Chapter 2

Mathematics-Related Beliefs and Affect

Gilah C. Leder

Abstract Influential approaches to the definition and operationalization of beliefs and other affective dimensions are described in this chapter. Snapshots of research findings are presented, commonly used measures are listed, and some technologically enhanced approaches to the measurement of beliefs and affect are discussed. Contributions, both theoretical and practical, to research on the affective domain by those concerned with gender issues are also referenced.

Keywords Affect · Beliefs · Gender · Mathematics · Measuring affect

2.1 About Beliefs, Affect, and Mathematics—Introduction

2.1.1 *Setting the Context*

Why, it could be asked, are both beliefs and affect in the title of this contribution? The inclusion of both terms warrants a special comment.

In his influential and frequently cited article Pajares (1992) argued convincingly that, within the field of educational psychology,

defining beliefs is at best a game of player's choice. They travel in disguise and often under alias - attitudes, values, judgments, axioms, opinions, ideology, perceptions, conceptions, conceptual systems, preconceptions, dispositions, implicit theories, explicit theories, personal theories, internal mental processes, action strategies, rules of practice, practical principles, perspectives, repertoires of understanding, and social strategy, to name but a few that can be found in the literature. (p. 309)

Mason (2004) similarly argued that it is often difficult to work out what beliefs actually cover and illustrated this by producing “an entire alphabet of associated interlinked terms”:

G. C. Leder (✉)

La Trobe University, Melbourne, VIC, Australia

e-mail: g.leder@latrobe.edu.au

Monash University, Melbourne, VIC, Australia

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A is for attitudes, affect, aptitude, and aims; B is for beliefs; C is for constructs, conceptions, and concerns; D is for demeanor and dispositions; E is for emotions, empathies, and expectations; F is for feelings; G is for goals and gatherings; H is for habits and habitus; I is for intentions, interests, and intuitions; J is for justifications and judgements; K is for knowing; L is for leanings; M is for meaning-to; N is for norms; O is for orientations and objectives; P is for propensities, perspectives, and predispositions; Q is for quirks and quiddity; R is for recognitions and resonances; S is for sympathies and sensations; T is for tendencies and truths; U is for understandings and undertakings; V is for values and views; W is for wishes, warrants, worlds, and weltanschauung; X is for xenophilia (perhaps); Y is for yearnings and yens; and Z is for zeitgeist and zeal. (Mason, 2004, p. 347)

How specialists in mathematics education defined beliefs was explored some years ago by Furinghetti and Pehkonen (2002). In their small study they aimed to clarify which elements were widely accepted as core identifiers or descriptors of beliefs. Specifically, the participants in the study were asked to indicate whether they agreed fully, agreed partially but leaning towards agreement, agreed partially, agreed partially but leaning towards disagreement, or disagreed fully with a series of characterizations of beliefs found readily in the relevant research literature and to elaborate on their answer on a five-point scale. “The first surprising result”, Furinghetti and Pehkonen wrote, “was that in the responses of the (18) specialists, there was no agreement” (p. 49). After a more nuanced analysis of the data gathered they concluded that

In characterizing beliefs and related concepts it is unlikely that complete agreement will be reached in the use of all the terms at issue. Nevertheless it can be asked that the authors of studies on beliefs reduce the terms and the concepts involved in their work to the minimum needed. Additionally, they have to make clear their assumptions, the meaning they give to basic words, and the relationship between the concepts involved. (p. 55)

Some years ago I trawled through the Conference Proceedings of two well established mathematics education research groups, the International Group for the Psychology of Mathematics Education [PME] and the Mathematics Education Research Group of Australasia [MERGA] for research on beliefs and mathematics education (see Leder, 2007). My concluding comments included:

- The mathematics education research community continues to explore how students’ and teachers’ beliefs affect mathematics learning and instruction.
- The use of the word belief/believe as a convenient synonym for other terms persists.
- Small sample, qualitative method studies, often of questionable generalizability, are common.

