Chapter 4

Mathematics and Common Sense—The Dutch School

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Abstract The old illusion in Dutch mathematics education was that the teacher could lead the students into a completely new world, ignoring all their prior knowledge and common sense. Nowadays, in many Dutch mathematics lessons, the teachers encourage their students to use their common sense. This is the result of a silent revolution in mathematics education. In this chapter I will offer a collage of the work of several mathematics educators, who have helped to put the common sense of students in the middle of Dutch mathematics education. We will meet students from age 6 to 16 working with whole numbers, fractions, geometry and exponential functions and we will discover how their common sense plays a crucial role in the development of their mathematical knowledge.

4.1 Introduction

A long time ago, at the start of my first mathematics lesson as a secondary school student, the mathematics teacher told us: “Forget what you know, here you will learn all sorts of new things.” As if we could delete all our prior knowledge and would not try to make sense of all these ‘new things’. This is an illusion. An echo of this view still survives in places. When I had a discussion with a mathematics teacher at the university and asked him how they orientate themselves on their students’ prior knowledge, he answered: “We just build the whole mathematics as new, so prior knowledge is not necessary.”

In the meantime, in primary schools and later in secondary schools, a silent revolution in mathematics education has taken place and nowadays the motto in Dutch mathematics education is ‘use your common sense!’ (see Dekker, 1996).

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4.2 Common Sense of Young Students

What is the common sense mathematical knowledge of students before they get any formal mathematics education? Marja van den Heuvel Panhuizen (1996) researched this question. She developed a paper-and-pencil test to find out how much common sense mathematical knowledge and capabilities young students at the age of 6 already have before they start the formal mathematical course at primary school. She gave the test to 441 students in 22 different primary schools. The teachers only gave a short oral instruction. The test was very visual and consisted for instance of adding up the dots on two dice in the context of a board game, counting down in the context of a missile launch, and subtracting without countable objects in the context of buying sunglasses with a given amount of pocket money. She also gave the test to a group of experts, consisting of school consultants, teacher educators, educational researchers, educational developers and a few primary school teachers. She compared their answers with the answers of the students. The experts clearly underestimated the knowledge of the students. The task of adding up the dots on a dice was expected by the experts to be answered correctly by 45% of the students, while the actual result was 80%. The counting down task was expected to be answered correctly by 25% of the students, while the actual result was 65%. And for the task on buying sunglasses the experts thought that close to 0% of the students would give a good answer, while the actual result was 40%!

The research findings of Marja van den Heuvel-Panhuizen were very striking and gained much attention. Several countries, including Germany, have repeated the research with similar outcomes.

So, students acquire more mathematical knowledge in realistic contexts of their life than we are aware of. And as we want to connect the mathematical knowledge we want them to learn with the knowledge they already have, we can use these contexts and make the mathematics in them the object of discussion in the classroom, as you can see in many mathematical learning materials for primary schools in the Netherlands.

4.3 A ‘Math Mom’ at Work with a Small Group

In many primary schools the parents are asked to participate in reading with some children. A mother of one of the children in a primary school in the centre of Amsterdam suggested to the school that she could do something with mathematics as a so-called ‘math mom’. The school reacted very positively and her daughter’s teacher suggested doing something extra with fractions. Many students find that subject problematic. The school has a mathematics textbook series based on Realistic Mathematics Education (RME), but the reality in it is often restricted to pictures and the tasks are often meant for individual work with paper and pencil. To compensate for

this, the math mom combined her knowledge about fractions in the context of pizzas (Treffers, Streefland, & De Moor, 1994) with her knowledge about collaborative learning (Dekker & Elshout-Mohr, 2004). On several afternoons, you could see her working with a small group of three children in Grade 4, where they played that they each were going to buy small pizzas to be shared by the four of them, mom included. The pizzeria had a special offer: three small pizzas for 5 euro. So, they had to divide. On the table, there were circles of cardboard of different colours, representing different types of pizzas; yellow could be cheese, red could be tomatoes or salami and green could be basil or olives. There were also scissors, pencils and rulers for the dividing.

The first child chose a type of small pizza and the math mom said that the three pizzas of this type would be served one after the other. The children discussed how to divide the pizzas fairly without having to wait for the next one to be served. The math mom intervened as little as possible. She only did so when she noticed that a child was not taking part in the discussion or was not 'heard'. The children decided that they would divide each pizza in four pieces and so they did. Each of the four persons ended up with $\frac{3}{4}$ pizza, and then the math mom asked them to write that down in a fraction problem that reflects this action. Again, they discussed it and then decided to write it as: $\frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4}$.

Then the second child chose another type of pizza and the math mom told them that now two pizzas would be served at the same time and the third one a bit later. Again, the children discussed how to divide fairly without waiting until the third one arrived. They decided that it made sense to give each participant half a pizza and divide the last one in four pieces later. Then they discussed what fraction problem reflected this division and they came up with: $\frac{1}{2} + \frac{1}{4} = \frac{3}{4}$.

The third child also chose a particular type of pizza, and the math mom told them that now all pizzas would be served at the same time. The children decided that they would each take one pizza and that each of them would give the math mom a quarter of their pizza: $1 - \frac{1}{4} = \frac{3}{4}$.

So, in each situation each person ended up with $\frac{3}{4}$ of a pizza. But each situation was a different one and so was the fraction problem that reflected this situation.

