Chapter 23

LEAP-2017 Centrifuge Test Simulation Using HiPER

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Abstract Simulations by effective stress analysis were carried out for nine centrifuge experiments of a gentle sloped ground. The constitutive equation is a hyperbolic model as a stress–strain relationship and a bowl model as a strain–dilatancy relationship. The experimental results largely vary depending on density and input acceleration. The analyses results can explain such a tendency of variability.

23.1 Outline of the Analysis Method

Analysis is done by FEM and the software used is “HiPER.” The integration scheme is implicit and the integration time interval is 0.002 s. The Newmark β a numerical integration method is adopted ($\beta = 1/4$). The method used for convergence is a modified Newton Raphson method. Convergence is determined as a relative force imbalance of $1.0\text{E-}3$ or less. The maximum number of iterations is four; if convergence is not reached, the unbalanced force is carried over to the next step.

To ensure analytical stability, a low stiffness proportional damping ($C = \beta K$, where K is stiffness matrix and $\beta = 1.0\text{E-}3$) is used.

The u-p formulation is used and excess pore water pressure is evaluated at nodes (the Sandhu method (Sandhu and Wilson 1969)).

23.2 Outline of the Constitutive Equations

In this section, the three-dimensional stress–strain–dilatancy relationship is explained. A hyperbolic model extending in three dimensions is used for the stress–strain relationship, while the strain–dilatancy relationship is modeled with a bowl

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function. The hyperbolic stress-strain model parameters are determined from dynamic deformation tests ($G/G_{\max}$ - γ , h - γ relationships). The bowl model parameters are determined from liquefaction resistance tests (from the relationship between stress ratio and number of cycles).

23.2.1 Hyperbolic Model and Its Parameters

In multidimensionalizing the hyperbolic model, the shear stress versus shear strain relationships for the shear component and the axial difference component, respectively, are defined by the following equations.

$$\tau_{xy} = \frac{G_{\max} \cdot \gamma_{xy}}{1 + \frac{\gamma_{xy}}{\gamma_r}}, \quad \tau_{yz} = \frac{G_{\max} \cdot \gamma_{yz}}{1 + \frac{\gamma_{yz}}{\gamma_r}}, \quad \tau_{zx} = \frac{G_{\max} \cdot \gamma_{zx}}{1 + \frac{\gamma_{zx}}{\gamma_r}} \quad (23.1a)$$

$$\begin{aligned} \frac{\sigma_x - \sigma_y}{2} &= \frac{G_{\max} \cdot (\epsilon_x - \epsilon_y)}{1 + \frac{(\epsilon_x - \epsilon_y)}{\gamma_r}}, \quad \frac{\sigma_y - \sigma_z}{2} = \frac{G_{\max} \cdot (\epsilon_y - \epsilon_z)}{1 + \frac{(\epsilon_y - \epsilon_z)}{\gamma_r}}, \\ \frac{\sigma_z - \sigma_x}{2} &= \frac{G_{\max} \cdot (\epsilon_z - \epsilon_x)}{1 + \frac{(\epsilon_z - \epsilon_x)}{\gamma_r}} \end{aligned} \quad (23.1b)$$

When solving two-dimensional problems, the hyperbolic model is used for the shear components $\tau_{xy} - \gamma_{xy}$ and the axial difference components $(\sigma_x - \sigma_y)/2 - \epsilon_x - \epsilon_y$.

Here, $G_{\max}$ is the initial shear modulus, and γ_r is the reference strain. γ_r is obtained from the shear strength τ_f by the following equation.

$$\gamma_r = \frac{\tau_f}{G_{\max}} \quad (23.2)$$

The h - γ relationship is given by the following equation.

$$h = h_{\max} \cdot \left(1 - \frac{G}{G_{\max}}\right) \quad (23.3)$$

where, h is hysteretic damping parameter, $h_{\max}$ is the maximum damping ratio and G is shear modulus.

