Chapter 4

The Impact of Hans Freudenthal and the Freudenthal Institute on the Project Mathe 2000

Erich Ch. Wittmann

Abstract This chapter is an attempt to describe the direct and indirect influence Hans Freudenthal and his institute had on the developmental research conducted by Mathe 2000. Special attention will be given to the balance of pure and applied mathematics in designing learning environments, where RME and Mathe 2000 differ to some extent, and to the role of mathematics in mathematics education.

Keywords Developmental research · Mathematics as a ‘design science’ · Structure-genetic didactical analysis · Mathe 2000

4.1 Introduction

In 1967 the department of mathematics at the University of Erlangen organised a colloquium in commemoration of the geometer K. G. Ch. von Staudt (1798–1867). Hans Freudenthal, an international expert also in the foundations of geometry, was one of the invited speakers, and I eagerly awaited to meet him for the following reason: I had just finished my studies as a prospective teacher and was going to submit my doctoral dissertation in the theory of infinite groups. As I was seriously considering the option of moving into mathematics education at a later point of my career, I had also started to read the literature in this rapidly growing field and while doing so I developed a strong aversion against the New Math movement, which at that time seemed to override the teaching of mathematics at both universities and schools.

Erich Ch. Wittmann: Project Mathe 2000

E. Ch. Wittmann (✉)
Technical University of Dortmund, Dortmund, Germany
e-mail: wittmann@math.tu-dortmund.de

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In this critical situation one of Freudenthal's papers (Freudenthal, 1963) was an enlightenment for me in several respects. The paper contained a convincing refutation of Bourbaki's architecture of mathematics as a basis for mathematics teaching. Moreover, the paper was written in a style I had never seen before: brilliant, witty and unconventional. For example, the hesitation of a mathematician to publish a paper according to its genesis was compared to the feelings of a man standing in the street in his underwear. The most important point, however, was the picture that was drawn of mathematical learning, namely as a process that passes through different stages, each one a necessary step for the next one. The paper emphasised mathematical activity as the crucial element of learning and described 'local ordering' as a reasonable alternative to readymade axiomatics.

At the colloquium, I had a chance to talk to Hans Freudenthal (Fig. 4.1), and here I learned of his fresh initiatives as the president of the International Commission on Mathematical Instruction (ICMI): the organisation of the First International Congress in Mathematics Education (ICME 1) in Lyon in 1969 and the foundation of an international journal in mathematics education (*Educational Studies in Mathematics Education*, first published in 1968).

What impressed and influenced me likewise was the work that Freudenthal initiated in 1971 at the IOWO¹ in Utrecht. Here he gathered a team of highly creative mathematics educators, among them Aad Goddijn, Fred Goffree, Martin Kindt, Jan de Lange, Ed de Moor, Leen Streefland, George Schoemaker, Adri Treffers, and later Marja van den Heuvel-Panhuizen.

The developmental research conducted at the IOWO (later re-named Freudenthal Institute) served as a source of inspiration for our project Mathe 2000 at the University of Dortmund in many respects. This project was founded in 1987 after Germany had overcome the painful stagnation caused by New Math. For good reasons Hans Freudenthal ranks as one of the five archfathers of Mathe 2000 (see <https://www.mathe2000.de/Projektbeschreibung>).

In what follows, the influence of 'Utrecht' on 'Dortmund' in four areas is described in a nutshell.

¹ Instituut voor de Ontwikkeling van het Wiskunde Onderwijs (Institute for the Development of Mathematics Education).

Fig. 4.1 Hans Freudenthal at the Staudt colloquium in 1967

4.2 Developmental Research

In the preface of the volume *Five Years IOWO* Hans Freudenthal stated:

IOWO is not a research institute; its members do not regard themselves as researchers but as producers of instruction, as engineers in the educational field, as curriculum developers. Engineering needs background research and can produce research as fall-out. Though both of them will be visible in the present account, its nucleus is our productive work, represented by a few specimens, and embodies our views on mathematics as a human activity and on curriculum development as a classroom activity, guided by curriculum developers, in close contact with all those interested in mathematics education. (IOWO, 1976, p. 189)

In developing didactical units, which nowadays are called learning environments, the Freudenthal Institute has set standards of quality. Many of these environments

are just brilliant, particularly those in geometry, and we adopted quite a number of them in the Mathe 2000-curriculum, with a clear preference for those that can be integrated with mathematical structures.

In addition to the contribution that each single learning environment means for teaching a certain content, the work of the Freudenthal Institute has also greatly inspired the view of mathematics education as a ‘design science’. The talk given by Edu Wijdeveld (IOWO, 1976, pp. 243–244) on the ‘dwarf village’ at the colloquium held in 1976 at the occasion of Hans Freudenthal’s retirement was particularly pertinent in this respect. The ‘dwarf village’ represents a substantial application of geometry that is related to combinatorial counting (group operating on a set). When my paper on mathematics education as a ‘design science’ appeared in English (Wittmann, 1995), I was happy to learn that Adri Treffers called it a “credo of our common work”.

As far as the intended close connection of curriculum development with teacher education is concerned, Mathe 2000 had perhaps better boundary conditions than the IOWO as Mathe 2000 did not form an institute of its own but was a kind of virtual project immersed in the official pre-service and in-service teacher education programmes offered by the University of Dortmund. Another clear advance of this structure was that Mathe 2000 was completely independent of any funding.

4.3 The View of Mathematics

Hans Freudenthal, a scholar with an enormous breadth of interests, has taught us to look at mathematics as a field of knowledge that is firmly integrated into our culture and determined by both external (‘applied’) and internal (‘pure’) factors. His masterpiece *Mathematics as an Educational Task* (Freudenthal, 1973) bears witness to this conviction. Richness of relationships (‘*Beziehungshaltigkeit*’) was a postulate to which Freudenthal frequently referred, and it included both structural relationships (‘vertical mathematisation’) and relationships with the real world (‘horizontal mathematisation’). In his book *Three Dimensions*, Adri Treffers elaborated this approach in detail (Treffers, 1978).

