regime because the plasma ions strongly defocus the positrons. Until recently it was not very clear how efficient positron acceleration at a high gradient could be carried out using highly nonlinear plasma wakes. Experiments at FACET showed that a certain positron beam current profile can lead to a loaded wake where the longitudinal electric field reverses sign (from decelerating to accelerating) in the middle of the single drive bunch [87]. This happens because the presence of the positrons pulls in the plasma electrons towards the axis. These plasma electrons can cross the axis in the middle of the drive bunch. Most of the electrons overshoot and set up a bubble like wake cavity but a significant fraction of the electrons are confined by the back of the positron beam close to the axis. This flattens the wake shape by beam loading [87]. A significant amount of positron charge can now be accelerated at the same electric field gradient producing a well-defined narrow energy spectrum. The energy extraction efficiency is similar to the electron bunch acceleration case described above.

In 2016, FACET ceased operation to make way for the LCLS II facility that will occupy the first 1 km space of the original SLAC linac tunnel. A new facility for advanced accelerator research, known as FACET II, is being constructed between the LCLS II linac and the LCLS linac. Together with the BELLA laser the FACET II facility [88] will arguably be the backbone of research facilities for short drive pulse driven plasma accelerator research for the next decade. The foremost physics challenge is the generation of collider quality transverse and longitudinal

emittance bunches and identifying the factors that cause emittance growth in plasma accelerators [88]. Transverse emittance growth may occur as a result of the hosing instability and also ion motion [31, 32]. Even if the electrons have a very small emittance inside the wake, such a beam must be extracted and matched either to another plasma acceleration stage or to conventional magnetic optics [89–91] to avoid emittance growth. Longitudinal emittance will depend on how small the energy spread of the beam can be, which in turn depends on optimizing the beam loading to give a constant accelerating field. Continued development of diagnostic techniques to visualize the fields of the highly nonlinear wake and the injected electrons is also needed. Issues of generating asymmetric emittance flat beams and spin polarized beams are still open questions. Creative solutions for the generation and acceleration of collider quality positron bunches using plasmas are needed. In the near future it is highly likely that a single stage of LWFA and a PWA will produce 10 GeV bunches with a percent level energy spread and a high efficiency.

12.1.6.2 Proton-Driven Plasma Wakefield Acceleration

The AWAKE experiment at CERN studies the driving of plasma wakefields with proton bunches [53, 90, 91]. AWAKE uses the (6–12)cm-long bunch delivered by the SPS with $(1\text{--}3)\times 10^{11}$, 400 GeV protons focused to a transverse rms size of $\sigma_r = 200\text{ }\mu\text{m}$. The plasma density is chosen so that the accelerating field can reach $\sim 1\text{ GV/m}$, i.e.: $n_0 > 10^{14}\text{ cm}^{-3}$ corresponding to $\lambda_p < 3\text{ mm}$. Since the proton bunch is long when compared to the wakefield period, the experiment relies on the seeded self-modulation (SSM) process [51] to reach these wakefield amplitudes.

A 10 m-long rubidium source was developed to produce a column of vapor with a very uniform density ($\delta n_{Rb}/n_{Rb} < 0.2\%$) and with sharp density ramps ($< 10\text{ cm}$) at the entrance and exit [92]. A 450 mJ, 100 fs laser pulse propagating within with the proton bunch creates a sharp ($< 100\text{ fs}$), relativistic ionization front that seeds the SSM process. With full ionization of the only electron of the rubidium atom outer shell, the plasma density longitudinal profile is identical to that of the rubidium vapor. The plasma radius is on the order of 1 mm. The plasma density is adjustable from 1×10^{14} to $10 \times 10^{14}\text{ cm}^{-3}$.

The occurrence of the SSM on the proton bunch is observed with three diagnostics [51]: a two-screen method [93], the time resolved emission of the optical transition radiation (OTR) emitted by the protons [94], and spectral analysis of the coherent transition radiation (CTR) emitted by the bunch train [95].

Preliminary experimental results obtained recently show clear evidence of the occurrence of SSM [51]. These results also show that the SSM leads to stable excitation of wakefields and that the process corresponds to the amplification of the seed wakefields with a final phase that is very weakly dependent on the variations of the bunch initial parameters. This was also shown in numerical simulations [52]. Excitation of seed wakefields was demonstrated with a low energy electron bunch [96].

