

Chapter 16

The Influence of Realistic Mathematics Education Outside the Netherlands—The Case of Puerto Rico



Omar Hernández-Rodríguez, Jorge López-Fernández,
Ana Helvia Quintero-Rivera and Aileen Velázquez-Estrella

Abstract In this chapter, we describe the genesis and evolution of Realistic Mathematics Education (RME) in Puerto Rico, and analyse the aspects that allowed or deferred its influence on local mathematics education. RME was introduced in Puerto Rico thanks to a group of mathematics professors at the University of Puerto Rico, Río Piedras Campus, who collaborated, first with staff from Wisconsin University and later more closely with a team of designers from the Freudenthal Institute. This was the beginning of a collaboration that lasted several years and accounted for the design and development of quality educational materials adapted to the Puerto Rican reality. The initial goal was to develop a curriculum for the elementary level, but it soon developed into a more comprehensive project *Las Matemáticas en Contexto en Puerto Rico* (MeC-PR) that included training for teachers and developers, implementation efforts, and research initiatives. RME in Puerto Rico went through interconnected, and sometimes overlapping, stages of design, training, implementation, and research. All of them left their mark in different areas such as educational practices, official documents, and research practices.

Keywords Realistic Mathematics Education · Mathematics in context · Puerto Rico

O. Hernández-Rodríguez (✉)

Department of Graduate Studies, School of Education, University of Puerto Rico, Río Piedras, Puerto Rico

e-mail: omar.hernandez4@upr.edu

J. López-Fernández · A. H. Quintero-Rivera

Department of Mathematics, University of Puerto Rico, Río Piedras, Puerto Rico

e-mail: jorgemar.lopez@gmail.com

A. H. Quintero-Rivera

e-mail: aquinter_2000@yahoo.com

A. Velázquez-Estrella

Carmen D. Ortiz School, Aguas Buenas, Puerto Rico

e-mail: aileen.velazquez@gmail.com

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16.1 Introduction

This chapter is divided into five sections that describe: (1) the beginnings of Realistic Mathematics Education (RME) in Puerto Rico; (2) the efforts for the development of educational materials based on RME principles and adapted to Puerto Rican culture; (3) the training of teachers for the implementation of the materials in their classrooms; (4) the incorporation of some elements of RME into the official documents of the Puerto Rican Department of Education (PRDE), the official government custodian of public mathematics education in Puerto Rico; (5) and the research efforts based on RME. The advances consider the significant developments experienced in RME in Puerto Rico, largely possible thanks to a collaborative effort between the Department of Graduate Studies of the College of Education and the Department of Mathematics at the University of Puerto Rico, Río Piedras Campus (UPR-RP). Finally, some concluding remarks are given to set the most likely directions for future endeavours. The authors took into consideration the contributions of many people related to RME: the people that brought the idea to Puerto Rico, the principals of the Regional Training Centres on Mathematical Instruction (CRAIM) that shaped and made possible training workshops for teachers, the people who promoted the inclusion of RME in official PRDE documents, the school teachers and university professors who participated in the design and development of the materials, and the teachers that later on used the materials in their classrooms, thus making a notable contribution to the validation of the materials, and, in doing so, giving an eloquent example of the effectiveness of the principles of RME when applied to mathematics teaching and learning.

16.2 The First Steps

RME was introduced in Puerto Rico in 1992 when Thomas A. Romberg, then director of the National Center for Research in Mathematical Sciences Education (NCRMSE) and professor of Curriculum and Instruction at the University of Wisconsin in Madison, visited Puerto Rico as an evaluator of the National Science Foundation for the Puerto Rico Statewide Systemic Initiative project. On that occasion, Romberg contacted Jorge López-Fernández to propose collaboration with him and Jan de Lange, director of the Freudenthal Institute (FI) at Utrecht University in the Netherlands, for the development of Spanish versions of the materials of the textbook series *Mathematics in Context* (MiC) (NCRMSE & Freudenthal Institute, 1997–1998), meant for the U.S. middle school (Grades 5–8), being developed at the University of Wisconsin-Madison together with the Freudenthal Institute. Romberg had become familiar with the RME approach developed at the FI at Utrecht University, and considered that it was consistent with the vision of the emerging standards recently developed and published by the National Council of Teachers of Mathematics (NCTM) and that it

could serve as a model for the middle grades curriculum for the United States (Webb & Meyer, 2007).

By that time, Jorge López-Fernández, a mathematician by training, was the director of the CRAIM, one of the few centres responsible for giving training in mathematics to Puerto Rican in-service teachers. By then, trainings given by CRAIM were focused mainly on the improvement of the mathematical content knowledge of teachers. Initially, most trainers were university professors in mathematics from different campuses of the University of Puerto Rico.

