Chapter 7

Graphing Linear Equations—A Comparison of the Opportunity-to-Learn in Textbooks Using the Singapore and the Dutch Approaches to Teaching Equations

Berinderjeet Kaur, Lai Fong Wong and Simmi Naresh Govindani

Abstract This chapter examines the opportunity-to-learn afforded by two textbooks, one using the Singapore approach and the other the Dutch approach for graphing linear equations. Both textbooks provide opportunities for students to connect mathematical concepts to meaningful real-life situations, practice questions for self-assessment, and reflect on their learning. However, the approaches presented in the two textbooks are different. The Dutch approach textbook has the same context for all the interconnected activities while in the Singapore approach textbook the activities are self-contained and can be carried out independently of each other. In addition, classroom activities, practice questions and prompts for reflection in the Dutch approach textbook provide students with more scope for reasoning and communication. From the reflections of two lead teachers using the Singapore approach textbook it is apparent that they see merit in the Dutch approach textbook, but feel that to adopt the Dutch approach they would need a paradigm shift and adequate support in terms of resources.

Keywords Opportunity-to-learn (OTL) · Dutch approach textbook · Singapore approach textbook · Graphing linear equations

B. Kaur (✉) · L. F. Wong
National Institute of Education, Singapore, Singapore
e-mail: berinderjeet.kaur@nie.edu.sg

L. F. Wong
e-mail: laifong.wong@nie.edu.sg

S. N. Govindani
Yishun Secondary School, Singapore, Singapore
e-mail: simmi_naresh_govindani@moe.edu.sg

© The Author(s) 2020
M. van den Heuvel-Panhuizen (ed.), *International Reflections on the Netherlands Didactics of Mathematics*, ICME-13 Monographs, https://doi.org/10.1007/978-3-030-20223-1_7

7.1 Introduction

Carroll (1963) was the first to introduce the concept of opportunity-to-learn (OTL). He asserted that an individual's learning was dependent on the task used and the amount of time devoted to learn. This concept has been particularly useful when comparing student achievement across countries, such as those carried out by studies like Trends in International Mathematics and Science Study (TIMSS). Several approaches have been used by researchers to assess OTL (Brewer & Stasz, 1996; Liu, 2009). Amongst the OTL variables considered by Liu (2009) are content coverage, content exposure, content emphasis and quality of instructional delivery and the OTL categories considered by Brewer and Stasz (1996) are curriculum content, instructional strategies and instructional resources.

The TIMSS textbook study (Foxman, 1999; Schmidt, McKnight, Valverde, Houang, & Wiley, 1997) is one example of examining the OTL based on studies of instructional materials such as textbooks. Recently, Wijaya, Van den Heuvel-Panhuizen, and Doorman (2015) showed how fruitful the concept of OTL is when they investigated the relation between the tasks offered in Indonesian mathematics textbooks and the Indonesian students' difficulties to solve context-based mathematics tasks.

Researchers have generally agreed that textbooks play a dominant and direct role in what is addressed in instruction. Robitaille and Travers (1992, p. 706) noted that a great dependence upon textbooks is "perhaps more characteristic of the teaching of mathematics than of any other subject". This is due to the canonical nature of the mathematics curriculum. Several researchers have noted that the textbooks teachers adopt for their teaching often result in dictating the content they teach and the teaching strategies they adopt (Freeman & Porter, 1989; Reys, Reys, & Chavez, 2004). Therefore, it is not surprising that textbooks may be used as proxies to determine students' OTL (Schmidt, McKnight, & Raizen, 1997; Tornroos, 2005). Inevitably if textbooks implementing a specific curriculum, such as the graphing of equations, differ, students using the respective textbooks will get different OTL (Haggarty & Pepin, 2002). This different OTL have often resulted in different student outcomes as there is a strong relation between textbook used and mathematics performance of students (see, e.g., Tornroos, 2005; Xin, 2007).