First acceleration experiments of low energy electrons (~ 15 MeV) externally injected into the wakefields are currently underway. In these experiments the electron bunch is purposely made longer than the plasma period in order to ease the temporal synchronization requirements between the witness electron bunch and the wakefields. Numerical simulation results show that a fraction of these electrons could emerge from the plasma with an energy in the GeV range and with a finite final energy spread ($\delta E/E \sim 10\%$) [90].

Future experiments will use a short witness electron bunch ($\sigma_z < \lambda_p$) that will load the wakefields in order to minimize the final the energy spread and at the same time preserve the emittance of the a large fraction of the bunch population [97] while gaining a few GeVs of energy.

The application of the proton-driven plasma wakefield accelerator is to fixed target experiments and to a possible very high energy electron/proton collider [98]. Electron/proton collision applications ease the requirements on the accelerated electron parameters since proton beams are not focused as tightly as beams of an electron/positron collider and do not require production of a high quality positron beam. Also, electron/proton collisions are used to study QCD physics in which interaction cross-sections tend to increase and not decrease with collision energy.

Self-modulation experiments with low energy electron bunches are also performed at DESY-Zeuthen [99].

12.2 Muon Collider

S. D. Holmes · V. D. Shiltsev

Both e^+e^- and $\mu^+\mu^-$ colliders have been proposed as possible candidates for a lepton collider to complement and extend the reach of the Large Hadron Collider (LHC) at CERN. The physics program that could be pursued by a new lepton collider (e^+e^- or $\mu^+\mu^-$) with sufficient luminosity would include understanding the mechanism behind mass generation and electroweak symmetry breaking; searching for, and possibly discovering, super symmetric particles; and hunting for signs of extra space-time dimensions and quantum gravity. However, the appropriate energy reach for such a collider is currently unknown, and will only be determined following initial physics results at the LHC. It is entirely possible that such results will indicate that a lepton collider with a collision energy well in excess of 1 TeV will be required to illuminate the physics uncovered at LHC. Such a requirement would require consideration of muons as the lepton of choice for such a collider.

The lifetime of the muon, 2 μ s in the muon rest frame, is just long enough to allow acceleration to high energy before the muon decays into an electron, a muon-type neutrino and an electron-type antineutrino ($\mu^- \rightarrow e^- \nu_\mu \bar{\nu}_e$). However, constructing and operating a muon based collider with useable luminosity requires surmounting significant technical challenges associated with the production, cap-

ture, cooling, acceleration, and storage of muons in the required quantities and with appropriate phase space densities. Over the last decade there has been significant progress in developing the concepts and technologies needed to produce, capture, cool, and accelerate muon beams with high intensities of the order of $O(10^{21})$ muons/year. These developments have established a multi-TeV Muon Collider (MC) in which μ^+ and μ^- are brought to collision at high luminosity in a storage ring as a viable option for the next generation lepton-lepton collider for the full exploration of high energy physics in the era following the LHC discoveries.

Muon colliders were proposed by Budker [100] in 1969 and later conceptually developed by a number of authors and collaborations (see comprehensive list of references in [101]). Figure 12.3 presents a possible layout on the Fermilab

Fig. 12.3 Schematic of a 4 TeV Muon Collider on the 6×7 km FNAL site

Table 12.2 The parameters of the low- and high-energy Muon Collider options

Parameter	Higgs factory	Low E	High E
Center-of-mass energy [TeV]	0.126	1.5	6
Luminosity [$\text{cm}^{-2} \text{s}^{-1}$]	$0.005 \cdot 10^{34}$	$4.5 \cdot 10^{34}$	$7 \cdot 10^{34}$
Number of bunches	1	1	1
Muons/bunch [10^{12}]	2	2	2
Circumference [km]	0.3	2.8	6.3
Focusing at IP β_*/σ_z [mm]	25/5	10/10	10/5
Beam energy spread dp/p (rms) [%]	0.003	0.1	0.10
Ring depth [m]	~ 10	13	~ 150
Proton driver pulse rate [Hz]	30	12	15
Proton driver power [MW]	≈ 4	≈ 4	≈ 2
Transverse emittance ε_T [$\pi \mu\text{mrad}$]	300	25	25
Longitudinal emittance ε_L [πmmrad]	1	72	72