From that initial contact, Jorge López-Fernández and Professor Víctor García-Muñiz were engaged in the translation to Spanish and cultural adaptation of the MiC units and the corresponding teacher guides developed at NCRMSE. The first materials for middle school (Grades 5–8) in Spanish based on RME principles were developed. In this effort, several people from different campuses of the University of Puerto Rico were integrated into the CRAIM team.¹ As part of the Puerto Rican participation in the development of the MiC units, CRAIM got special permission to use the materials for teacher training seminars and workshops as long as there was no commercial version available. This access to using the experimental materials with teachers gave a significant boost to RME in Puerto Rico.

From the beginning, it became clear that the units developed at the FI were of exceptional quality. This led the CRAIM principals to implement the teaching of these units at various schools in Puerto Rico and investigate their effect on students. In 1995, with financial and logistical support from the Encyclopaedia Britannica Educational Corporation, a CRAIM team piloted some of the recently created materials with students from a second unit² school in a rural area of the municipality of Yabucoa. Participants in the experimental group were disadvantaged students and surprisingly outperformed their peers in the regular stream. This led Jorge López-Fernández to consider establishing a direct partnership with the people who developed the materials for MiC.

16.3 A Productive Collaboration

After the MiC project with Romberg, around 1995 a second collaboration ensued between the FI and CRAIM, with the purpose of developing a curriculum for elementary school. In fact, it was desirable to co-develop instructional materials with the FI to incorporate RME's philosophic views to the design of mathematics materials fitting Puerto Rican culture. The public policy of the time supported the solution of problems as a strategy to teach mathematics, however, teachers had difficulties implementing it. Most of the teachers required for the didactical material to be full

¹ Among these, Ana Helvia Quintero-Rivera and René Hernández-Toledo, mathematics professors at respectively the UPR Río Piedras and Cayey campuses, stood out.

² Locally, the term 'second unit' is used to describe schools with all grades from elementary to intermediate (Grades K–9).

of interesting and concise contexts avoiding general and open-ended tasks. In addition, findings from previous pilot testing show MiC material requires students to do extensive reading, an obvious deterrent to its use. A more piecemeal approach became the goal, and it was achieved thanks to the collaboration between the FI and CRAIM, which lasted several years. Eventually the Mathematics Program of the PRDE became interested in the project and decided to give financial support.

The project was then called *Las Matemáticas en Contexto en Puerto Rico*³ (MeC-PR) and its goal was to develop a curriculum for the elementary level (Grades K–6), based on the principles of RME, to meet the needs of the Puerto Rican school system. The plan to undertake the task was two-fold: to create a collaborative development team with developers from the FI and CRAIM, and to train Puerto Rican teachers, developers and researchers on RME. Koeno Gravemeijer led the Dutch team and Jorge López-Fernández the Puerto Rican team that participated in the design of the curriculum materials. The Puerto Rican team consisted of mathematics professors and teachers from public schools.

The purpose of the initial stage was to form a critical mass of developers able to undertake a series of projects to create materials based on Gravemeijer's textbook series *Rekenen & Wiskunde*, but taking into consideration the normative documents of PRDE mathematics education and the very particular contexts suitable for Puerto Rican students. CRAIM's staff encountered many problems that had to be overcome. For example, the materials as well as the teacher guides were in Dutch. Non-specialised translators were contracted to translate some of the students' materials; in fact, they were Puerto Rican students that learned the language while holding internships in Dutch universities. In addition, the Dutch team developed executive summaries in English that served as guides to start the development of the units. However, the most difficult and important issue was the need to transform the paradigms of Puerto Rican educators, given that the RME principles and design methods were different from the ones used in the development of the Puerto Rican curriculum. It was a slow process that took several years. The first production of the Puerto Rican team was the creation of the principles of MeC-PR to be used as reference for the design and development of the materials, and also for the training of teachers who would implement the materials in their classrooms later on. Ana Helvia Quintero-Rivera and Jorge López-Fernández developed five learning principles and five teaching principles based on those exposed by Treffers (1991).

The general principles, succinctly presented in Table 16.1, were stated in *Los Principios Generales del Aprendizaje y de la Enseñanza* (López-Fernández & Quintero-Rivera, 1995), a document widely used to promote the MeC-PR goals and philosophy. In the document, these principles are explained and illustrated with examples on how to use them in training workshops for teachers, and for the design and development of curriculum materials.

After the initial statement of principles and goal, several years of intense labour followed under the guidance of Koeno Gravemeijer, who was instrumental in overcoming the many difficulties that this project encountered. A lot of time was invested

³ Mathematics in Context in Puerto Rico.