7.2 A Study of Teaching Graphing Linear Equations in Textbooks Using the Singapore and Dutch Approach

7.2.1 Objective of This Chapter

The objective of this chapter is to examine the OTL related to graphing linear equations in two textbooks, one of which is using a Singapore approach and the other using a Dutch approach. The book using the Singapore approach is *Discovering*

Fig. 7.1 Textbook *Discovering Mathematics 1B* (Chow, 2013) with approval stamp

Mathematics 1B (Chow, 2013) and the book using the Dutch approach is *Mathematics in Context* (Wisconsin Center for Education Research & Freudenthal Institute, 2010).

7.2.2 Backgrounds of the Contexts of Textbooks Examined

The textbook *Discovering Mathematics* (Chow, 2013) adopts a Singapore approach. It is one of the approved texts that schools may adopt for their instructional needs. Textbooks in Singapore that are approved by the Ministry of Education have an approval stamp, as shown in Fig. 7.1.

These textbooks are closely aligned to the intended curriculum (mathematics syllabuses) issued by the Ministry of Education in Singapore for all schools. The framework for the school mathematics curriculum in Singapore is shown in Fig. 7.2.

Fig. 7.2 Framework of the school mathematics curriculum (Ministry of Education, 2012)

The primary goal of the curriculum is mathematical problem solving and five inter-related components, namely concepts, skills, processes, metacognition and attitudes, contribute towards it.

The *Discovering Mathematics* textbook includes clear and illustrative examples, class activities and diagrams to help students understand the concepts and apply them. Essentially the textbook advocates a teaching for problem solving approach. In this conception of teaching problem solving, the content is taught for instrumental, relational and conventional understanding (Skemp, 1976) so that students are able to apply them to solve problems associated with content. This is clearly evident from the key features of the textbook, which are a chapter opener, class activities, worked examples to try, exercises that range from direct applications in real-life situations to tasks that demand higher-order thinking.

The *Mathematics in Context* textbook (Wisconsin Center for Education Research & Freudenthal Institute, 2010) written for the U.S. middle school reflects the Dutch approach of Realistic Mathematics Education (RME) (Van den Heuvel-Panhuizen & Drijvers, 2014). The textbook manifests the core teaching principles of RME which are:

- The activity principle—students are active participants in the learning process;
- The reality principle—mathematics education should start from problem situations and students must be able to apply mathematics to solve real-life problems;
- The level principle—learning mathematics involves acquiring levels of understanding that range from informal context-related solutions to acquiring insights into how concepts and strategies are related;
- The intertwinement principle—mathematics content domains such as number, geometry, measurement, etc. must not be treated as isolated curriculum chapters, but be integrated in rich problems;

- The interactivity principle—learning mathematics is a social activity; and
- The guidance principle—teachers should have a proactive role in students’ learning and programmes should be based on coherent long-term teaching-learning trajectories (Van den Heuvel-Panhuizen & Drijvers, 2014, pp. 522–523).

7.2.3 Framework for Analysing the OTL in the Textbooks

The analysis of textbooks can not only be carried out in several ways, but has also evolved with time. This is evident from research studies related to TIMSS. Schmidt et al. (1997) involved in TIMSS, initially focused on examining the content of textbooks, but later Valverde, Bianchi, Wolfe, Schmidt, and Houang (2002) expanded the examination to (i) classroom activities proposed by the textbook, (ii) amount of content covered and mode of presentation—abstract or concrete, (iii) sequencing of content, (iv) physical attributes of the textbooks such as size and number of pages, and (v) complexity of the demands for student performance. Furthermore, non-canonical aspects of mathematics may also be examined. For example, Pepin and Haggarty (2001) in their study on the use of mathematics textbooks in English, French and German classrooms adopted an approach that focused not only on the topics (content) and methods (teaching strategies), but also the sociological contexts and cultural traditions manifested in the books.

In this chapter, we examine the OTL related to graphing linear equations in two textbooks, one of which is using a Singapore approach and the other using a Dutch approach. Our investigation is guided by the following questions:

What are the similarities and differences in the two textbooks with respect to

- sequencing of content in the chapter on graphing equations
- classroom activities proposed by the chapter on graphing equations
- complexity of the demands for student performance in the chapter on equations?