site of a MC that would fully explore the physics responsible for electroweak symmetry breaking. Such a MC requires a center-of-mass energy ($\sqrt{s}$) of a few TeV and a luminosity in the $10^{34} \text{ cm}^{-2} \text{ s}^{-1}$ range (see Table 12.2 for the list of parameters). The MC consists of a high power proton driver based, e.g., on the “Project X” SRF-based 8 GeV 2–4 MW H^- linac [102]; pre-target accumulation and compressor rings where very high intensity 1–3 ns long proton bunches are formed; a liquid mercury target for converting the proton beam into a tertiary muon beam with energy of about 200 MeV; a multi-stage ionization cooling section that reduces the transverse and longitudinal emittances and creates a low emittance beam; a multistage acceleration (initial and main) system—the latter employing Recirculating Linear Accelerators (RLA) to accelerate muons in a modest number of turns up to 2 TeV using superconducting RF technology; and, finally, a roughly 2-km diameter Collider Ring located some 100 m underground where counter-propagating muon beams are stored and collide over the roughly 1000–2000 turns corresponding to the muon lifetime.

12.2.1 Technical Motivations

Synchrotron radiation (proportional to the fourth power of the Lorentz factor γ^4) poses severe limitations on multi-TeV e^+e^- colliders, namely they must have a linear, not circular, geometry. Practical acceleration schemes then require a facility tens of kilometers long. Furthermore, beam-beam effects at the collision point induce the electrons and positrons to radiate, which broadens the colliding beam energy distributions. Since $(m_\mu/m_e)^4 = 2 \times 10^9$, all of these radiation-related effects can be mitigated by using muons instead of electrons. A multi-TeV $\mu^+\mu^-$ collider can be circular and therefore have a compact geometry that will fit on existing accelerator sites, and may be significantly less expensive than alternative machines.

The center-of-mass energy spread for a 3-TeV $\mu^+\mu^-$ collider, $dE/E < 0.1\%$, is an order of magnitude smaller than for an e^+e^- collider of the same energy. Additionally, the MC needs lower wall plug power and has a smaller number of elements requiring high reliability and individual control for effective operation [103].

An additional attraction of a MC is its possible synergy with the Neutrino Factory concept [104]. The front-end of a MC, up to and including the initial cooling channel, is similar (perhaps identical) to the corresponding Neutrino Factory (NF) front-end [105]. However, in a NF the cooling channel must reduce the transverse emittances ($\varepsilon_x, \varepsilon_y$) by only factors of a few, whereas to produce the desired luminosity, a MC cooling channel must reduce the transverse emittances (vertical and horizontal) by factors of a few hundred and reduce the longitudinal emittance ε_L by a factor $O(10)$. Thus, a Neutrino Factory could offer the opportunity of a staged approach to a Muon Collider, and also the opportunity of shared R&D.

12.2.2 Design Concepts

Since muons decay quickly, large numbers of them must be produced to operate a muon collider at high luminosity. Collection of muons from the decay of pions produced in proton-nucleus interactions results in a large initial phase volume for the muons, which must be reduced (cooled) by a factor of 10^6 for a practical collider. Without such a cooling, the luminosity reach will not exceed $O(10^{31} \text{ cm}^{-2} \text{ s}^{-1})$, a substantial limitation on the discovery reach of the MC. The technique of ionization cooling [106] is proposed for the $\mu^+\mu^-$ collider [107, 108]. This technique is uniquely applicable to muons because of their minimal interaction with matter.

Ionization cooling involves passing the beam through some material absorber in which the muons lose momentum essentially along the direction of motion via ionization energy loss, commonly referred to as dE/dx . Both transverse and longitudinal momentum are reduced via this mechanism, but only the longitudinal momentum is then restored by reacceleration, leaving a net loss of transverse momentum (transverse cooling). The process is repeated many times to achieve a large cooling factor. The energy spread can be reduced by introducing a transverse variation in the absorber density or thickness (e.g., a wedge) at a location where there is dispersion (a correlation between transverse position and energy). This method results in a corresponding increase of transverse phase space and represents in an exchange of longitudinal and transverse emittances. With transverse cooling, this allows cooling in all dimensions. The cooling effect on the emittance is balanced against stochastic multiple scattering and Landau straggling, leading to an equilibrium emittance.

Theoretical studies have shown that, assuming realistic parameters for the cooling hardware, ionization cooling can be expected to reduce the phase space volume occupied by the initial muon beam by a factor of 10^5 – 10^6 . A complete

Fig. 12.4 Cooling-channel section. Muons lose energy in lithium hydride (LiH) absorbers (blue) that is replaced when the muons are reaccelerated in the longitudinal direction in radio frequency (RF) cavities (green). The few-Tesla superconducting (SC) solenoids (red) confine the beam within the channel and radially focus the beam at the absorbers. Some representative component parameters are also shown (from [101])

cooling channel would consist of 20–30 cooling stages, each stage yielding about a factor of 2 in 6D phase space reduction—see Fig. 12.4.