Table 16.1 Learning and teaching principles as stated in the document *Los Principios Generales del Aprendizaje y de la Enseñanza* (López-Fernández & Quintero-Rivera, 1995)

Learning principles	Teaching principles
Learning is a constructive process, that is, learning is built. Students learn by building their own knowledge, that is, relating new ideas and concepts to the body of knowledge they already have	Teaching must come from students (exploring their informal or prior mathematical knowledge) and must originate in the consideration of specific situations that arise in contexts of interest to students. Overall teaching should follow the outline concrete, pre-formal, formal stages
Knowledge is achieved over long periods of time and across levels of abstraction that become progressively higher	Education should be planned to provide the circumstances for the student to develop their mathematical knowledge through progressively higher levels of abstraction. It must be specifically designed to 'integrate' vertically the elements of mathematics education in order to provide opportunities for the introduction of models, notations, conceptual schemes, symbols, etc., that promote the transition from lower to higher levels of knowledge
The students' reflection about their own and others' reasoning promotes the learning of mathematics and elevates the levels of abstraction of the knowledge acquired	The mathematics curriculum should provide students with multiple opportunities to reflect on the learning of mathematics and to anticipate the mathematical development that still lies ahead. Teachers should make use of challenging situations and conflicting problems to make the students reflect on the nature and the consequences of the mathematical knowledge acquired. Student productions that result from such reflections allow the teacher to determine with more certainty the development reached by the students
Rather than an individually based activity, learning is an activity of social nature. The social and cultural contexts stimulate and guide learning	Education should have an interactive character that promotes the exchange of ideas between students with each other, and between students and the teacher
Learning must be structured and schematic. Learning requires the structuring of data and mathematics skills in a coherent whole. Learners should be able to connect between different areas of the curriculum	Teachers should provide environments that allow students to discover and build relationships between different areas of study of the curriculum. Such linkage should promote the development of connections between the normative documents of mathematics education and individual students. The integration of knowledge must be based on the consideration of actual or contextual situations taken from the everyday world of students, which empower them to use informal strategies in the search for mathematical connections

in educating CRAIM's staff in the theoretical aspects of RME and studying, in seminars and special meetings, different features of RME and the ways in which to apply it to Puerto Rico. CRAIM principals were convinced that RME offered realistic possibilities to develop and execute a solid curriculum.

One example of such difficulties was related to the fact that in Puerto Rico it is expected that students learn the digit-based algorithms (known in Puerto Rico as 'column algorithms') for addition and subtraction of natural numbers very early in elementary school. This expectation remains, even though mathematics education research shows that direct exposure to these algorithms fosters serious conceptual errors related to order of magnitude and the decimal representations of numbers. In contrast, students who precede the study of these algorithms with activities based on informal arithmetic and counting strategies, such as rounding to the nearest multiple of five or ten, counting by doubles, among others, end up understanding the digit-based algorithms for addition and subtraction better and faster (Cobb et al., 1991).

To solve the situation, an RME context was adapted at the suggestion of the Dutch team: the cookie factory. At the factory, cookies are sold individually, in packages of ten, in boxes of 10 packages of 10 cookies, and so on. Children had to pack or unpack boxes to solve contextual problems. Horizontal and vertical mathematisation were present in the situations proposed to students (López-Fernández & Velázquez-Estrella, 2007, 2011). The metaphor made it to the textbook materials and research was carried out afterwards to ascertain the possible cognitive advantages related to the discussion and solution of problems that arise in the context of the cookie factory, which naturally led to the digit-based addition and subtraction algorithms. This is exemplary in at least one way; RME incorporates the use of familiar contexts (which could be and most likely are culturally dependent) with the ideas of modelling (descriptive and prospective) as a vehicle for applying Freudenthal's principles to present coherent mathematics education units. In practice, educational systems where innovations are proposed have their own views, judgments and prejudices. For example, in Puerto Rico, if the column algorithms are not present in the arithmetic lessons for the second grade, teachers and the official educational system will not accept such lessons as adequate for teaching. We opted to follow the expected presence of these algorithms, but formulated them in robust MeC-PR units that made their teaching more meaningful and improved significantly student understanding of the algorithms as follow-up research has shown.

The collaboration between the FI and the CRAIM staff finally gave results. The first MeC-PR products for students were thirteen textbooks, two for each elementary Grade 1–6 and one for kindergarten, all including their corresponding teacher guides. The RME materials were designed to minimise printing costs for the production of the student units. All materials can be accessed online and can be made available for students in the form of low cost pamphlets. Furthermore, the MeC-PR materials admit the possibility of continuing renewal and improvement by allowing for the creation of new contexts of current interest. The possibilities for improvement are truly amazing.

It should be appended that in the development of the materials for kindergarten, collaboration was sought by the CRAIM team from the Autonomous University

of Barcelona at Bellaterra, led by Joseph Maria Fortuny Aymeni. Traditionally (in Puerto Rico as well as in the Netherlands), kindergarten materials are usually in the hands of both mathematics educators and early childhood experts. The Dutch materials of interest to the CRAIM principals, at the time, did not include kindergarten topics. Monserrat Torra Bitlloch, from the Barcelona Team, worked with Puerto Rican kindergarten teachers to develop the materials for that grade. A by-product of this partnership was the design of a master's degree in the area of elementary mathematics in context for schoolteachers. The entire curriculum was designed and some of the intended virtual courses were prepared.