The respective textbook materials examined are Chap. 12, titled “Coordinates and Linear Functions” from the Singapore approach textbook *Discovering Mathematics* (Chow, 2013) and the chapter “Graphing Equations” (Kindt et al., 2010) from the Dutch approach textbook *Mathematics in Context* (Wisconsin Center for Education Research & Freudenthal Institute, 2010).

Table 7.1 Sequencing of content in the two textbooks

<p>Singapore approach in the textbook <i>Discovering Mathematics 1B—Coordinates and Linear Functions</i></p> <p>Construct the Cartesian coordinate system in two-dimensions and state coordinates of points on it → Plot a graph of a set of ordered pairs as a representation of a relationship between two variables → recognise the idea of functions → recognise linear functions in various forms and draw their graphs → find the gradient of a linear graph as a ratio of the vertical change to the horizontal change</p>
<p>Dutch approach in the textbook <i>Mathematics in Context—Graphing Equations</i></p> <p>Locate points using compass directions and bearings → locate points on a coordinate system in the context of a forest fire → Starting from steps along a line, investigate the concept of slope → Use the y-intercept as a reference point for graphing linear functions → Draw lines for given equations and write equations for drawn lines → Develop formal algebraic methods for solving linear equations through visualizing frogs jumping towards or away from a path → Learn a formal way of solving equations by simultaneously changing the diagrams and the equations the diagram → Write down the operations performed to keep track of the steps taken in solving an equation → Describe and graph problem situations, which are solved by locating the point of intersection → Combine the graphical method to find a point of intersection with the use of equations → Relate the method for solving frog problems to finding the point of intersection of two lines by linking the lines in the graph to their equations using arrows → Connect the graphical and algebraic method explicitly → Explore the relationship between parallel lines and graphs of lines without intersection points</p>

7.3 Data and Results

7.3.1 *The Sequencing of the Content on Graphing Equations in the Two Textbooks*

In this section, we tabulate the content in the chapters on graphing equations in the two textbooks. This will allow us to draw out the similarities and differences. Table 7.1 shows the flow of content in the Singapore and the Dutch approach textbook respectively.

From Table 7.1 it is apparent that in the two textbooks the sequence of the content is dissimilar. The books take significantly different pathways in developing the content. In the Singapore approach textbook, students are directly introduced to the terminology (such as Cartesian coordinate system, x - and y -axis, origin, x - and y -coordinates, etc.) and concepts of the topic through some class activities or investigative work. Worked examples are provided next and these are then followed by practice questions on three different levels—simple questions involving direct application of concepts are given on Level 1; more challenging questions on direction application on Level 2; and on Level 3 questions that involve real-life applications, thinking skills, and questions that relate to other disciplines. This is the sequence for each sub-unit, and the chapter ends with a summary, a revision exercise, a real-life context that relates to the topic, and students' reflection.

In the Dutch approach textbook, a real-life context (such as a forest fire) is first introduced and students continuously formalise their knowledge, building on knowledge from previous units (and sub-units). Regarding the context, students gradually adopt the conventional formal vocabulary and notation, such as origin, quadrant, and x -axis, as well as the ordered pairs notation (x, y) . In each sub-unit, a summary is provided and some questions are given for students' self-assessment, followed by further reflection.

7.3.2 Classroom Activities Proposed on Graphing Equations in the Two Textbooks

In this section, we tabulate the classroom activities as intended by the two textbooks for the development of knowledge related to the graphing of (linear) equations. Table 7.2 shows the flow of activities in the two chapters in the Singapore and the Dutch approach textbooks respectively. In the Singapore approach textbook, the content is organised as units while in the Dutch approach textbook the content is organised in sections.