Such a cooling method seems relatively straightforward in principle, but has proven quite challenging to implement in practice. One of the main issues is breakdown suppression and attainment of high accelerating gradients in normal-conducting RF cavities immersed in strong magnetic fields. The International Muon Ionization Cooling Experiment (MICE [109]) at RAL (UK) was set to test an ionization cooling channel cell consisting of a sequence of LiH absorbers and 201 MHz RF cavities within a lattice of solenoids that provide the required focusing in a 200 MeV muon beam [110]. The initial results indicate anticipated significant emittance reduction $O(10\%)$ in the “no re-acceleration” configuration [111] and, therefore, can be considered as the first experimental proof of the ionization cooling concept.

12.2.3 Technology Development

Multi-MW target R&D has greatly advanced in recent years, and has culminated in the Mercury Intense Target experiment (MERIT [112]) which has successfully demonstrated a Hg-jet injected into a 15 T solenoid and hit by an intense proton beam from the CERN PS. A high- Z target is chosen to maximize $\pi^\pm$ production. The solenoid radially confines essentially all the $\pi^\pm$ coming from the target. The Hg-jet choice avoids the shock and radiation damage related target-lifetime issues that arise in a solid target. The jet was viewed by high speed cameras which enabled measurement of the jet dynamics. MERIT results suggest this technology could support beam powers in excess of 4 MW. More advanced solutions for multi-MW targets are under considerations, too, such as granular waterfall targets [113].

Significant efforts are presently focused on high gradient normal conducting RF cavities operating in multi-Tesla magnetic fields as required in the bunching, phase rotation, and cooling channel designs. Closed 805 MHz RF cells with thin Be windows have initially shown significant reduction of maximum RF gradient in a 3 T field—12 MV/m vs. 17 MV/m specified. Further R&D as part of the U.S. based Muon Accelerator Program (MAP) has experimentally demonstrated some 50 MV/m gradients in the RF cavities with high pressure hydrogen gas [114] and in the Be-coated vacuum cavities [115].

Several self-consistent concepts based on different technologies have recently emerged for the MC six-dimensional cooling channel which plays a central role in reaching high luminosity. To achieve the desired mixing of transverse and longitudinal degrees of freedom, the muons must either pass through a series of wedge absorbers in a ring [116] or be put onto a helical trajectory, e.g., as in a “Helical Cooling Channel” [117] or a “FOFO-snake” [118]. The design simulations of the channels are not yet complete and the main challenges are attainment of sufficiently large dynamic apertures, taking into account realistic magnetic fields, RF cavities and absorbers, optimization of the B-fields in RF cavities and technological complexity. The design of the final cooling stages is particularly challenging as it requires very high solenoid fields (up to ~30 T have been considered [119]). The final MC luminosity is proportional to this field. High-field superconducting magnets for the collider ring and for the cooling have been actively developed [120], including feasibility studies of a high temperature superconductor (HTS) option for the 25–50 T final cooling solenoids [121].

A Recirculating Linac with SC RF cavities (e.g. 1.3 GHz ILC-like cavities) is a very attractive option for acceleration of muons from the low energies emerging from the cooling sections to the energy of the experiments. The recirculating linac offers small lengths and low wall plug power consumption but requires small beam emittances.

Recently, realistic collider ring beam optics has been designed which boasts a very good dynamic aperture for about $dP/P = \pm 0.5\%$ and small momentum compaction [122, 123]. The distortions due to the beam-beam interaction will need to be studied as well as practical issues of the machine-detector interface.

Representative performance parameters for a multi-TeV Muon Collider are given in Table 12.2. These parameters are based on the design concepts described above and represent reasonable extrapolations of technologies currently under development. The luminosities displayed are appropriate for the physics research programs that would be undertaken at such a facility [124–127].

