Correction to: Nonlinear, Nonlocal and Fractional Turbulence

Correction to:

P. W. Egolf, K. Hutter,

Nonlinear, Nonlocal and Fractional Turbulence,

<https://doi.org/10.1007/978-3-030-26033-0>

Below corrections are updated in book.

Text book cover image,

third line from bottom: ...with permission. ©2019 American Institute of Physics

Page v, 2nd line: ...of irregularity, with all its facets, is a main topic. . .

5th line: . . . , the main ingredients in life, art, music, etc.

7th line . . . order and disorder, with different

8th line . . . tempi and intensities, gives our. . .

11th line . . . (energy input), it gives. . .

20th line . . . (from its big bang), but even the creation. . .

Page vi, 17th line He, at . . .

The updated online versions of the chapters can be found at

https://doi.org/10.1007/978-3-030-26033-0_1

https://doi.org/10.1007/978-3-030-26033-0_2

https://doi.org/10.1007/978-3-030-26033-0_3

https://doi.org/10.1007/978-3-030-26033-0_5

https://doi.org/10.1007/978-3-030-26033-0_6

https://doi.org/10.1007/978-3-030-26033-0_7

https://doi.org/10.1007/978-3-030-26033-0_8

https://doi.org/10.1007/978-3-030-26033-0_9

https://doi.org/10.1007/978-3-030-26033-0_10

https://doi.org/10.1007/978-3-030-26033-0_11

https://doi.org/10.1007/978-3-030-26033-0_12

<https://doi.org/10.1007/978-3-030-26033-0>

18th line ...his time, did not distinguish between art and science.

Page vii, Pic. 3, Tab. Capt.: In June 1889, the famous impressionist artist Vincent van Gogh painted the picture “starry night” by oil on canvas.

1st line: In the literature on turbulence, Leonardo da Vinci’s drawings...

Page x, 3rd line: This is all well known, as is the fact...

16th line: ... a flux law, in analogy to Fick’s law...

Page xi, 5th line: In April 1985, one of us (Peter William Egolf – PWE), deriving the *Difference-Quotient Turbulence Model* (DQTM), got automatically involved into the problem of nonlocality and started to tackle it.

11th line: ... engineering experts, applying often stronger...

16th line: ...and, thereby, describes a *hereditary*...

25th to 27th line: ...approach, because in the mathematical formulation the positions in between these three distinct points are of no importance.

34th line: Based on these new ideas, the first author...

Page xii, 10th and 11th line: ...the entire spectrum and, for large wave numbers, contains Kraichnan’s correct part as a special case.

Page xiii, 33rd line: ...”constitutive equation”, is reviewed and...

40th to 42nd line: Many zero-equation turbulence models of Prandtl and his contemporary scientists and engineers are reviewed, and their advantages and disadvantages are discussed.

Page xiv, 13th line: After its development in the middle of the 1980s and its first publication in...

40th line: ... constant in the thermal quasi-equilibrium enstrophy range.

Page xv, 21st & 22nd line: ... their care and positive influence on our work-life balances, which is...

26th line: Good support of production was also given by Shivakumar Krithika.

Page xviii and page 188: 9.2.1 Jets in Quiescent Surroundings

and page 220: 9.2.2 Jets in Parallel Co-flows

Pages xix and 338: Justification of the Quadratic Form of the Energy as a Function of Space and Velocity Coordinates

Page xix and page 358: 11.2.1 What is a Critical...

and page 372: 11.2.7 Turbulent Phase Transition with its Two Phases...

Page xx and page 413: Appendix E: Fourier Transform for q -Generalized Energy. . .

Page xxi: $\bar{a}(\bar{t})$ Mean acceleration of a fluid lump on the axis of a two-dimensional turbulent jet

$\bar{\mathbf{A}}_2$ Averaged second Rivlin-Ericksen tensor

Page xxii: b Half-width of turbulent shear flows, e.g., axi-symmetric jets

c Constant in characteristic function of Lévy flight distribution

Page xxiii: da Differential surface element

dv Differential volume element

${}_L D_+^\alpha$ Left-handed Liouville fractional derivative of order α

${}_L D_\pm^\alpha$ Left- and right-handed Liouville fractional derivative of order α

$\hat{\epsilon}(k)$ Energy density in Fourier space of ϵ at wave number k

E_ϵ Turbulent kinetic energy of a two-dimensional flow

E_H Turbulent kinetic energy of a three-dimensional flow

$\check{E}_\epsilon$ Generalized energy of a two-dimensional flow

$\hat{\check{E}}_\epsilon(k)$ Fourier transform of the generalized energy $\check{E}_\epsilon$