From Table 7.2 it is apparent that there are distinct differences in the classroom activities proposed in the two books. Activities in the Singapore approach textbook facilitate the learning of mathematical concepts through exploration and discovery. Some of these activities provide students with opportunities to use ICT tools that encourage interactive learning experiences. While these classroom activities are structured systematically, each activity is complete of itself, and can be carried out independently from the others. There is no one context that runs through all the activities in the chapter. However, in the Dutch approach textbook, students are introduced to the context of locating forest fires from fire towers and this context is used in the activities throughout the chapter. These classroom activities require students to apply their existing knowledge before introducing the formal mathematical concepts, thus providing students with opportunities to make connections between the new concepts and previous knowledge and with applications in real-life situations as well.

7.3.3 Complexity of the Demands for Student Performance on Graphing Equations in the Two Textbooks

In the two textbooks, classroom activities and practice questions comprise questions of two types. The first type is merely about the recall of knowledge and development of skills. These questions contain verbs such as 'find', 'write down', and 'plot/draw'. The second type involves higher-order thinking and these questions ask students to 'explain', 'justify', and 'interpret'. The verbs in the questions refer to the level of cognitive activity the students are invited to be engaged in.

Table 7.2 Classroom activities in the two textbooks

Singapore approach in the textbook <i>Discovering Mathematics 1B – Coordinates and Linear Functions</i>	
Unit 12.1. Cartesian coordinate system:	
State the coordinates of given points on the Cartesian plane and the quadrants in which the points lie → Plot points on the Cartesian plane → Play a battleship game that involves the use of Cartesian coordinates	
Unit 12.2. Idea of a function:	
Use a function machine to understand the concept of function and represent a function using verbal, tabular, graphical and algebraic forms → Practise the different ways a function can be represented → Associate ordered pairs with points on a coordinate plane to represent a relationship between two variables	
Units 12.3. Linear functions and their graphs:	
Recognise linear functions and draw graphs of linear functions	
Unit 12.4. Gradients of Linear Graphs:	
Learn the idea of gradient of a straight line as the ratio of the vertical change to the horizontal change → Interpret the meanings of positive, negative, zero and undefined gradients → Recognise how the graph of the linear function $y = ax + b$ changes when either a or b varies → Understand the physical interpretation of the gradient of a linear graph as the rate of change	
<hr/>	
Dutch approach in the textbook <i>Mathematics in Context—Graphing Equations</i>	
Section A. Where there's smoke:	
Use compass directions and then degree measurements to describe directions → Plot lines that intersect to locate forest fires on a map → Use coordinates to locate these fires on a computer screen that uses a four-quadrant coordinate grid → Explore how the coordinates (x, y) change as the fire moves in different directions → Use equations of vertical/horizontal lines to describe the movement of fires along the vertical/horizontal lines → Draw vertical/horizontal lines described by equation to represent firebreaks → Use inequalities to describe regions	
Section B. Directions as pairs of numbers:	
Use direction pairs to describe directions and discover that more than one direction pair can describe the same direction → Explore the use of direction pairs on a coordinate grid → Investigate direction pairs that describe the same direction and different directions → Investigate direction pairs that are opposite and discover that they form one line → Learn and apply the concept of slope through describing a direction as the slope of a line using a single number, the ratio of the vertical component to the horizontal component → Determine the slope of a graphed line and graph a line give the slope and a starting point → Determine the slope of graphed lines → Use the slope to determine the point at which two non-parallel lines meet	
Section C. An equation of a line:	
Investigate how to move along a straight line by taking steps in horizontal and vertical directions on a graphing programme → Use horizontal and vertical steps to informally investigate the equation of a line in slope-intercept form → Learn and interpret the meanings of the parameters in an equation in slope-intercept form → Draw the lines using the slope and y-intercept → Write equations for graphed lines → Investigate the equations of parallel lines → Investigate the relationship between the slope of a line and the angle that the line makes with the positive x -axis → Learn the relationship between the slope and the tangent of the angle that a line makes with the positive x -axis	
Section D. Solving equations:	
Investigate a context involving jumping frogs and compare the effect of different jump lengths on the distance that two frogs travel from starting points → Compare and use diagrams and equations to determine the unknown length of a frog jump → Solve equations of the form $ax + b = cx + d$ with reference to the 'frog problem' → Solve 'frog problems' that involve jumps in opposite directions → Use diagrams to represent expressions and equations and solve an equation → Use a number line to represent and solve equations → Perform the same operation on each side of an equation to solve it	
Section E. Intersecting lines:	
Estimate the coordinates where two lines intersect and use the equations for the lines to check the estimate → Solve equations to determine the coordinates of the point where two lines intersect → Solve problems involving point of intersection for pairs of lines → Compare and make connections between the graphical and algebraic methods of solving linear equations	