12.2.4 Advanced Muon Collider Concepts

In the last few years several advanced muon collider concepts were proposed. An alternative low emittance muon source based on near-threshold production of muons in the reaction $e+e- \rightarrow \mu+\mu-$ was considered in [128]. The scheme

relies on availability of high intensity beam of 45 GeV positrons hitting solid, liquid or crystal target of Be, C or diamond. The resulting emittance of the muon beam is very small and allows direct acceleration with extensive ionization cooling. Synchrotron radiation of high-energy muons channelling in between crystal planes results in very small emittances, too, and opens opportunities for crystal-based muon colliders. Given natural advantages of muons, such as absence of nuclear interaction characteristic of protons and greatly reduced synchrotron radiation compared to electrons, the muons are particle of choice for ultra-high gradient acceleration in crystals, originally proposed in [129]. Such colliders with gradients $O(0.1\text{--}1\text{ TeV/m})$ can potentially reach c.o.m energies hundreds of times higher than in the LHC collisions, though, by necessity, with lower luminosities due to practical limits on the facility total electrical power consumption $O(100\text{ MW})$ [130]. Of course, significant R&D is needed to demonstrate feasibility of the channelling acceleration in crystals or, as a first step, in carbon nanotubes [131].

References

1. Tajima, T., Dawson, J.M.: Phys. Rev. Lett. 43 (1979) 267.
2. Chen, P., et al.: Phys. Rev. Lett. 54 (1985) 693.
3. O'Connell, C.L., et al.: Phys. Rev. ST Accel. Beams 9 (2006) 101301.
4. Caldwell, A., et al.: Nature Phys. 5 (2009) 363.
5. Ruth, R., et al.: Particle Accelerators 17 (1985) 171.
6. Tsung, F., Narang, R., Mori, W.B., Joshi, C., Fonseca, R.A., Silva, L.O.: Phys. Rev. Lett. 93 (2004) 185002.
7. Mangles, S.P.D., et al.: IEEE Trans. Plasma Sci. 36 (2008) 1715.
8. Geddes, C.G.R., Nakamura, K., Plateau, G.R., Toth, Cs., Cormier-Michel, E., Esarey, E., Schroeder, C.B., Cary, J.R., Leemans, W.P.: Phys. Rev. Lett. 100 (2008) 215004.
9. Suk, H., Barov, N., Rosenzweig, J.B., Esarey, E.: Phys. Rev. Lett. 86 (2001) 1011.
10. Esarey, E., Hubbard, R.F., Leemans, W.P., Ting, A., Sprangle, P.: Phys. Rev. Lett. 76 (1997) 2682.
11. Faure, J., et al.: Nature 444 (2006) 737.
12. Pak, A., Marsh, K.A., Martins, S.F., Lu, W., Mori, W.B., Joshi, C.: Phys. Rev. Lett. 104 (2010) 025003.
13. Oz, E., et al.: Phys. Rev. Lett. 98 (2007) 084801.
14. Clayton, C.E., et al.: Phys. Rev. Lett. 70 (1993) 37.
15. Katsouleas, T., Wilks, S., Chen, P., Dawson, J.M., Su, J.J., et al.: Part. Accel. 22 (1987).
16. Tzoufras, M., et al.: Phys. Plasmas 16 (2009) 056705.
17. Guo-Zheng Sun, Ott, E., Lee, Y.C., Guzdar, P.: Phys. Fluids 30 (1987) 526.
18. Wilks, S.C., Dawson, J.M., Mori, W.B.: Phys. Rev. Lett. 61 (1988) 337.
19. Esarey, E., Ting, A., Sprangle, P.: Phys. Rev. A 42 (1990) 3526.
20. Faure, J., et al.: Phys. Rev. Lett. 95 (2005) 205003.
21. Butler, A., Spence, D.J., Hooker, S.M.: Phys. Rev. Lett. 89 (2002) 185003.
22. Lu, W., Huang, C., Zhou, M., Mori, W.B., Katsouleas, T.: Phys. Rev. Lett. 96 (2006) 165002.
23. Ralph, J.E., Marsh, K.A., Pak, A.E., Lu, W., Clayton, C.E., Fang, F., Mori, W.B., Joshi, C.: Phys. Rev. Lett. 102 (2009) 175003.
24. Leemans, W.P., et al.: Nature Phys. 2 (2006) 696.
25. Ralph, J.E., et al.: Phys. Plasmas 17 (2010) 056709.
26. Kneip, S., et al.: Phys. Rev. Lett. 103 (2009) 035002.