$\check{E}_H$ Generalized energy of a three-dimensional flow

Page xxiv: $g(x, x')$ Nonlocality function, response function

$g_H(x, x')$ Hamba's response function

$g_K(x, x')$ Kraichnan's response function

G_{21} Shear modulus of plane shear flows

Page xxv: $\mathbf{H}$ Hessian tensor of the pressure field

k - l Two-equation turbulence model for the transport of turbulent kinetic energy k and Kolmogorov length l

k - ω Two-equation turbulence model for the transport of turbulent kinetic energy k and vorticity ω

k' Lowest occurring wave number modulus

k'' Highest occurring wave number modulus

$\mathbf{K}$ Bachelor's diffusivity tensor ("coefficient")

Page xxvi: m_p Scalar magnetization

m_0 Mass flux of axi-symmetric turbulent jet in the orifice

$n_{\min}$ Eddy density of the eddies on the lower side of x_2^* ; see Fig. 7.5

$n_{\max}$ Eddy density of the eddies on the upper side of x_2^* ; see Fig. 7.5

Page xxvii: $\bar{p}_{\max}$. . . of the smaller scale. . .

$\vec{p}_B$ Momentum of particle B on trajectory x_B

q Index to identify Tsallis's thermodynamic quantities

Page xxviii: s Step size in a Lévy flight distribution

$\hat{S}_{\pm}$ Positive and negative shift operator

Page xxix: $\tilde{t}$. . . the orifice and moving on the jet axis

$\bar{u}$ Downstream velocity of a fluid lump moving on the axis of a turbulent axisymmetric jet

Page xxx: $\hat{u}_{\vec{n}}^{(\kappa', \kappa'')}$. . . and highest wave number $\vec{k}'' = \kappa''(n_1, n_2, n_3)$

$\hat{u}_{\vec{k}}$ Fourier. . .

Page xxxi: $\bar{x}$ Downstream position of a fluid lump moving on the axis of a turbulent axisymmetric jet

x_0 Core length of a turbulent cylindrical jet

$\bar{x}_0$ Streamwise position of a fluid lump moving on the axis of a turbulent axisymmetric jet at the end of the theoretical core

$\bar{x}_1$ Streamwise position of a fluid lump moving on the axis of a turbulent axisymmetric jet

Page xxxii : α Order of Liouville fractional derivative

α Exponent of a power law describing the workman's sizes

α Critical exponent for a quantity that approaches criticality from above

α' Critical exponent for a quantity that approaches criticality from below

β Factor for normal turbulence intensity.

β^* Exponent. . . to Re_{crit}

γ Critical exponent for a quantity that approaches criticality from above

γ Reduction factor for normal turbulence intensities. . .

γ Scale factor in orthotropic turbulence intensity para..

Page xxxiii: γ' Critical exponent for a quantity that approaches criticality from below

ϵ_m Eddy viscosity of turbulent momentum transfer

Page xxxiv : ν Critical exponent for a quantity that approaches criticality from above

ν' Critical exponent for a quantity that approaches criticality from below

ξ Integration variable in Liouville fractional derivative

Page xxxv: σ_q Standard deviation of q -Gaussian distribution

τ_0 Viscous shear stress of flow at the wall

Page 3, 6th line: ... boundary, as it was derived with Prandtl's mixing-length...

11th line: ... working lifetime prior to 1988, was the most important...

12th line: Turbulence, by its field quantities, ...

14th and 15th line: Chaos and turbulence, in all its facets, fascinates...

Page 7: Works of additional scientists and engineers, who studied fluid flows theoretically, numerically or at laboratory scale and contributed significantly to the field of turbulence, will be cited below.

Page 8: Clay,: [http... prize-problems](http://prize-problems) (2000)

Page 16, 17th line: The two terms describing the divergence of $\langle g' \vec{u}' \rangle$ and of $\langle \vec{g}' \otimes \vec{u}' \rangle \dots$

Page 17, 16th line: ...contains the substantial derivative...

Page 18, 10th line: Therefore, six unknown components of this tensor must...

Page 19, 17th line: $\langle \text{mom. eq.} \rangle \cdot \langle \vec{u} \rangle = 0 \neq \langle \text{mom. eq.} \cdot \vec{u} \rangle = 0$ (20a-c)

Page 20, 6th line: ... more convenient than Eq. (21), but mathematically...

Page 21, 28th line: $\langle \text{mom. eq.} \rangle \otimes \dots$ (29)

Page 31, 21st line: 4. Mass of constituents.