In this section, we focus on questions of the second type present in classroom activities and practice questions. Table 7.3 shows the key words and questions stemming from the classroom activities, practice questions and prompts for reflection in the two chapters in the Singapore and the Dutch approach textbook respectively.

From Table 7.3, it is apparent that the classroom activities, practice questions and prompts for reflection in both textbooks do engage students in higher-order thinking. In the Singapore approach textbook questions/instructions such as “What can you observe about the relationship of ...?”, “What can you say about ...?”, “Interpret ...”, “Explain ...”, and “Describe ...” encourage students to integrate information, choose their own strategies, and explain how they solved a problem. However, in the Dutch approach textbook, in addition to the questions/instructions found in the

Table 7.3 Complexity of cognitive demands for student performance in questions in the two textbooks

Approach in textbook	
Singapore approach in the textbook Discovering Mathematics 1B— <i>Coordinates and Linear Functions</i>	Dutch approach in the textbook Mathematics in Context— <i>Graphing Equations</i>
<ul style="list-style-type: none"> • What can you observe about the relationship of ...? • What can you say about ...? • Can we use the equation to ...? Explain briefly. • Describe a real-life example where 2 variables are in a linear relationship and draw a graph to represent the relationship. • Interpret the physical meaning of ... 	<ul style="list-style-type: none"> • Explain why or why not. • What can you say about ...? • Describe what happens ... • How do you ...? Explain your answer. • What do you notice in your answers ...? • Explain how you can conclude this from ... • How did you find out? • What is the simplest way to ...?
	<ul style="list-style-type: none"> • Explain what each of ... refers to. • Explain the formula. • Does the formula work for ...? • What is the importance of ... for the graph? • Why do you think it is called the ...? • Justify your answer. • What is ... if ...? • Do you agree ...? Explain. • Write down your thinking about this problem. Share your group's method with the other members of your class. • How can you be sure that your answers are correct?
<p>For reflection</p> <ul style="list-style-type: none"> • Describe in your own words ... • Describe two quantities which have a linear relationship in your daily life. 	<p>For reflection</p> <ul style="list-style-type: none"> • Compare the two ways... • How can similar triangles be used to find the slope of a line? • Describe in your own words what is meant by the word... • Explain why it is important to ... • Think about the three different methods for ... What are the advantages and disadvantages of each method? • Graphs and equations can be used to describe lines and their intersections. Tell which is easier for you to use and explain why.

Singapore approach textbook, there are further questions/instructions such as “What is the simplest way to ...?”, “What if ...?”, “Do you agree ...?”, “How can you be sure ...?”, “Write down your thinking ...”, and “Share your method ...”. These encourage students to analyse, interpret, synthesise, reflect, and develop their own strategies or mathematical models. Therefore, it may be said that the classroom activities, practice questions and prompts for reflection in the Dutch approach textbook span a wider range of higher-order thinking when compared with the Singapore approach textbook.

7.4 Findings and Discussion

In the last section, we examine both the textbooks in three main areas, namely (1) sequencing of content, (2) classroom activities, and (3) complexity of the demands for student performance proposed in the chapter on graphing equations in the two textbooks. Our data and results show that there are similarities and differences in all three of the above areas.

7.4.1 Sequencing of Content

Both the Singapore approach and Dutch approach textbooks provide opportunities for students to connect the mathematical concepts to meaningful real-life situations, practice questions for self-assessment, and reflect on their learning. However, the approaches presented in the two textbooks are different.