27. Clayton, C.E., et al.: Phys. Rev. Lett. 88 (2002) 154801.
28. Muggli, P., et al.: Phys. Rev. Lett. 93 (2004) 014802.
29. Esarey, E., et al.: IEEE Trans. Plasma Sci. 24 (1996) 252.
30. Zhou, M.: UCLA Ph.D. Thesis (2008).
31. Huang, C., et al.: Phys. Rev. Lett. 99 (2007) 255001.
32. Rosenweig, J.B., et al.: Phys. Rev. Lett. 95 (2005) 195002.
33. Wang, S., et al.: Phys. Rev. Lett. 88 (2002) 135004.
34. Johnson, D., et al.: Phys. Rev. Lett. 97 (2006) 175003.
35. Hogan, M., et al.: Phys. Rev. Lett. 95 (2005) 054802.
36. Chen, P., et al.: Phys. Rev. Lett. 56 (1986) 1252.
37. Bane, K., et al.: IEEE Trans. Nucl. Sci. NS-32 (1985) 3524.
38. Muggli, P., et al.: Phys. Rev. ST Accel. Beams 13 (2010) 052803.
39. Lu, W., et al.: Phys. Plasmas 12 (2005) 63101.
40. Lu, W., et al.: Phys. Rev. Lett. 96 (2006) 165002.
41. Lu, W., et al.: Phys. Plasmas 13 (2006) 56709.
42. M. Tzoufras et al.: Phys. Rev. Lett. 101(2008) 145002.
43. Blue, B., et al.: Phys. Rev. Lett. 90 (2003) 214801.
44. Blue, B., et al.: Laser Part. Beams 21 (2003) 497.
45. Lee, S., et al.: Phys. Rev. E 64 (2001) 04550.
46. Lotov, K.V., et al.: Phys. Plasmas 14 (2007) 023101.
47. Muggli, P., et al.: Phys. Rev. Lett. 101 (2008) 055001.
48. Wang, X., et al.: Phys. Rev. Lett. 101 (2008) 124801.
49. G. Xia et al.: Proceedings IPAC2010, Kyoto, Japan, June 2010, p. 4395
50. Kumar, N., Pukhov, A., Lotov, K.: Phys. Rev. Lett. 104 (2010) 255003
51. P. Muggli et al., Plasma Physics and Controlled Fusion, 60(1) 014046 (2017).
52. N. Savard et al.: in Proc. North American Particle Accelerator Conf. (NAPAC'16), Chicago, IL, USA, Oct. 2016, paper WEPOA01, pp. 684, 2017, M. Moreira, Phys. Rev. Accel. Beams 22, 031301 (2019)
53. AWAKE Collaboration: *Plasma Phys. Control. Fusion* **56** (2014) 084013
54. A. Caldwell et al.: Physics of Plasmas 18 (2011) 103101
55. Joshi, C., et al.: Nature 311 (1994) 525.
56. Amiranoff, F., Bernard, D., Cros, B., Jacquet, F., Matthieussent, G., Mine, P., Mora, P., Morillo, J., Moulin, F., Specka, A.E., Stenz, C.: Phys. Rev. Lett. 74 (1995) 5220.
57. Darrow, C., et al.: Phys. Rev. Lett. 56 (1986) 2629.
58. Everett, M., et al.: Nature 368 (1994) 527.
59. Tochitsky, S., et al.: Phys. Rev. Lett. 92 (2004) 095004.
60. Joshi, C., et al.: Phys. Rev. Lett. 47 (1981) 1285.
61. Mori, W.B., Decker, C.D., Hinkel, D.E., Katsouleas, T.: Phys. Rev. Lett. 72 (1994) 1482.
62. Coverdale, C.A., Darrow, C.B., Decker, C.D., Mori, W.B., Tzeng, K.-C., Marsh, K.A., Clayton, C.E., Joshi, C.: Phys. Rev. Lett. 74 (1995) 4659.
63. Modena, A., Najmudin, Z., Dangor, A.E., Clayton, C.E., Marsh, K.A., Joshi, C., Malka, V., Darrow, C.B., Danson, C., Neely, D., Walsh, F.N.: Nature 377 (1995) 606.
64. Faure, J., Glinec, Y., Pukhov, A., Kiselev, S., Gordienko, S., Lefebvre, E., Rousseau, J.-P., Burgy, F., Malka, V.: Nature 431 (2004) 541.
65. Geddes, C.G.R., Toths, C., van Tilborg, J., Esarey, E., Schroeder, C.B., Bruhwiler, D., Nieter, C., Cary, J., Leemans, W.P.: Nature 431 (2004) 538.
66. Mangles, S.P.D., Murphy, C.D., Najmudin, Z., Thomas, A.G.R., Collier, J.L., Dangor, A.E., Divall, E.J., Foster, P.S., Gallacher, J.G., Hooker, C.J., Jaroszynski, D.A., Langley, A.J., Mori, W.B., Norreys, P.A., Tsung, F.S., Viskup, R., Walton, B.R., Krushelnick, K.: Nature 431 (2004) 535.
67. Tsung, F.S., Lu, W., Tzoufras, M., Mori, W.B., Joshi, C., Vieira, J.M., Silva, L.O., Fonseca, R.A.: Phys. Plasma 13 (2006) 56708.
68. Pukhov, A., Meyer-Ter-Vehn, J.: Appl. Phys. B 74 (2002) 355.
69. Gordienko, S., Pukhov, A.: Phys. Plasmas 12 (2005) 043109.