Three parameters are required to construct the hyperbolic model; $G_{\max}$, $h_{\max}$, and γ_r . Of these, $G_{\max}$ and γ_r are functions of effective stress. If $G_{\max}$ and γ_r at a certain reference effective stress σ'_{mi} are $G_{\max i}$ and γ_{ri} , then $G_{\max}$ and γ_r satisfy the following equations.

Table 23.1 Parameters of the hyperbolic model

Parameters	Physical meaning of parameters
$G_{\max}$	Initial shear modulus. $G_{\max} = \rho V_s^2$
$h_{\max}$	Maximum damping ratio. As $h_{\max}$ increases the nonlinearity becomes stronger
γ_r	Reference strain. $\gamma_r = \tau_p/G_0$ The shear strain when $G/G_{\max} = 0.5$

Fig. 23.1 Parameters of hyperbolic model: $h_{\max}$, γ_r

$$G_{\max} = G_{\max i} \left(\frac{\sigma'_m}{\sigma'_{mi}} \right)^{0.5}, \quad \gamma_r = \gamma_{ri} \left(\frac{\sigma'_m}{\sigma'_{mi}} \right)^{0.5} \quad (23.4)$$

As the effective stress varies, at each incremental calculation step, these parameters are calculated as they vary with time based on the above equations. At the same time, the shear stress versus shear strain relationship in Eq. 23.1 varies with time in accordance with the effective stress.

Applying the Masing rule to the hyperbolic model results in excessive hysteretic damping. Therefore, hysteretic damping h is adjusted using the method described by Ishihara et al. (1985).

The three parameters of the hyperbolic model ($G_{\max i}$, $h_{\max}$, γ_{ri}) are defined in Table 23.1 and Fig. 23.1. As shown in Eq. 23.4 above, $G_{\max}$ and γ_r depend on the effective stress. The formulation $G_{\max i}$, γ_{ri} is used when the mean effective stress σ'_m is 1.0 kN/m^2 . These values can be simply determined from the stiffness reduction curve ($G/G_{\max}$ versus γ relationship) or the damping increase curve (h – γ relationship) obtained from the dynamic deformation tests.

23.2.2 Bowl Model (Dilatancy Model) and Its Parameters

To model deformation in three dimensions, in addition to the shear strains (γ_{zx} , γ_{yx} , γ_{xy}) representing simple shearing deformation and the axial shear strain differentials ($\varepsilon_x - \varepsilon_y$, $\varepsilon_y - \varepsilon_z$, $\varepsilon_z - \varepsilon_x$) representing axial deformation differential,

definitions of resultant shear strain Γ and cumulative shear strain G^* are needed. They are defined by the following equations.

$$\Gamma = \sqrt{\gamma_{zx}^2 + \gamma_{zy}^2 + \gamma_{xy}^2 + (\varepsilon_x - \varepsilon_y)^2 + (\varepsilon_y - \varepsilon_z)^2 + (\varepsilon_z - \varepsilon_x)^2} \quad (23.5)$$

$$\begin{aligned} G^* &= \sum \Delta G^* \\ &= \sum \sqrt{\Delta \gamma_{zx}^2 + \Delta \gamma_{zy}^2 + \Delta \gamma_{xy}^2 + \Delta (\varepsilon_x - \varepsilon_y)^2 + \Delta (\varepsilon_y - \varepsilon_z)^2 + \Delta (\varepsilon_z - \varepsilon_x)^2} \end{aligned} \quad (23.6)$$

The bowl model (as proposed by Fukutake and Matsuoka 1989, 1993) focuses on the resultant shear strain Γ and the cumulative shear strain G^* . Dilatancy ε_v^s results from a certain soil particle repeatedly dropping into the valley between other soil particles (negative dilatancy), and rising up on other soil particles (positive dilatancy). This mechanism is represented by the superposition of positive and negative dilatancy using the following equation.