In the Singapore approach textbook, students learn the topic in a structured and systematic manner—direct introduction of key concepts, class activities that enhance their learning experiences, worked examples, followed by practice questions and question that allow students to apply mathematical concepts. The application of the mathematical concepts to real-world problems takes place after the acquisition of knowledge in each sub-topic, and reflection of learning takes place at the end of the whole topic.

In the Dutch approach textbook, students learn the mathematical concepts in the topic in an intuitive manner, threaded by a single real-life context. Students learn the concepts through a variety of representations and make connections among these representations. They learn the use of algebra as a tool to solve problems that arise in the real world from a stage where symbolic representations are temporarily freed to a deeper understanding of the concepts. The application of the mathematical concepts to real-world problems takes place as the students acquire the knowledge in each sub-topic, and reflection of learning also takes place at the end of each sub-topic.

7.4.2 *Classroom Activities*

The classroom activities proposed in both the Singapore approach and Dutch approach textbooks provide opportunities for students to acquire the mathematical knowledge through exploration and discovery. ICT tools are also used appropriately to enhance their interactive learning experiences.

However, the classroom activities proposed in the Singapore approach textbook are typically each complete in themselves and can be carried out independently from the others. There is no one context that runs through all these activities. In the Dutch textbook approach, the context introduced at the beginning of the chapter is used in the classroom activities throughout the chapter. These classroom activities require students to apply their existing knowledge before introducing the formal mathematical concepts, thus providing students with opportunities to make connections between the new concepts and previous knowledge and with applications in real-life situations as well.

7.4.3 *Complexity of the Demands for Student Performance*

In both the Singapore approach and the Dutch approach textbooks, classroom activities and practice questions comprise questions that (1) require recall of knowledge and development of skills, and (2) require higher-order thinking and make greater cognitive demands of the students. The student learning process is facilitated with questions such as “What can you observe?”, “What can you say?”, “Explain”, “Why do you think?” and “What if?”.

However, the classroom activities, practice questions and prompts for reflection in the Dutch approach textbook provide students with more scope for reasoning and communication and promote the development of the disciplinary orientation of mathematics. There are further questions/instructions that encourage students to analyse, interpret, synthesise, reflect, and develop and share their own strategies or mathematical models.

7.5 Reflections of Two Singapore Mathematics Teachers

Two mathematics teachers who are co-authors of this chapter and are using the Singapore approach textbook in their schools, studied of both textbooks the chapter on graphing equations. Their reflections on these chapters were guided by the following questions:

- How do you teach graphing equations to your students?
- Has the Dutch approach textbook provided you with an alternative perspective?

- Would the Dutch approach work in Singapore classrooms? What would it take for it to work in Singapore classrooms?

7.5.1 Profiles of the Two Teachers

Both teachers, Wong Lai Fong (WLF) and Simmi Naresh Govindani (SNG), are lead mathematics teachers. They have been teaching secondary school mathematics for the past two decades. As lead teachers, they have demonstrated a high level of competence in both mathematical content and pedagogical and didactical content knowledge. In addition to their teaching duties they are also responsible for the development of mathematics teachers in their respective schools and other dedicated schools. Teacher WLF teaches in an average ability band school while Teacher SNG teaches in a lower ability band school compared to that of Teacher WLF.

7.5.2 How Do You Teach Graphing Equations to Your Students?

WLF:

Typically, when teaching the topic of graphing equations, I adopt the following sequence. First, I use a real-life example to illustrate the use of the mathematical concepts. Next, I engage students in learning experiences that provide them with opportunities to explore and discover the mathematical concepts, with appropriate scaffolding using questions of higher cognitive demands that require students to reason, communicate and make connections. Lastly, I induct my students in doing practice questions varying from direct application of concepts to application of concepts to real-life problems.