Page 33, 21st line: $\tau_{21} = -\rho(\overline{\mathbf{T}}_R)_{21} = \dots$ (48a-c)

Page 35, last line: ... effects, from Eq. (16b) it follows that

Page 37, 3rd line: $\tau_{21} = -\rho \dots$ (65a,b)

23rd line: ... interpreted as a derivative somewhere between...

Page 38, last line: ... boundary regions, because there the eddy...

Page 39, 11th line: Turbulence is rotational and, therefore, flows are...

Page 42, 16th line: This was already noticed by Taylor, who found...

Page 47, 16th line: $\dots \frac{1}{v_T}$ (106a,b)

Page 49, 5th & 6th line: ... Prandtl number is defined as the ratio...

Page 52, 10th & 11th line: ...is applied to linear and nonlinear eddy viscosity models.

Page 54, 20th line: $\overline{u'_2 u'_1} \neq \propto \frac{d\bar{u}_1}{dx_2}, \quad \alpha < \infty, \quad \text{Re} \rightarrow \infty$ (131a,b)

25th line: ... eg., Eqs. 53a,b,69,71, and 89a,b). *It immediately...*

Page 55, 21st line: 3. The fallacies of turbulence modeling.

25th line: ...the role of higher-order moments. ...

26th line: ...influence of higher-order moments. ...

26th line: ..., then higher-order turbulence modeling. ...

27th line: ...superior to lower-order methods.

28th line: ..., applied to higher-order moment closures, ...

Page 56, 8th line: ... 1960's and 1970's, because of the ambition to treat complex geometries, was mainly. ...

Page 57, 17th line: ...in the Reynolds stresses a higher-order term by products of. ...

Page 62: $u^+(y^+)|_{y^+=0} = 0$ (147)

Page 63, 8th line: ...with decreasing thickness as a function of increasing Reynolds number.

$B = A \left[\log_e \left(\frac{4}{\lambda} \right) - 1 \right], \quad A = \frac{1}{\kappa}$ (158a,b)

Page 65: Table 5.3 : ...applying Lie group methods, shows that for turbulent shear flows any function and, thus, any scaling law is admissible. ... dependent coefficients (incomplete similarity) is obtained.

Page 66: Toward the wall, approximately the same laws are valid for plane (plate or channel) and cylindrical (pipe) geometries.

Page 67 : $\lim_{\text{Re} \rightarrow \infty} \frac{\bar{u}_1(\text{Re}, x_2)}{U} = 1$ (162)

Page 68 : ..., using Lie group analysis, he demonstrated that any function is fully admissible.

Page 71: Eisner, F., Reibungswiderstand... (1932) (in German)

Page 72: Kevlahan, N.K.R., Farge. ...

Page 76, 38th line: ...trajectory of the real particle, contributing to fluctuations and..

Page 79, 23rd line: ...length of the overall flow field L and its related turnover time T ?

Page 80, 16th line: A first class of nonlocal models. ...

22nd line: ... (173a-c)

Page 84, 20th & 21st line: After multiplication of both sides of this equation with dx_2 and introducing the differences. . .

Page 91, 2nd line: . . .which is the Hessian tensor of the pressure. . .

6th line: . . ., it follows from Eq. (213a) that

Page 92, 5th line: Note the formal analogy with the one-dimensional version given by Eq. (207).

Page 94, 20th line: . . .where the D/Dt is the substantial time derivative. The. . .

Page 95, 8th line : . . .been neglected, because in turbulence. . .

Page 98, 10th line: By integrating Eq. (238), it follows that

Page 99, 1st line: $\hat{u}_{1a} - \hat{u}_{1b} = - \int_{\tau}^0 \frac{\partial p}{\partial x}(s) ds + \frac{1}{Re} \int_{\tau}^0 \Delta \hat{u}_1 ds,$ (239)

4th line: . . . quantities yields the non-normalized

Page 102, 28th line: . . .the local strain rate ($\sim$ local mean velocity gradient) is. . .

Page 104: Liepmann, H.W.: Mécanique. . .

Page 107, 5th line: . . .will be determined for a wake flow behind a cylinder. . .

Page 108: At this stage, in April 1985, Egolf was somewhat sceptical to take this particular result for granted and to accept it as a basis for further investigation.