$$\varepsilon_v^s = \varepsilon_\Gamma + \varepsilon_G = A \cdot \Gamma^{1.4} + \frac{G^*}{C + D \cdot G^*} \quad (23.7)$$

Here A , C , and D are parameters. ε_G is monotonic negative dilatancy (compressive strain). It is irreversible and represented as a hyperbolic function with respect to G^* . ε_Γ is cyclic positive dilatancy (swelling strain) and is reversible and represented as an exponential function with respect to Γ . The ε_G component is the master curve and is the component that determines the basic dilatancy during cyclic shearing, while the ε_Γ component is a oscillating component associated with it. $1/D$ is the asymptotic line to the hyperbolic curve, corresponding to a relative density of 100%.

The mechanism of such a bowl model is the movement of soil particles in seven-dimensional strain space with γ_{xy} , γ_{yz} , γ_{zx} , $(\varepsilon_x - \varepsilon_y)$, $(\varepsilon_y - \varepsilon_z)$, $(\varepsilon_z - \varepsilon_x)$, and ε_v^s as axes.

Figure 23.2 illustrates the case of unidirectional cyclic shearing.

Next, the consolidation term is taken into consideration in the stress-strain relationship and effective stress is modeled under undrained (constant volume) conditions.

The volumetric strain ε_v^s represented by Eq. 23.7 is the dilatancy component due to shearing, but an additional volumetric strain due to change in the effective stress σ'_m has to be considered. The total volumetric strain increment of the soil $d\varepsilon_v$ is then given by the following equation, where the shear component is $d\varepsilon_v^s$ and the consolidation component is $d\varepsilon_v^c$.

Fig. 23.2 Dilatancy in unidirectional cyclic shearing

$$d\varepsilon_v = d\varepsilon_v^s + d\varepsilon_v^c \quad (23.8)$$

The consolidation term $d\varepsilon_v^c$ is given by the following equations, assuming one-dimensional consolidation conditions.

$$d\varepsilon_v^c = \frac{0.434 \cdot C_s}{1 \cdot e_0} \cdot \frac{d\sigma'_m}{\sigma'_m} \quad (\text{for } d\sigma'_m < 0) \quad (23.9)$$

$$d\varepsilon_v^c = \frac{0.434 \cdot C_c}{1 \cdot e_0} \cdot \frac{d\sigma'_m}{\sigma'_m} \quad (\text{for } d\sigma'_m > 0) \quad (23.10)$$

Here, C_s is the swelling index, C_c is the compression index, and e_0 is the initial void ratio. If undrained conditions ($d\varepsilon_v = 0$) are assumed in Eqs. 23.9 and 23.10, the following equation is obtained.

$$d\varepsilon_v^s + \frac{0.434 \cdot C_s}{1 + e_0} \cdot \frac{d\sigma'_m}{\sigma'_m} = 0 \quad (23.11)$$

If the mean effective stress in the initial shearing stage is σ'_{m0} , and if the above equation is integrated under the initial conditions ($\sigma'_m = \sigma'_{m0}$), the following equation is obtained.

$$\varepsilon_v^s + \frac{C_s}{1 + e_0} \cdot \log \frac{\sigma'_m}{\sigma'_{m0}} = 0 \quad (23.12)$$

From Eq. 23.12, the effective stress under undrained shear σ'_m is given by the following equations.

$$\sigma'_m = \sigma'_{m0} \cdot 10^\alpha, \quad \alpha \equiv \frac{-\varepsilon_v^s}{C_s/(1 + e_0)} \quad (23.13)$$

Therefore, substituting α from the above equation, the effective stress reduction ratio is given by the following equation:

$$\left(\frac{\sigma'_{m0} - \sigma'_m}{\sigma'_{m0}} \right) = 1 - 10^\alpha \quad (23.14)$$

In order to suppress the occurrence of dilatancy under small shear amplitude, a spherical region with shear strain radius $\Gamma = R_e$ is considered within the strain space, and within this region there is no $d\varepsilon_G$. R_e is given as follows, with reference to Fig. 23.3.