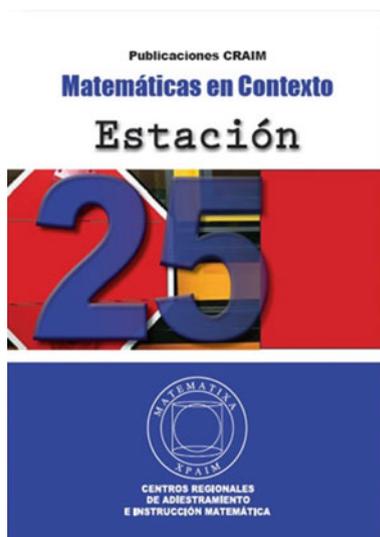
A few years later, the units went through two stages of revisions. Due to official requirements, the materials had to be aligned to the PRDE mathematics standards released in 2000 and 2007. This represented a problem since in some instances there were huge cultural differences between our mathematics education traditions (influenced mostly by the U.S. Department of Education) and those of the Dutch. On these revisions, materials were edited to a form that made it easier for them to be used as handbooks; the teacher's guides were also revised. These efforts represent an honest way to gain a hold and take possession of the intense and deep didactical tradition of the Netherlands. The textbooks were converted to 'stations', a kind of workbooks with more focalised topics and few objectives. Twenty-eight stations were developed for grades from kindergarten to third grade. All the stations had a teacher's guide with explanations on the use of the models and its connection to MeC-PR principles of learning. Testing and assessment tools were also developed for these stations. Figure 16.1 shows the cover for Station 25 for third grade, and Fig. 16.2 the translation to English of an example of the use of ratio tables to solve problems related to the farm context presented in the station (Centros Regionales de Adiestramiento e Instrucción Matemática, 2011, p. 15).

The Dutch experience had other important effects. Through the influence of Martin Kindt of the FI, it inspired the production of educational materials for talented students. A series titled *Tesoros de la Matemática* (Centros Regionales de Adiestramiento e Instrucción Matemática, 2008) as produced with around fifteen titles. The authors of the series units were renowned local mathematicians from different universities in Puerto Rico. These didactical materials made an honest attempt to keep the mathematical formalities at a distance while conveying the mathematical ideas and intuitions to the young minds interested in finding out about the frontline discoveries of this discipline. They were inspired by Hilbert's old dictum that affirms that a mathematical theory is not to be considered complete until you have made it so clear that you can explain it to the first man whom you meet on the street.

Along with these productions, there were the accompanying trainings of teachers who were to implement the materials in their classrooms. There were also teams of people interested in showing the positive effects of the materials in student learning.

As part of the outreach activities of CRAIM, some of the Grade 1–3 materials were translated into Haitian Creole to be used in teacher training and subsequently with students in Haiti. Also, all of the materials for Grade K–3 have been translated into English to be eventually used with the population of bilingual students in Puerto

Fig. 16.1 Cover of Station 25 (Centros Regionales de Adiestramiento e Instrucción Matemática, 2011)



Rico, who typically come to the island having completed part of their studies in the mainland United States.

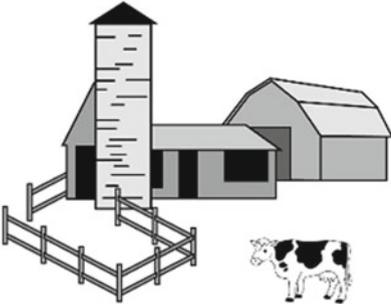
16.4 Training of Local Staff and Teacher Leaders

Training started right from the beginning of the collaboration with the FI. Jan de Lange, Martin Kindt, Els Feijs, Marja van den Heuvel-Panhuizen, Nisa Figueiredo, Jaap den Hertog, and Koeno Gravemijer were members of the FI team. Some of them came to Puerto Rico to lead workshops on RME principles and methodologies useful in the design of teaching materials. Simultaneously, members of the CRAIM team visited the FI at Utrecht University. Gradually selected teachers from different geographical areas of Puerto Rico were invited to participate in the training sessions. The first effort was to form a team of teacher leaders that would, at a later stage, help the CRAIM staff to develop and pilot the materials. Many materials were locally developed to be used as guides for the training of workshops leaders. The book *Children Learn Mathematics* (Van den Heuvel-Panhuizen, 2001) based on the Dutch TAL project that developed teaching-learning trajectories for primary school and was led by Marja van den Heuvel-Panhuizen, was translated to Spanish⁴ and used to train workshop leaders.

An important result of this effort was that a team of workshop leaders was formed to provide training to other teachers. Initially, the focus was on strategies for using

⁴ Later Spanish versions of these teaching-learning trajectories were also published by Correo del Maestro (see Van den Heuvel-Panhuizen, 2010; Van den Heuvel-Panhuizen & Buys, 2012).