The children clearly enjoyed this 'play', and the math mom did as well. But as she also wanted them to reflect on the work, she asked them at the end to individually describe in a little story one of the situations, visualise it in a drawing and write down the problem that fits this situation. Most children chose their 'own' division, but some described them all and with beautiful drawings, even a comic was drawn; and with the appropriate problems (Fig. 4.1). Mathematics can be fun, especially when you bring to life the situations in which it can develop.

Fig. 4.1 Visualisation of $1 - \frac{1}{4}$

4.4 A Russian Pioneer Within the Dutch School

Tatiana Ehrenfest-Afanassjewa (1876–1964), a Russian mathematician, came to the Netherlands in 1912 with her husband Paul Ehrenfest, the physicist, who had got a job at the University of Leiden. Tatiana had a deep interest in mathematics education. In Russia, she had developed an introductory course for geometry. The course was based on the idea that students already have developed intuitive geometrical notions in reality. These notions form the start of the course. In her *Übungensammlung* (Ehrenfest-Afanassjewa, 1931, p. 27), she described all sorts of problems to be analysed by the students. Problems like:

Warum läuft der Mond uns nach? Warum laufen uns die nahen Gegenstände rascher vorbei, als die entfernten, wenn wir etwa in einem Zuge fahren? – Eine schematische Zeichnung machen.¹

In the Netherlands, Tatiana started to invite mathematics teachers and mathematicians into her home in Leiden to discuss mathematics education and her ideas about it (De Moor, 1993). Most of her guests were ‘shocked’ by her radical ideas, but some

¹Why does the moon run after us? Why do near objects pass us more rapidly than distant ones when we ride, say, in a train? Make a schematic drawing (translated by the author).

of them became really interested and tried to put her ideas into practice. Dina van Hiele-Geldof, for example, developed her lessons on tiles and geometry and analysed the classroom conversations and student work wonderfully in her dissertation (Van Hiele-Geldof, 1957). There was also a German mathematician who had come to the Netherlands and was impressed by Tatiana's revolutionary ideas: Hans Freudenthal. He discussed these ideas in her house and in public (Ehrenfest-Afanassjewa & Freudenthal, 1951). Later he stimulated developers of learning materials to use her ideas and now in many Dutch mathematics school books for secondary school subjects like looking along lines, or vision geometry, form a substantial part of geometry. The following exercise is used even in primary schools. It is taken from the Ehrenfest-Afanassjewa's (1931, p. 25) *Übungensammlung*:

Es sollen sich drei Schüler längs einer Geraden vor die Klasse aufstellen – ohne irgend welche Hilfsmittel zu gebrauchen; ein vierter Schüler soll sie, ebenfalls ohne Hilfsmittel, kontrollieren. – Worauf beruht die Möglichkeit dieser Aufgabe?²

4.5 A World of Packages

Wim Sweers was a teacher at a secondary school where many students had problems with mathematics. The subject he focused on was three-dimensional mathematical shapes, which is always a very difficult subject. Walking through the school and thinking about problems to introduce the subject, he noticed a glass case with many beautiful packages, made by his own students! He realised that they had to do a lot of implicit mathematics to create these packages. He decided to bring the world of packages into the classroom and so he did. He asked his students to collect different empty packages from home and he invited them to sort them according to function, material, colour and also according to the mathematical shapes which he wanted to teach them. Then the students were invited to deconstruct the packages and reconstruct them again, and so they discovered all sorts of characteristics of the shapes. And step by step he guided his students from the world of packages into the world of mathematical shapes, which were now much more concrete to them than they had been. He got the feeling that his students would now be able to develop some spatial insight.

This strong idea has found its way into Dutch mathematics school books and also into mathematics schoolbooks abroad, for instance in Portugal.

²Three students should stand in a straight line in front of the class without using any tools; a fourth student should check them, also without any tools. What makes this problem possible? (translated by the author).

4.6 A Real Problem in the Classroom

During one of my visits to a mathematics lesson by teacher Lidy, she introduced a problem about cleaning a brush after painting. She said that she had painted her garden fence and that she wanted to clean the brush. She found only a little bit of turpentine in a bottle and she wondered what she could do best: pour the turpentine in a jam jar and clean the brush, or first divide the turpentine over two jam jars and clean the brush in the first jar and then in the second. The students worked in small groups and the teacher gave them all the information they wanted. They concluded to their own surprise that it was better to divide the turpentine over two jam jars. Then the teacher went on and asked them what would happen if the turpentine were divided over more cups, and more and more... and so she was planning to draw her students into the world of formal mathematics with the number 'e' at the horizon.

One small group of students was arguing a lot. They wondered whether it would work to clean a brush in just a little bit of turpentine. They remembered a test they did in chemistry where they put sugar in water and noticed at a certain moment that the sugar did not dissolve any more. They thought that could also happen when the quantity of turpentine was too small. I witnessed their discussion. These students clearly stopped in front of the door of formal mathematics. Their common sense prevented them from passing it. Later I told a colleague from abroad about this conversation and she reacted that the teacher should have told the students that in the mathematics lesson it is all about mathematics. I heard an echo of an old message... I also told teacher Lidy about my observation and she answered: "I should have discussed the problem with the chemistry teacher first."

Lidy made my heart glow. She clearly is a member of the Dutch School!

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