Figure 23.3a shows the liquefaction resistance curve and the relationship between liquefaction resistance and the lower limit value X_l . X_l is the liquefaction resistance

(a) Liquefaction resistance curve and lower limit of liquefaction resistance X_l

(b) Effective stress path and X_l (c) Skeleton curve of hyperbolic model and R_e

Fig. 23.3 Lower limit of liquefaction resistance X_l and shear strain R_e . (a) Liquefaction resistance curve and lower limit of liquefaction resistance X_l . (b) Effective stress path and X_l . (c) Skeleton curve of hyperbolic model and R_e

after a very large number of cycles, in other words, it represents the stress ratio at which liquefaction does not occur even however many cycles occur. Also, for simplicity, it is assumed that the excess pore water pressure does not arise ($P_w = 0$) for repeated stress ratios less than X_l . From Fig. 23.3b, c, setting the shear strain when the stress ratio is X_l equal to R_e , then R_e is given by the following equation.

$$R_e = \frac{X_l \sigma'_{m0}}{G_{max} - \frac{X_l \sigma'_{m0}}{\gamma_r}} \quad (23.15)$$

Here σ'_{m0} is the mean effective stress in initial shear. When the amplitude of the stress ratio is X_l or less, positive excess pore water pressure does not arise.

There are six parameters in the bowl model (A , C , D , $C_s/(1 + e_0)$, $C_c/(1 + e_0)$ and X_l) as shown in Table 23.2. The values of these parameters are determined from a liquefaction resistance curve.

Table 23.2 and Fig. 23.4 give the meanings of the bowl model parameters. These parameters are determined by fitting to the liquefaction resistance curve.

Table 23.2 Parameters of bowl model

Parameter	Physical meaning of parameters
A	Parameter representing the swelling component ε_r of the dilatancy components. The larger the absolute value of A , the greater the cyclic mobility
C, D	Parameters representing the compression component ε_G of the dilatancy components. $1/C$ is the slope of the dilatancy in the initial stage of shear. $1/D$ is calculated from the minimum void ratio $e_{\min}$ on the hyperbolic asymptotic line (maximum amount of compression)
$C_s/(1 + e_0)$	C_s is the swelling index; e_0 is the initial void ratio
$C_c/(1 + e_0)$	C_s is the compression index; e_0 is the initial void ratio
X_l	The lower limit value of the liquefaction resistance. In the relationship between stress ratio τ/σ'_{m0} and the number of cycles N_c , it is represented by τ/σ'_{m0} when N_c is sufficiently large. When $\tau/\sigma'_{m0} > X_l$ excess pore water pressure arises

Fig. 23.4 Liquefaction resistance curves and bowl model parameters C and X_l

23.3 Element Test Simulation

23.3.1 Determining Parameters by Test Simulation

Hyperbolic Model

The initial shear stiffness $G_{\max}$ is calculated from the following equation based on research by Liu et al. (2017).

$$G_{\max} = 6.35 \frac{1}{0.3 + 0.7e^2} \sigma_m^{0.5} \quad (23.16)$$

Here γ_r and $h_{\max}$ are standard values for sand based on the shear strain dependence of stiffness ratio and damping for Toyoura sand ($G/G_{\max}-\gamma$, $h-\gamma$ relationship). Since the experiment of Ottawa sand was not carried out, it replaced it with Toyoura sand experiment. Figure 23.5 shows the $G/G_{\max}-\gamma$, $h-\gamma$ relationship used in calculations.

Fig. 23.5 $G/G_{\max}$ – γ , h – γ relationship used in calculations (hyperbolic model)

Bowl Model

The parameter D is determined from the following equation since $1/D$ is the asymptote of dilatancy. The average of $e_{\min}$ is 0.49.

$$\frac{1}{D} = \frac{e_0 - e_{\min}}{1 + e_0} \quad (23.17)$$

The lower limit value of liquefaction resistance X_l is determined based on the liquefaction resistance curve. The liquefaction resistance curve is calculated using values for standard sand for the other parameters. Parameter C is adjusted while comparing the test results and the calculation results so as to match the whole liquefaction resistance curve with the measured values.