3. How much milk does the dairy farm sell?



a. Don Julio, a farmer, has 40 cows. If one cow produces 15 liters of milk a day, how many liters in total does he get from his 40 cows?

b. The milk industry pays \$75.00 for every 100 liters of milk. How much money does Don Julio receive daily from his milk?

Price	\$75												
Liters of Milk	100												

c. Don José, another farmer, gets 750 liters of milk from milking his cows. How many cows does he have?

Cows	1												
Liters of Milk	15												

d. The milk factory pays the farmer \$75.00 for every 100 liters. How much does he receive for 750 liters?

Price	\$75												
Liters of Milk	100												

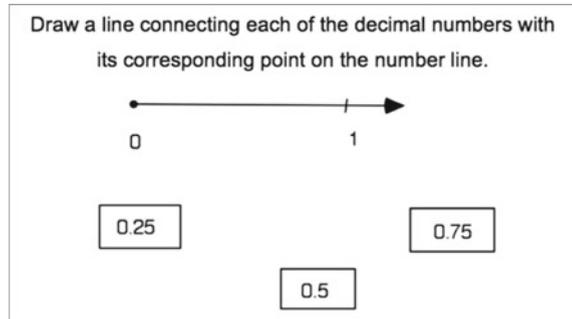
e. If Don Juan receives \$375 from the milk industry, how many liters did he sell?

Price	\$75												
Liters of Milk	100												

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Fig. 16.2 Sample of ratio tables used to solve problems related to the farm context (Centros Regionales de Adiestramiento e Instrucción Matemática, 2011, p. 15)

Fig. 16.3 Task that revealed teachers' mathematics shortcoming



the MeC-PR materials in the school context. But soon it became clear that much more was needed. The need to have teachers participate ‘as students’ in working out together the details of the units was recognised immediately. Such sessions were followed by detailed discussions around the mathematics of the units and a reflection on the use of paradigmatic situations and the use of apparent areas of conflict. It was clear that these units were not the type of materials that you could place in the hands of teachers and expect them to follow (or even understand) the underlying principles of RME. During these discussions, the teachers asked the leaders to work the problems and tasks presented in the MeC-PR materials with them. As the problems and tasks were solved it became evident that some teachers did not understand some of the mathematical concepts they were supposed to teach. For example, when asked to place the numbers 0.25, 0.5 and 0.75 on the corresponding points on the positive ray of the real line, many K–3 elementary teachers did not take into consideration the decimal position and order of magnitude implied by such decimal expressions, ending this with ordering like: $0.5 < 0.25 < 0.75$ (Fig. 16.3).

Surprising as this may appear, it is important to note that this is not only a problem of Puerto Rican teachers. Similar situations have been documented in several studies with teachers and pre-service teachers from other parts of the world (Humarán-Martínez, 2012; Ma, 1999; Simon, 1993; Zazkis & Campbell, 1996). Interaction with teachers revealed that mathematical learning has been dedicated to rules and algorithms, and that their conceptual knowledge was very weak.

To overcome these situations, CRAIM’s staff introduced changes. Instead of focusing on the strategies to teach the MeC-PR materials, the leaders of the workshops solved the MeC-PR activities with the teachers, clarifying and building their conceptual knowledge. After working the MeC-PR material, reflection about how the strategies helped them to build their own understanding was stimulated. Teachers discussed how these strategies should be used or modified when teaching their students.

Another shift was necessary when workshop leaders visited the classroom of the teachers participating in the workshops and noticed they were only able to ‘integrate’ the MeC-PR materials in the mainstream of their classes in a piecemeal manner, without allowing the materials to become the centre of their lessons. As the situation was

discussed between CRAIMS's principals and the workshop leaders, it was evident that integrating the MeC-PR materials in the curriculum was not an easy task. The main difficulties arose from the fact that the PRDE mathematics curriculum follows a very well established and rather monolithic approach that makes it particularly difficult to readily apply the new materials, given the need for time for repetition and revisiting the discussion of topics of the curriculum at different grades in order to allow for integration, coherence and verticalisation of seminal mathematical ideas. These experiences were used to feed the design process and introduce improvements to the materials.

The most challenging and important issue to be overcome was the change of paradigm that Puerto Rican educators had to make, given that the RME principles and design methods were different from the ones used in the development of the Puerto Rican curriculum. There was a major balancing act to be completed: promoting teachers' inventiveness on how to work the MeC-PR materials while following the official curriculum.

From 2004 to 2008 intensive training sessions were scheduled during the summer months with follow-ups in Saturday meetings during the regular academic year. Hundreds of teachers participated and some of them were selected to implement materials in their classrooms. For example, teachers from the Antonio S. Pedreira School in the San Juan School District were selected to implement the MeC-PR curriculum. All support and materials were given to help teachers in this enterprise.