SNG:

Usually when I teach this topic I would first of all use a real-life example to explain the concept of location. To do so, I use the Battleship puzzle (available as a physical board game as well as in an online version) to provide my students with a learning experience and set the context for learning the topic. This puzzle facilitates students in plotting points using coordinates (x, y) . Next, I would explain the concept of gradient by linking it to steepness and gentleness of slope of a straight line. An interactive worksheet or an ICT enabled lesson would be used to scaffold learning. Lastly, the concept of equation of a straight line would be explained by plotting points (on graph paper) which lie on a straight line. Students would be engaged in looking for patterns to arrive at the relation between x and y coordinates of any point on a given line. I would highlight and show that every point on the line satisfies the equation and points not on the line do not satisfy the equation. In all of the above, I would ask pertinent

questions during the course of the lesson, to check for students' understanding. In addition, I would use mathematical tasks to engage students in reflecting on their learning and addressing students' misconceptions/errors.

7.5.3 Has the Dutch Approach Textbook Provided You with an Alternative Perspective?

WLF:

The Dutch approach has provided me with an alternative perspective where a topic can be taught with the introduction of a real-life context. Moving from informal to formal representations, this approach encourages student to continuously formalise their mathematical knowledge, building on what they already know in real-life and previous topics through mathematical reasoning and communication, thus creating an appreciation and making meaning of what they are learning and how it will be a tool to solve problems that arise in the real world.

SNG:

Yes, the Dutch approach is very interesting because it provides for mathematical reasoning and communication in the classroom throughout the process of learning. Also, teachers are able to help their students in monitoring success and correct errors when appropriate, thus promoting metacognition. These are some of the 21st century competencies that we would like our students to acquire.

7.5.4 Would the Dutch Approach Work in Singapore Classrooms? What Would It Take for It to Work in Singapore Classrooms?

WLF:

When adopting the Dutch approach, the role of a teacher is impetus. The teacher must possess sound pedagogical and didactical content knowledge in order to facilitate student learning with effective questions that promote thinking and make higher-cognitive demands on the students. Through classroom discourse, the teacher has to listen closely to students' responses and observe for evidence of students' understanding of the mathematics. Besides the mathematical knowledge, the teacher must also possess knowledge of the real-life context so as to help students connect to the context through appropriate questions and discussions.

In order to adopt the Dutch approach in the Singapore classrooms, perhaps a paradigm shift in the teachers' mindset on how mathematics learning takes place is necessary—from 'content to application' to 'content through application'. There may

not be a drastic change in the teaching approach or strategies as learning experiences that promote mathematical reasoning and communication are currently taking place in the Singapore classrooms. With appropriate modification to our existing teaching resources, accompanied with well-designed textbooks and teachers' guides, there is definitely a chance of successful implementation of the Dutch approach in our local classrooms.

SNG:

Singapore mathematics teachers may not be adequately skilled in carrying out such lessons. Hence, there is a need for teacher training and a mindset change to explore and embrace the change. Well-designed textbooks and teacher guides could help to alleviate some of the issues.

7.6 Concluding Remarks

This chapter shows how the teaching of graphing equations differs in the Singapore approach and the Dutch approach textbooks. Needless to say, this is the case as both the books are based on different ideas of how best to teach mathematics or how best teachers may facilitate the students' learning of mathematics. It is clearly evident that teachers using the Singapore approach teach for problem solving in which they move 'from content to application', while in the Dutch approach, following the core teaching principles of RME (Van den Heuvel-Panhuizen & Drijvers, 2014) the students are taught 'content through application'. From the reflections of the two lead teachers teaching in Singapore schools and using the Singapore approach it is apparent that they see merit in the Dutch approach but feel that for teachers to adopt the Dutch approach, a paradigm shift in the minds of teachers and adequate support in terms of resources would be necessary.