The parameters for Ottawa sand set by this procedure are given in Table 23.3.

23.3.2 Results of Element Test Simulations

Figure 23.6 shows the simulations of liquefaction resistance curves. They explain the experiment results for the three selected densities.

Figure 23.7 shows the effective stress path and stress–strain relationship at these three different densities. In the dense case, strong cyclic mobility is present.

Table 23.3 Parameters for Ottawa sand

		Parameter of constitutive equation													
		Hypabolic model							Bowl model						
No	$\sigma_{mo}(\text{kN/m}^2)$	e	ρ_d (t/m^3)	V_s (m/s)	G_0 (kN/m^2)	Reference strain (γ_r)	G_{0i} (kN/m^2)	γ_{ri}	$h_{\max}$ (%)	A	C	D	C_s ($1 + e_0$)	C_c ($1 + e_0$)	X_I
1	100	0.515	1.7442	274	130751	0.0005	13075	5.0E-5	27	-1.5	35.0	61	0.0060	0.0061	0.20
2	100	0.542	1.7126	271	125585	0.0005	12558	5.0E-5	27	-0.8	9.0	30	0.0060	0.0061	0.16
3	100	0.585	1.6656	266	117689	0.0005	11769	5.0E-5	27	-0.6	6.0	17	0.0060	0.0061	0.09

Fig. 23.6 Liquefaction resistant curves

23.4 Centrifuge Simulation

23.4.1 Determination of Parameters

The model parameters (that is to say the G/G_0 - γ , h - γ relationship and the liquefaction resistance curve) are determined by element test simulations of Phase I except for G_0 and D . G_0 and D are reset by the following procedure for each centrifuge test. From the density of sand (Table 23.4), the relative density D_r is obtained, and the void ratio e is calculated from D_r . The initial shear stiffness G_0 is calculated from Eq. 23.16 based on the research of Liu et al. 2017. G_{0i} is the value at unit mean stress ($\sigma_m = 1.0 \text{ kN/m}^2$). Parameter D is determined from Eq. 23.17 since $1/D$ is the asymptote of dilatancy. The average of $e_{\min}$ is 0.49.

Table 23.5 shows the parameters set by this method. The Poisson ratio of the ground is set to 0.33. The bulk modulus of water is $K_w = 2.2\text{E} + 6 \text{ kN/m}^2$. For soil permeability, $k = 0.015 \text{ cm/s}$ is used from experimental results.

23.4.2 Summary of the Numerical Simulations

The FEM mesh is shown in Fig. 23.8. The FEM model has 2013 nodes and 1920 elements. A plane strain four-noded quadrilateral element with displacements and pore pressure degrees of freedom at each node is employed. Four integration points

Table 23.5 Parameters of constitutive equation

Parameter of constitutive equation													
					Hyperbolic model			Bowl model					
		pd (kg/m3)	Dr	e	G_{oi} kN/m ²	γ_{ri}	h_{max} %	A	C	D	$\frac{C_s}{(1+e_0)}$	$\frac{C_c}{(1+e_0)}$	X_l
1	CU-2	1605.8	0.485	0.632	10954	5.0E-5	27	-0.6	6.0	11	0.0060	0.0061	0.09
2	Ehime-2	1656.55	0.660	0.584	11790	5.0E-5	27	-0.6	6.0	17	0.0060	0.0061	0.09
3	KAIST-1*	1701.2	0.805	0.544	12525	5.0E-5	27	-0.8	9.0	29	0.0060	0.0061	0.16
4	KAIST-2	1592.5	0.437	0.645	10736	5.0E-5	27	-0.6	6.0	11	0.0060	0.0061	0.09
5	KyU-3	1637	0.594	0.602	11467	5.0E-5	27	-0.6	6.0	14	0.0060	0.0061	0.09
6	NCU-3	1652	0.645	0.588	11715	5.0E-5	27	-0.6	6.0	16	0.0060	0.0061	0.09
7	UCD-1	1665	0.688	0.576	11929	5.0E-5	27	-0.6	6.0	18	0.0060	0.0061	0.09
8	UCD-2	1658	0.665	0.582	11814	5.0E-5	27	-0.6	6.0	17	0.0060	0.0061	0.09
9	ZJU-2	1606	0.486	0.632	10958	5.0E-5	27	-0.6	6.0	12	0.0060	0.0061	0.09