In 2009, Doctor Omar Hernández-Rodríguez, from the College of Education of the University of Puerto Rico, Río Piedras Campus (UPR-RP), started to use the CRAIM materials to train K–6 teachers from the Mathematics and Science Partnerships Project (MSP) that he directed. Aileen Velázquez Estrella and María del Pilar Díaz, two of the CRAIM staff who were also public-school teachers and users of MeC-PR materials, trained a dedicated group of K–3 level teachers in the use of these materials in the classroom.

Graduate students from the College of Education were integrated into the training sessions and acted as research assistants. They interviewed some of the participant teachers, led focal groups, and visited the classrooms of the teachers using the MeC-PR materials. Several video recordings of the leaders of the workshops and the participating teachers' classrooms were completed. A new wave of research initiatives started to emerge. These will be described in more detail in the research section of this paper.

16.5 A Parallel Effort that Led to Official Recognition

By the time, in 1995, that the FI and CRAIM collaboration started, Professor Leonardo Torres Pagan, then a high school mathematics teacher, developed ten units using the RME principles. These units were part of an academic endeavour to fulfil his master's degree requirements (Torres-Pagán, 1997). Unlike the ones developed

by the CRAIM team, his units were aimed at high school students, specifically tenth grade.

The units, of a contextual nature, were intended to “present new ways of working with mathematics” (Torres-Pagán, 1997, p. 45). These were adapted from the examples found by Torres-Pagán on the articles published by FI staff in international journals. Each unit presented a context, and students were required to use mathematical concepts to solve problems related to the context. For example, students were asked to submit proposals to develop the economy of a Caribbean island through eco-tourism. They also had to submit statistical data to support their development plan. By 1997, Torres-Pagán came to meet Professor Gail Burrill, then collaborating with Romberg in the MiC project. Positively impressed by the quality of his activities, Burrill invited Torres to attend her lectures at the annual convention of the NCTM. Torres-Pagán also came to meet Professor Glenda Lappan who worked in the Connected Mathematics project. Burrill and Lappan gave Torres-Pagán advice on how to improve the materials he developed. It is important to mention that Gail Burrill was elected president of the NCTM and also participated in the top-level project Core-Plus Mathematics. All the projects, Mathematics in Context, Connected Mathematics, and Core-Plus Mathematics, were funded by the National Science Foundation and were intended to develop instructional materials aligned to the NCTM standards that were released in 1989 (see Hirsch, 2007).

By 2000, Torres-Pagán presented a proposal to PRDE to extend the use of his materials to a total of ten schools. The PRDE Office of Federal Affairs supported the proposal. Training was given to all participating high school teachers (Grade 10–12); the materials were reproduced and distributed to all participating schools. However, the materials did not have the expected effect. The main reasons were: the pressure on teachers regarding the issue of how to address the curricular contents in harmony with the standardised test then used for students’ assessment; the little amount of contact between the director of the proposal and the participating teachers (Torres-Pagán was also a teacher and had to attend his own classroom); and, to a lesser extent, the lack of alignment with the contents of the Puerto Rican curriculum.

By 2006, Torres-Pagán became director of the PRDE’s Mathematics Program office. The previous director, Professor Leida Negrón, had initiated collaboration with MeC-PR, funding the development of materials and elementary teachers’ training. She also included some ideas of RME in the curriculum framework published in 2003 endorsed by the PRDE (Departamento de Educación de Puerto Rico, 2003). For example, it stated the importance of the use of contexts for mathematics learning, included the idea that the teaching/learning of mathematics is a social process, and included many references to RME authors such as Freudental and Treffers. This curriculum framework is the one currently being enforced in Puerto Rico.

When Torres-Pagán assumed the PRDE’s Mathematics Program director’s position, he continued the collaboration with MeC-PR and promoted the principles of RME as the theoretical framework of the PRDE Mathematics Program. For example, the use of contexts familiar to students was included in the *Circular Letter Number 1* of the academic year 2007–2008 (Departamento de Educación de Puerto Rico, 2007), which states public policy about the teaching of mathematics in public

schools. From that position, Torres-Pagán also sponsored López-Fernández' proposals for the design and development of curriculum materials from kindergarten through third grade. As for the administrative aspects, he directly supervised the bureaucratic processes to ensure the contracting of services between the UPR-RP and PRDE.

From the very beginning, educational research was present in CRAIM efforts. First to establish the effect of the translated MiC materials for Grades 5–8 created at the University of Wisconsin by Romberg's and the FI team, then to establish the perceptions of teachers on the possibilities and difficulties in the implementation of RME materials, and more recently in a series of projects that try to determine the way mathematics is taught in schools and how the process can be improved using the foundation and principles of RME. The next section describes the major achievements of research.