Page 112: $\frac{d\delta_{21}}{dt} = - \lim_{\substack{\Delta t \rightarrow 0 \\ \Delta x_2 \rightarrow 0}} \frac{\arctan\left(\frac{\delta x_1}{\Delta x_2}\right)}{\Delta t}$ (272)

Page 117: Throughout this page, the averaged quantities $\overline{\Delta x_1}$ and $\overline{\Delta x_2}$ must be replaced by Δx_1 and Δx_2 .

$$\frac{d\delta_{T21}}{dt} = - \lim_{\substack{\Delta t \rightarrow 0 \\ \overline{\Delta x_2} \rightarrow \max}} \frac{\arctan\left(\frac{\delta \bar{x}_1}{\Delta x_2}\right)}{\Delta t} \quad (290)$$

Because the fluid motion fluctuates, the average deformation $\delta \bar{x}_1$ of a fluid element is considered.

Page 117, 19th line: . . .spatial difference Δx_2 and the mean velocity difference. . .

Page 121, 16th line: $\bar{u}_2^* = \text{const} \times \bar{u}_1^*.$ (298)

Page 125, Table 7.1, ninth line, second row: Density of molecules n

Table 7.1, ninth line, third row: Distribution of eddy density n

Page 126, 2nd line: Consider a random walk with steps of. . .

Page 129, 21st line: In the static picture, describing. . .

22nd line: . . . describing probability distributions, we used. . .

Page 130, 2nd line: . . .such as turbulent momentum or particle concentration, are also . . .

Page 135: The size of eddies of this class is defined by Kolmogorov's dissipation length scale (see Sect. 7.2.4.11).

Page 141: The special case of Kolmogorov/Obukhov, for infinite Reynolds number flow, ($a=b=2$, $d=D=3$) leads to

Page 144: $\hat{p}_n = \rho q_0 \left(\frac{a}{b^{(2+d-D)/3}} \right)^n u_0 \left(\frac{1}{b^n} \right)^{(1+D-d)/3} = \dots$ (398a,b)

Page 151: $\kappa_{k,\infty} = \frac{\hat{P}_{k,\infty}}{\hat{P}_0} = \dots, \quad \kappa_{m,\infty} = \frac{\hat{P}_{m,\infty}}{\hat{P}_0} = \dots$ (429a-d)

Page 152, 8th line: This is the DQTM which—at the time of its derivation in 1985—was developed by different reasoning.

Page 154, 20th line: . . . gorov microscales vanish.

Page 160, 6th line: . . . and consider the Liouville weight $\omega(\lambda, \tau) = \tau^{\lambda-1}$ ($0 < \lambda < 1$), . . .

Page 163-165: By replacing the two derivatives in Prandtl's mixing-length model (Eq. 53b) by the left- and right-handed Liouville fractional derivative each, . . .

In equations (468b), (470), (473) and (474) the spatial derivatives are the left- and right-handed Liouville fractional derivative ${}_L D_{\pm}^{\alpha}$.

Page 165, second last line: . . . leads to the same result, makes it. . .

Pages 165, 167 and 168: In equations (477), (481) and (482) the derivatives are $\frac{\partial \bar{u}}{\partial x_2}$.

Pages 168, 8th line: *We thus have proven that the DQTM. . .*

Page 171: Einstein, A.: Zur. . .Ann. Phys. **19**, 371 (1906). (in German)

Page 172: Osterwalder, K.: Methoden. . .Mathematics and Physics (1980). . .

Page 175, last line: . . . and the dimensionless Reynolds shear stress and the normal turbulence intensities are

Page 179: $\bar{u}_1 = U_G - \bar{u}_1^*, \quad \bar{u}_1^* \geq 0, \quad \bar{u}_2^* = \bar{u}_2,$ (500a-c)

Page 181, 11th line: $\frac{\partial}{\partial x_1} \left(\frac{\bar{u}_1^*}{U_G} \right) - \frac{\partial}{\partial x_2} \left(\frac{\overline{u_2' u_1'}}{U_G^2} \right) = 0$ (503a,b)

Page 182, 17th line: Inserting Eqs. (491a) and. . .

Page 186: . . . to obtain with $\sigma=db/dx_f$ and Eq. (500a),. . .

Page 186, 2nd line: By substituting (522a,d, and e), it follows that

18th line: ... to obtain, with $\sigma=db/dx_l$ and Eq. (500a), ...

Page 190: ..., where a decomposition of the downstream velocity $\bar{u}_1$ into a horizontal flow...

Page 191, 1st line: $p_1 f_1 - p_0 \eta \frac{df_1}{d\eta} + \frac{1}{\beta \eta} \frac{d(\eta f_2)}{d\eta} = 0$ (544)

Page 192, 12th line: The Reynolds shear stress decreases inversely...

Page 192: With Eqs. (554a-d) the momentum equation (549) is simplified to the ODE

Page 200, 15th line: ... propositions:

Page 210, 19th line: ...relation, which is stated as:

Page 216, figure 9.20: c) d)

Page 217, 6th line: ... (647a-c)

11th line: ... (648a,b)

Page 222, 8th line: Next, it must be proven that...