70. Lu, W., Tzoufras, M., Joshi, C., Tsung, F.S., Mori, W.B., Vieira, J., Fonseca, R.A., Silva, L.O. *Phys. Rev. ST Accel. Beams* 10 (2007) 061301.
71. Martins, S.F., Fonseca, R.A., Lu, W., Mori, W.B., Silva, L.O.: *Nature Phys.* 6 (2010) 311.
72. Froula, D.H., Clayton, C.E., Döppner, T., Marsh, K.A., Barty, C.P.J., Divol, L., Fonseca, R.A., Glenzer, S.H., Joshi, C., Lu, W., Martins, S.F., Michel, P., Mori, W.B., Palastro, J.P., Pollock, B.B., Pak, A., Ralph, J.E., Ross, J.S., Siders, C.W., Silva, L.O., Wang, T.: *Phys. Rev. Lett.* 103 (2009) 215006.
73. Osterhoff, J., Popp, A., Major, Zs., Marx, B., Rowlands-Rees, T.P., Fuchs, M., Geissler, M., Hörlein, R., Hidding, B., Becker, S., Peralta, E.A., Schramm, U., Grüner, F., Habs, D., Krausz, F., Hooker, S. M., Karsch, S.: *Phys. Rev. Lett.* 101 (2008) 085002.
74. Perry, M.D., Mourou, G.: *Science* 264 (1994) 917.
75. Umstadter, D., Chen, S.-Y., Maksimchuk, A., Mourou, G., Wagner, R.: *Science* 273 (1996) 472.
76. Clayton, C.E., Ralph, J.E., Albert, F., Fonseca, R.A., Glenzer, S.H., Joshi, C., Lu, W., Marsh, K.A., Martins, S.F., Mori, W.B., Pak, A., Tsung, F.S., Pollock, B.B., Rosse, J.S., Silva, L.O., Froula, D.H.: *Phys. Rev. Lett.* 105 (2010) 105003.
77. Rouse, A., et al.: *Phys. Rev. Lett.* 93 (2004) 135005.
78. Fuchs, M., et al.: *Nature Phys.* 5 (2009) 826.
79. Schenvoigz, H.P., et al.: *Nature Phys.* 4 (2008) 130.
80. Leemans, W.P., Esarey, E. *Physics Today* 62 (2009) 44.
81. Rosenzweig, J.B., et al.: *Phys. Rev. Lett.* 61 (1988) 98.
82. Barov, N., et al.: *Phys. Rev. Lett.* 80 (1998) 81.
83. Nakanishi, H., et al.: *Phys. Rev. Lett.* 66 (1991) 1870.
84. Hogan, M., et al.: *Phys. Plasmas* 7 (2000) 2241.
85. Joshi, C., et al.: *Phys. Plasmas* 14 (2007) 055501.
86. Muggli, P., et al.: *IEEE Trans. Plasma Sci.* 27 (1999) 791.
87. Blumenfeld, I., et al.: *Nature* 445 (2007) 741.
88. Hogan, M., et al.: *New J. Phys.* 12 (2010) 055030.
89. Muggli, P., et al.: *New J. Phys.* 12 (2010) 045022.
90. E. Gschwendtner et al.: *Nucl. Instr. and Meth. in Phys. Res.* **A829** (2016) 76.
91. A. Caldwell et al.: *Nucl. Instr. and Meth. in Phys. Res.* **A829** (2016) 3.
92. E. Öz et al.: *Nucl. Instr. and Meth. in Phys. Res.* **A740** (2014) 197, E. Öz et al.: *Nucl. Instr. and Meth. in Phys. Res.* **A829** (2016) 321.
93. M. Turner et al.: submitted to *Nucl. Instr. Meth. Phys. Res. A*, (2017), M. Turner et al.: *Nucl. Instr. and Meth. in Phys. Res.* **A829** (2016) 314, M. Turner et al.: *Nucl. Instr. and Meth. in Phys. Res.* **A854** (2017) 100.
94. K. Rieger et al.: *Review of Scientific Instruments* **88** (2017) 025110.
95. M. Martyanov et al.: in preparation, F. Braumueller et al., *Nucl. Instr. and Meth. in Phys. Res. A*, 909, 76 (2018).
96. Y. Fang et al.: *Phys. Rev. Lett.* **112** (2014) 045001.
97. V. K. Berglyd Olsen et al.: accepted for publication in *Phys. Rev. Accelerators and Beams* (2017).
98. A. Caldwell et al.: *Eur. Phys. J.* **C76** (2016) 463.
99. A. Martinez de la Ossa et al.: AIP Conference Proceedings 1507 (2012) 588, O.Lishilin et al.: *Nucl. Instr. and Meth. in Phys. Res.* **A829** (2016) 37.
100. G. Budker: *Proc. 7th Intern. Conf. High Energy Accel.*, Yerevan, (1969) 33.
101. S. Geer: *Annu. Rev. Nucl. Part. Sci.* 59 (2009) 347–365.
102. S.D. Holmes, in: *Proc. 2010 Intern. Part. Accel. Conf.*, Kyoto, Japan, (2010) 1299.
103. V. Shiltsev: *Mod. Phys. Lett. A* 25 (2010) 567–577.
104. S. Geer: *Phys. Rev. D* 57 (1998) 6989.
105. *The Neutrino Factory Intern. Scoping Study Accelerator Working Group Report*, J. Instrum. 4 (2009)P07001.
106. Yu. Ado, V. Balbekov: *Atomnaya Energiya*, 31 (1971) 40; transl. in: *Sov. Atomic Energy* 31 (1971) 731.