References

- Brewer, D. J., & Stasz, C. (1996). *Enhancing opportunity to learn measures in NCES data*. Santa Monica, CA: RAND.
- Carroll, J. (1963). A model of school learning. *Teachers College Record*, 64, 723–733.
- Chow, W. K. (2013). *Discovering mathematics 1B* (2nd ed.). Singapore: Star Publishing Pte Ltd.
- Foxman, D. (1999). *Mathematics textbooks across the world: Some evidence from the third international mathematics and science study*. Slough, UK: National Foundation for Educational Research (NFER).
- Freeman, D., & Porter, A. (1989). Do textbooks dictate the content of mathematics instruction in elementary school? *American Educational Research Journal*, 26, 403–421.
- Haggarty, L., & Pepin, B. (2002). An investigation of mathematics textbooks in England, France and Germany: Some challenges for England. *Research in Mathematics Education*, 4(1), 127–144.

- Kindt, M., Wijers, M., Spence, M. S., Brinker, L. J., Pligge, M. A., Burrill, J., et al. (2010). Graphing equations. In Wisconsin Center for Education Research & Freudenthal Institute (Eds.), *Mathematics in context*. Chicago, IL: Encyclopaedia Britannica, Inc.
- Liu, X. (2009). *Linking competencies to opportunities to learn: Models of competence and data mining*. New York, NY: Springer.
- Ministry of Education Singapore. (2012). *O-Level, N(A) Level, N(T) level mathematics teaching and learning syllabuses*. Singapore: Author.
- Pepin, B., & Haggarty, L. (2001). Mathematics textbooks and their use in English, French and German classrooms: A way to understand teaching and learning cultures. *ZDM—Mathematics Education*, 33(5), 158–175.
- Reys, B. J., Reys, R. E., & Chávez, O. (2004). Why mathematics textbooks matter. *Educational Leadership*, 61(5), 61–66.
- Robitaille, D. F., & Travers, K. J. (1992). International studies of achievement in mathematics. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 687–709). New York, NY: Macmillan Publishing Company.
- Schmidt, W. H., McKnight, C. C., & Raizen, S. (1997a). *A splintered vision: An investigation of U.S. science and mathematics education*. Boston, MA: Kluwer Academic Publishers.
- Schmidt, W. H., McKnight, C. C., Valverde, G. A., Houang, R. T., & Wiley, D. E. (1997b). *Many visions, many aims: A cross-national investigation of curricular intentions in school mathematics*. Dordrecht, the Netherlands: Kluwer Academic Publishers.
- Skemp, R. (1976). Relational understanding and instrumental understanding. *Mathematics Teaching*, 77, 20–26.
- Tornroos, J. (2005). Mathematics textbooks, opportunity to learn and student achievement. *Studies in Educational Evaluation*, 31(4), 315–327.
- Van den Heuvel-Panhuizen, M., & Drijvers, P. (2014). Realistic mathematics education. In S. Lerman (Ed.), *Encyclopedia of mathematics education* (pp. 521–525). Dordrecht, the Netherlands: Springer.
- Valverde, G. A., Bianchi, L. J., Wolfe, R. G., Schmidt, W. H., & Houang, R. T. (2002). *According to the book. Using TIMSS to investigate the translation of policy into practice through the world of textbooks*. Dordrecht, the Netherlands: Kluwer Academic Press.
- Wijaya, A., Van den Heuvel-Panhuizen, M., & Doorman, M. (2015). Opportunity-to-learn context-based tasks provided by mathematics textbooks. *Educational Studies in Mathematics*, 89, 41–65.
- Wisconsin Center for Education Research & Freudenthal Institute. (2010). *Mathematics in context*. Chicago, IL: Encyclopaedia Britannica Inc.
- Xin, Y. P. (2007). Word problem solving tasks in textbooks and their relation to student performance. *Journal of Educational Research*, 100(6), 347–359.

Open Access This chapter is distributed under the terms of the Creative Commons Attribution 4.0 International License (<http://creativecommons.org/licenses/by/4.0/>), which permits use, duplication, adaptation, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, a link is provided to the Creative Commons license and any changes made are indicated.

The images or other third party material in this chapter are included in the work's Creative Commons license, unless indicated otherwise in the credit line; if such material is not included in the work's Creative Commons license and the respective action is not permitted by statutory regulation, users will need to obtain permission from the license holder to duplicate, adapt or reproduce the material.