16.6 Research as an Integral Part of RME in Puerto Rico

As previously mentioned, the quality of the work of the FI was evident from the very beginning. It was CRAIM's interest to adequately document the effectiveness of the materials. The first research effort was with the MiC units for Grades 5–8 created at the University of Wisconsin by Romberg's and the FI team and translated to Spanish by CRAIM team. This project was sponsored by Encyclopaedia Britannica Educational Corporation in 1994. All participants were fifth-grade students of the School Segunda Unidad of Yabucoa, both regular stream and Title I students. The Title I programme was then intended to provide special services to students from low-income homes showing academic deficiencies. Title I students were randomly divided into two groups. The experimental group used the MeC-PR materials, while the control group used the traditional materials associated with the official curriculum developed by PRDE. Students from the experimental group obtained the highest scores on the standardised mathematics test when compared to similar students from the same school district. All students from the experimental group obtained 85% or more on the mathematics test, and only three students had less than 90%. The following year, all Title I students of the experimental group, that is, using the MeC-PR materials, became regular stream students. The other students in the pilot project had similar scores on the standardised tests mentioned before. This results pleasantly surprised CRAIM team since, normally, students that are trained with materials designed to develop high mathematical thinking skills do not often get particularly high scores on standardised tests that measure routine skills for numerical computations (such as those used in 1995). However, participant students from the experimental group developed such an interest in mathematics that they managed to improve in all areas, including computational skills.

The quality of the MiC materials along with the results from Yabucoa's students encouraged CRAIM principals to develop a complete curriculum for elementary mathematics based on RME principles.

In a similar track, the research of Torres-Pagán had the purpose of showing the effectiveness of the RME-based educational materials. The lessons were tested with a group of fifty students from tenth grade. The students took a test before and after treatment and the averages were compared with an equivalent group receiving the same content using the curriculum from the PRDE (Torres-Pagán, 1997). The difference on the posttest between groups was statistically significant. Despite the reported success, the lessons were not well received by PRDE administrators.

Another research activity was conducted at the School Carmen D. Ortiz in Aguas Buenas, Puerto Rico. The context of the cookies factory was used with second grade students for the development of digit-based addition and subtraction in separate columns (in which every digit takes the value of the column). At the request of teachers, members of CRAIM modelled the use of the materials, and interviewed the children. The effectiveness of the strategy was evident. Even after a year, the children mentioned the context to solve addition and subtraction problems as a point of reference. Results showed that students who went through the arithmetic counting and rounding activities in contextual settings (such as the one of the cookies factory) at an early stage were more prone to learning the algorithms faster than those who jump right into the study of such algorithms. This research is based on similar investigations carried out by Koeno Gravemeijer and others regarding this topic done in the United States. A description of the contexts and results of the research was published in professional journals from Spain and the United States (López-Fernández & Velázquez-Estrella, 2007, 2011).

By 2004, Emely Fernández-Dávila, then a graduate student at the School of Education of the UPR-RP, became interested in determining the impact of the training offered by CRAIM on teachers. Specifically, she wanted to determine how teachers conceived RME as an educational tool, the needs identified to try to bring the materials to the classroom and assessing positive and negative aspects of the implementation. She gathered information by interviewing two participant teachers, the director, and the coordinator of the workshops. To triangulate the data, she analysed the evaluations of the workshops given during the summer of 2004. A complete revision of the literature was carried out, a 130-page report was composed and an executive summary of 30 pages was written. Results indicated that teachers had some doubts about how to implement RME materials in their classes. They felt that activities modelled by international designers were significantly different to those developed locally. Evidence indicated that teachers had problems understanding Freudenthal's guided-reinvention principle and how to apply it with students. The pressure to cover all the curriculum material to be evaluated in the standardised tests, and the time-consuming process to implement the new materials were deterrents to transfer the materials to the classroom. The recommendations called for increasing research projects and the importance of disseminating its findings. The importance of maintaining a link between teachers, trainers and developers was also stated.

In 2008, Jorge López-Fernández and Omar Hernández-Rodríguez met each other and their affinity allowed them to establish collaboration between the Department of Mathematics and the Department of Graduate Studies of the School of Education, both of the UPR-RP. A joint seminar to study issues related to mathematics educa-

tion was established with the participation of graduate students from mathematics and mathematics education. Simultaneously, graduate students participating at the seminar became familiar with RME principles and methodologies. The main idea was, and still is, to do research to make evident the way Puerto Rican students learn, and to determine how to improve through local solutions. A new wave of efforts on research started to emerge. Some of the results have been published in *Puentes*, a journal edited by graduate students of mathematics and mathematics education, the purpose of which is to disseminate the seminar assistants' findings.

By 2010, Hernández-Rodríguez started to use the MeC-PR materials to train teachers from the Mathematics and Science Partnerships Program (MSP-San Juan) he directed. A group of teachers from the metropolitan area was selected with the purpose of being trained and to transfer RME to their classroom. Graduate students from the School of Education of the UPR-RP were integrated as research assistants to help with the collection of information to be used on further research. Interviews with the workshops leaders and the participating teachers were carried out, videos of model lessons and teachers' class sessions were recorded, and a focus group to explore teachers' perceptions were performed.