* dense

$$G_{oi} = 6.35 \frac{1}{0.3 + 0.7e^2} 1.0^{0.5}$$

$$\frac{1}{D} = \frac{e - e_{min}}{1 + e}$$

Fig. 23.8 Boundary conditions and time history output points (black square box acceleration, red square box pore water pressure)

are used in each element. The u - p formulation is used. The excess pore water pressure is evaluated at the nodes (the Sandhu method).

The nodes located at the base of the model are fully constrained in the x and y directions while the nodes on the side walls are constrained laterally. The nodes on the free surface allow full drainage and have a fixed pore water pressure.

Figure 23.9 shows the initial stress calculated by linear analysis. The initial stress is determined by linear self-weight analysis. For stiffness, the value at 4 m of depth is used.

In the dynamic analysis, horizontal acceleration and vertical acceleration were input simultaneously.

23.4.3 Results of Centrifuge Simulations

Horizontal displacement time histories of the ground surface at the center are shown in Fig. 23.10. The amount of lateral spreading of the ground surface was of the same

Fig. 23.9 Initial stress calculated by linear analysis

order in experiment and analysis. For UCD3, Ehime2, and CUD1, the simulated values and experimental values show good correspondence. The analysis values are rather smaller for CU 2, ZJU 2, and NCU 3. The experimental show large variation depending on density and input acceleration. The simulations successfully explain this tendency.

Figure 23.11 shows fivefold enlarged deformation plots and contours of excess pore water pressure ratio. For KAIST1, KyU3, and UCD1, liquefaction does not occur and deformations are small. In the other six cases, the ground liquefies and deforms in the downstream direction.

(a) Simulations

Fig. 23.10 Horizontal displacement time histories of ground surface at the center. (a) Simulations, (b) experiments (displacements from ACC and surface marker)

According to Fig. 23.12, shear strain accumulates in the direction in which the initial shear stress acts. The reverse warping tendency of the stress–strain relationship due to cyclic mobility is not noticeable.

Generally, horizontal deformation is large in the upper layer at the center of the slope. Vertical deformation is dominated by sinking on the upper side of the slope. Deformation peaks at the end of the excitation, and then recovers slightly during the process of dissipating excess pore water pressure. Excess pore water pressure is fully dissipated after 600 s.

Fig. 23.11 Fivefold enlarged deformation and excess pore pressure ratio distribution (contour) at 25 s. (a) CU2, (b) Ehime2, (c) KAIST1, (d) KAIST2, (e) KyU3, (f) NCU3, (g) UCD1, (h) UCD3, and (i) ZJU2

Fig. 23.11 (continued)

23.5 Conclusion

We simulated the lateral spreading of sloping ground using a hyperbolic model and a bowl model for comparison against experimental results. The parameters of the constitutive equation were determined based on simulation of element tests. Experimental results show large variation with density and input acceleration. The simulations successfully explain this tendency to variability. The lateral spreading of the ground surface was of the same order in the experiments and simulations.

Fig. 23.12 Stress–strain and effective stress path at P3. (a) CU2, (b) Ehime2, (c) KAIST1, (d) KAIST2, (e) KyU3, (f) NCU3, (g) UCD1, (h) UCD3, and (i) ZJU2

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