Page 224, 5th line: $f_{21} = \dots$ (684)

Page 226, 13th line: ...turbulence, the DQTM, was derived, contains...

Page 234: Applying these two equations to (720) and dividing the resulting equation by u^{*2} yields

Page 237, 2nd line: $\dots = \sqrt{\frac{\beta}{4-\beta}},$

Page 240: Table 9.3: $g_1 = 1/2 \langle 1 + \sqrt{(1-\chi/\chi)} \tan \{ \arctan [\sqrt{\chi/(1-\chi)}] \eta \} \rangle$

Page 248, Fig. Capt.: The flow was calculated with a LES dynamic...

Page 249, 9th line : ... Reynolds shear stress, τ_{21} ,

10th line: $\tau_{\text{tot}} = \tau_0 + \tau_{21} = \mu \frac{du_1}{dx_2} - \rho \overline{u'_2 u'_1}$. (782a,b)

Page 251, 12th line: $\tau_{21} = \dots$ (796a,b)

15th line: In terms of the self-similar coordinate ...

Page 258, 2nd last line: ... in earlier equations) $' := d/d\psi$, lead to ...

Page 260, 4th line: In Eqs. (727a,b), replacing $f_1(1)$ by $f_1(0)$ and substituting these two equations into (844a), it follows that

Page 270, 20th line: ...olds numbers up to 35,259,000, which is the highest Re value...

Page 271, 23th line:kind of flows why in this book a deficit power law...

Page 279, Capt. Fig. 9.55:inverse Reynolds number].

Page 310, 13th line: ... (1015a-c)

Page 310: Replace in Eqs. (1013a,b) $\hat{u}_{in}^-$ by $\hat{u}_{i,n}^-$

Page 311, 5th line : ... , (1018a,b)

Page 313, 10th line: ... very close to “1” (see Sect. 10.4) and power law exponents close to -3, are expected to occur.

22nd & 23rd line: ... *of modeling turbulent fields.*

$$\text{Page 322: } \dots - \lambda_2 \left[\int_{\Gamma} x^2 p(x) dx - \sigma^2 \right], \quad (1066)$$

$$\text{Page 323: } p(x) = A \exp \left(-\frac{x^2}{2\sigma^2} \right) \quad (1067)$$

$$\text{Page 325: } \Phi[P] = k \frac{1 - \int_0^{\infty} P(x) dx}{q-1} - \dots \quad (1080)$$

Page 334, in the figure: ...

Page 335: A comparison of Eq. (1114a) with the maximum value q reveals Eq. (1114c).

Page 343: (a) $E_e = E$ is the sum of energy contributions E_n : $E = \sum_{n=0}^N E_n$ (see Eq. (1011f))

Page 346: ⁶Temperatures of turbulence were also proposed by Chorin (1994) and Castaigne (1996).

Page 347, 13th line: .. and Tsallis et al. (1995)

Page 350: Chorin, A.J., Vorticity and Turbulence. Applied Mathematical Sciences 103. Springer edition, New York, Inc. (1994), ISBN 0-387-94197-5

Page 362: These limit the translational variance for size-scaled eddies to the distance of periodic length 2λ .

Page 364, 13th line: ... (1181a-c)

Page 365, 26th line: Still a further quantity, that diverges as s_{1c} is approached, may...

Page 366, 18th line: Following Eq. (1187a-c), the compressibility diverges...

24th line: ... (1193a-c)

Page 367, 3rd line: ... (1194a-c)

6th line: ... (1195a-c)

16th line ... (1197a-c)

Page 368, 1st line: . . .	(1198a-c)
Page 370, 12th line: . . . ,	(1210)
Page 376, 19th line: . . .	(1228a-c)
Page 376, 21st line: . . .	(1229a-c)
Page 378, 17th line: . . .	(1241a,b)
Page 378, 19th line: . . .	(1242a,b)
Page 378, 21st line: . . .	(1243a-c)
Page 379, 27th line: . . . ,	(1245a,b)
Page 383, 8th line: . . .	(1256a-d)
Page 384, 14th line: . . .	(1261a,b)
Page 388, 1st line: . . . domains with disordered and the white with ordered spins.	
Page 396, 2nd last line: . . .proportion is, which in analogy, we call the <i>marvelous</i> . . .	
Page 401: Song, F., Karniadakis,. . .	
Page 413: . . .	(D.45a-c)
Page 415: . . .	(E.7a-c)