Ortiz-Fernández (2015) conducted an analysis of the recordings of the participation of teachers on the focus group and of the videos of four elementary school teachers (Grades K–3) when teaching their classes. In his analysis, he used the guidelines of Godino's onto-semiotic approach to mathematics education (Godino, Batanero, Font, 2007). Discursive and operational productions were analysed to get evidence of epistemic, cognitive, mediational, interactional and emotional elements. He found that teachers had an adequate theoretical knowledge of the discipline, which was evidenced as a holistic view of the learning process. During classes, he observed some specific practices from the MeC-PR trainings, such as the bus that picks up and drops off passengers at bus stops. The activity was conducted with concrete materials developed by the teachers themselves. The use of an environment that allows students manipulation of concrete objects was a common practice of the participant teachers. Activities to motivate students' participation and learning of mathematics were also reported. Although the document evidences, to a certain extent, the use of RME principles and methodologies, a deep analysis of students' mathematical productions was not given.

Meanwhile, Hernández-Bosch (2015) determined the knowledge about the principles and methodologies that still exists in the participants of the 2010 training sessions. He proposed deep interviews with five teachers who participated in training and are still teaching in elementary school. The purpose was to establish how they were using the strategies, models and contexts studied in RME training in their current practice.

On other dissemination efforts, Ana Helvia Quintero-Rivera published the book *Matemáticas con Sentido* (Quintero, 2010) with the purpose to describe RME principles and methodologies. The book is intended for in-service and pre-service elementary teachers and includes ample discussion of activities that promote a deep understanding of mathematics. All the work presented has its roots in the findings over the many years that Doctor Quintero-Rivera has participated on research and

development on MeC-PR materials. The book was translated by World Scientific and is available to the English-speaking public under the title *Math Makes Sense! A Constructivist Approach to the Teaching and Learning of Mathematics* (Quintero & Rosario, 2016).

The authors of this chapter published a book titled *Sentido Numérico: Más Allá de Los Números* (López-Fernández, Quintero-Rivera, Hernández-Rodríguez, & Velázquez-Estrella, 2016) the purpose of which is to state the importance of the development of number sense at school level. Through a taxonomy of the current models used by the RME, readers will encounter plenty of examples to develop number sense in a comprehensive way.

16.7 Concluding Remarks

In this chapter, the authors report on the beginnings of RME in Puerto Rico; the efforts made for the development of educational materials adapted to the particular culture, resulting in MeC-PR; the training of teachers for the implementation of the materials in their classrooms; the incorporation of some elements of RME into the official documents of the Puerto Rican Department of Education (PRDE), and the research efforts based on RME. Achievements are mainly due to CRAIM; however, the efforts of others are reported.

At some point, all the elements pointed to the possibility that the MeC-PR could become the spearhead of mathematics education in Puerto Rico. Public policy existed, the educational materials were developed, training was given and there was an entire infrastructure to disseminate MeC-PR. What factors blocked the continuity of scaling up the project? A first element was the change in Puerto Rico's governing party. The PRDE is strongly associated with the governing party. So once there is a political switch, all programme directors usually change. As a result, Professor Torres ceased to be the Mathematics Program director in 2009. Piloting and implementing the MeC-PR materials required additional effort and accompaniment of specialists. Both are expensive and impossible without adequate funding. Given that the new PRDE Program of Mathematics Director was not as enthusiastic about the MeC-PR, the funds were not allocated.

Yet, the most difficult and important issue to be overcome is the change of paradigm that Puerto Rican educators had to make, given the RME principles and design methods, which are different from the ones used in the development of the Puerto Rican curriculum. Indeed, the educational community assigned too much importance to the PRDE mathematical standards, which are for their part aligned to the U.S. standards. The perceived lack of alignment with the educational materials created with the PRDE standards led teachers, supervisors and other officials to discard MeC-PR materials.

We have two major tasks in order to implement the MeC-PR materials. The first one is promoting a change in the professors and professionals that work curriculum development towards RME. Our work with the graduate students at the UPR College

of Education is a first step in that direction. Yet we have to work with the professors of mathematics education in the other universities.

In the meantime, we should promote teachers' inventiveness on how to adapt the MeC-PR materials to follow the official curriculum. Indeed, the teachers' manual deals with this issue. The manual suggests ways of working the PRDE standards at the same time that the MeC-PR material is used.

Changing paradigm is not an easy task. Once seen in practice, an example of the new paradigm helps in the transformation. So, another task is to develop a school that follows the MeC-PR materials. We expect that this school will have outstanding results in the PR standard exams, as did the group of Yabucoa. We can then invite PRDE officials and professors of mathematical education to study in practice the model of the new paradigm in action. This might be a third line of action.

As can be seen, we still have an agenda for promoting RME in Puerto Rico. Advances may be possible if people from different governmental and private offices were to consolidate a collaboration project.

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