107. A. Skrinsky, V. Parkhomchuk: Sov. J. Nucl. Phys. 12 (1981) 3.
108. D. Neuffer: Part. Accel. 14 (1983) 75.
109. R. Sandstrom (MICE Collab.): AIP Conf. Proc. 981 (2008) 107.
110. M. Bogomilov et al. (The MICE collaboration), Phys. Rev. Accel. Beams 20 (2017) 063501
111. Rogers, C. T. et al, in Proc. 2017 IPAC (Copenhagen, Denmark), (2017) 2874.
112. H. Kirk, et al. (MERIT Collab.), in: Proc. 2007 IEEE Part. Accel. Conf. (Albuquerque, NM, USA), (2007) 646.
113. H.J. Cai, et al.: Phys. Rev. Accel. Beams 20 (2017) 023401.
114. M. Chung, et al, Phys. Rev. Lett. 111 (2013) 184802.
115. D. Bowring, et al, in: Proc. 2016 IPAC (Busan, Korea), (2016) 444.
116. R. Palmer, et al: Phys. Rev. ST Accel. Beams 8 (2005) 061003.
117. Ya. Derbenev, R. Johnson: Phys. Rev. ST Accel. Beams 8 (2005) 041002.
118. Y. Alexahin, AIP Conference Proceedings 1222, no.1 (2010) 313.
119. D. Neuffer, et al, JINST 12 T07003 (2017)
120. G. Apollinari, S. Prestemon, A. Zlobin, Annu. Rev. Nucl. Part. Sci. 2015.65:355-377
121. V. Kashikhin, et al, IEEE Trans. Appl. Superconductivity, 18, no. 2 (2008) 938
122. Y. Alexahin, E. Gianfelice-Wendt, in: Proc. 2009 IEEE Part. Accel. Conf. (Vancouver, Canada), (2009) 3817.
123. M.H. Wang, et al, JINST 11 P09003 (2016).
124. E. Eichten, A. Martin, Physics Letters B 728 (2014) 125
125. R. Brock, et al, arXiv:1401.6081
126. A. Conway, et al, arXiv:1405.5910.
127. N. Chakrabarty, et al, Physical Review D, 91(1), 015008 (2015).
128. M. Antonelli, et al, NIM-A 807 (2016) 101.
129. T. Tajima, M. Cavenago Phys. Rev. Lett. 59 (1987) 1440.
130. V. Shiltsev, Physics-Uspekhi 55.10 (2012) 965.
131. X. Zhang, et al, Phys. Rev. Accel. Beams 19, 101004 (2016)

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