As Freudenthal and the IOWO wanted to establish a distinct counterpart to New Math, it is understandable that a clear emphasis was put on horizontal mathematisation and that the choice of the term ‘Realistic Mathematics Education’ (RME) was a deliberate one. In Mathe 2000, however, we had the feeling that in the later development of the Freudenthal Institute too much emphasis was put on applications; see, for example, the textbook series *Mathematics in Context* (National Center for Research in Mathematical Sciences Education & Freudenthal Institute, 1997–1998). However, it is only fair to acknowledge that there are also publications by members of the Freudenthal Institute that are extremely interesting in terms of mathematical structures (see, for example, Kindt & De Moor, 2012) and that helped us to shape our own more balanced conception.

4.4 A Genetic View of Teaching and Learning

In Freudenthal's view the learner has no choice but to 're-invent' mathematics under appropriate guidance by starting as a child from most elementary experiences and managing more and more complex structures with growing expertise. Mathematical knowledge can never be transmitted top-down in a readymade form. Even the most perfect lecture can become vital for a student only if he or she makes sense of it by actively re-constructing in personal terms what has been proposed. Hans Freudenthal radically objected to the idea of a didactical transposition from the level of specialists to lower levels. In his talk at the Carbondale Conference on Geometry he put this view in his typical language (Freudenthal, 1971, p. 435):

Geometry is endangered by dogmatic ideas on mathematical rigor. They express themselves in two different ways: absorbing geometry in a system of mathematics like linear algebra, or strangulating it by rigid axiomatics. So, it is not one devil menacing geometry as suggested in the title of my paper. There are two. The escape that is left is the deep sea. It is a safe escape if you have learned swimming. In fact, that is the way geometry should be taught, just like swimming.

This genetic view is reflected in a series of studies of learning processes conducted at the Freudenthal Institute. Two of them have been of particular importance for Mathe 2000.

In the last e-mail I received from Hans Freudenthal in March 1990, a few months before he passed away, he mentioned a study conducted by a young lady by the name of Marja van den Heuvel-Panhuizen who at that time was unknown to me. This study turned out as fundamentally important for Mathe 2000 for the following reason. According to a long tradition not only in Germany, but also in other European countries, the number space 0–20 and the addition table were introduced in Grade 1 step by step: the students worked with numbers up to 5 or 6, then some months with the numbers up to 10, and only in the last months of the school year the whole space from 0 to 20 was addressed. In Volume 1 of our *Handbook of Productive Practice* (Wittmann & Müller, 1990) we proposed to substitute this step-by-step introduction and the corresponding step-by-step introduction of the addition table by a holistic approach, in which the numbers 0–20, and later the addition table, are introduced in one step. We had a hard time to defend this approach against many critics. Marja van den Heuvel-Panhuizen's empirical findings with her MORE entry-test showed convincingly that the knowledge about numbers that school beginners bring to school is substantial and at the same time strongly underestimated by teachers (Van den Heuvel-Panhuizen, 1996, Chap. 5). These findings, which were corroborated by similar findings in some other European countries, supported the holistic approach of Mathe 2000 on all grounds and helped us to defend our position which in German mathematics education is now widely shared.

In Volume 2 of our *Handbook of Productive Practice* (Wittmann & Müller, 1992) we suggested a similar paradigm shift away from the strong fixation on standard algorithms towards various ways of calculating that are based on the arithmetical laws and represent a kind of early algebra; '*halbschriftliches Rechnen*' (*semiformal*

strategies) in the German tradition). We also suggested to use these semiformal strategies as a basis for deriving the standard algorithms (Wittmann & Müller, 1992). We were lucky once more that this approach was strongly supported by another great achievement of a member of the Freudenthal Institute. Adri Treffers' research on 'progressive schematisation' (Treffers, 1987) showed convincingly how naturally students perform the transition from semiformal strategies to the standard algorithms. For us progressive schematisation is so important that we chose it as one of the ten didactical principles on which the conception of Mathe 2000 is based.

4.5 Mathematics Education as a Research Domain

Based on his experiences at the IOWO, Hans Freudenthal has always looked sceptically at mathematics education as a research field, but nevertheless wrote his book *Weeding and Sowing* as a kind of prologue for an emerging research field (Freudenthal, 1978). In this book, he clearly separated the research he had in mind from the research on teaching and learning that is conducted by psychologists, pedagogues, sociologists and other generalists who do not and cannot take the content properly into account. For Freudenthal the didactical analysis of the subject matter was the most important source for designing learning environments and curricula. In this respect his book *Didactical Phenomenology of Mathematical Structures* (Freudenthal, 1983) is of overriding importance. This book is one of the basic references of our attempt to shed some new light on didactical analyses.

In a recent paper, I have tried to combine what I have learned both from Hans Freudenthal and Jean Piaget, another archfather of Mathe 2000 (Wittmann, 2018). My intention with this paper, titled "*Structure-genetic Didactical Analyses—'Empirical Research of the 'First Kind'*", is also to broaden the scope for empirical evidence. Mathematics, if properly understood, provides not only the contents for teaching but incorporates also processes that are crucial for the teacher-student interaction. After all, mathematics itself is the result of learning processes. When once asked what his motifs as a mathematician were for engaging in mathematics education Hans Freudenthal replied: "I want to understand better what mathematics is." I believe that vice versa it is worthwhile for mathematics educators to engage more deeply in mathematics in order to understand better what mathematics education is or should be about.

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