Thus Pajares’ observation about educational researchers’ all too frequent use of belief as a generic term seemingly applies as well to researchers in mathematics education. It is worth noting that the review of the contents of the two sets of proceedings also identified a number of painstakingly crafted and theoretically driven studies which pointed to new avenues for capturing students’ and teachers’ multifaceted beliefs and their impact on mathematical learning behaviours. From Liljedahl and Hannula’s (2016) review of work presented at PME’s annual conferences in the decade 2006–2015 it is apparent that the diversity of research on affective issues

has continued. Captured in that chapter are examples of work embodying loosely defined concepts as well as research designed and executed within a carefully elaborated theoretical framework.

2.1.2 *Defining Beliefs—A Continuing Story*

In the mathematics education literature, beliefs have been conceptualized and operationalized in different, and evolving, ways. This is illustrated by a sample of excerpts published a decade apart.

According to McLeod (1992):

beliefs are largely cognitive in nature, and are developed over a relatively long period of time.... (W)e can think of beliefs, attitudes, and emotions as representing increasing levels of affective involvement, decreasing levels of cognitive involvement, increasing levels of intensity of response and decreasing levels of response stability. (p. 579)

Nuanced differences in affective subdomains have been described by Goldin (2002, p. 61):

(1) Emotions (rapidly changing states of feeling, mild to very intense, that are usually embedded in context), (2) attitudes (moderately stable predispositions toward ways of feeling in classes of situations, involving a balance of affect and cognition), (3) beliefs (internal representations to which the holder attributes truth, validity, or applicability, usually stable and highly cognitive, may be highly structured), and (4) values, ethics, and morals (deeply-held preferences, possibly characterized as “personal truths”, stable, highly affective as well as cognitive, may also be highly structured).

A decade later Hannula (2012) argued that terminological ambiguity has continued to be a problem and that a new framework might be needed to move the field forward:

The review of research on mathematics-related affect indicates that the dominant framework of beliefs, emotions and attitudes is not sufficiently broad to incorporate all research in this area. More specifically, embodied perspectives and strong social theories go beyond that frame.... Many frameworks aim to theorise the interaction between different aspects. (p. 155)

Ways in which the call to re-examine prevailing assumptions has been taken up can be seen in, for example, the recently published book *From beliefs to dynamic affect systems in mathematics education*, edited by Pepin and Roesken-Winter (2015). “Systems thinking”, they explain,

is based on the belief that the component parts of a system can best be understood in the context of relationships with each other and with other systems rather than in isolation – in other words, on the belief that ‘the whole is greater than the sum of its parts’.... Linking this to ‘affect systems’, we can see why there have been so many debates over the last few decades about what constitutes ‘emotions’, ‘attitudes’, ‘beliefs’ and ‘values’, and there are no clear definitions available to date. Could it be that emotions, attitudes, beliefs and values each constitute a system (e.g. in an individual, or in a collective/group) and that these systems are

indeed inter-related, or ‘nested’ within any one person/group, albeit nurtured by the context? (pp. xv–xvi)

It is perhaps no coincidence that, referring not to beliefs about mathematics but to mathematics anxiety, Dowker, Sarkar, and Looi (2016) have argued for a more intensive focus on the putative link between different factors and specific aspects of mathematics anxiety. They concluded:

Our biggest need for further learning may involve not so much any specific aspect, as the ways in which the aspects relate to one another. How do the social aspects relate to the neural aspects? How do either or both of these relate to changes with age? How might appropriate treatment be related to age and to the social and cognitive characteristics of the individuals. (Dowker et al., 2016, p. 12)

Explanations for the continuing focus among researchers and practitioners on the affective domain abound. Some may be encouraged by Kramers’ (n.d.) acknowledgement that the physical sciences, too, are plagued by a lack of precision. “In the world of human thought generally, and in physical science particularly”, he argued, “the most important and fruitful concepts are those to which it is impossible to attach a well-defined meaning”.

The continuing preoccupation with the affective domain is arguably also fuelled by the attention given to it in influential overviews and comparisons of international student achievement data such as those reported in the Trends in International Mathematical and Science Study [TIMSS] and the Programme for International Student Assessment [PISA]. The following is taken from a PISA publication:

Individuals’ attitudes, beliefs and emotions play a significant role in their interest and response to mathematics in general, and their employment of mathematics in their individual lives. Students who feel more confident with mathematics, for example, are more likely than others to use mathematics in the various contexts that they encounter. Students who have positive emotions towards mathematics are in a position to learn mathematics better than students who feel anxiety towards that subject. Therefore, one goal of mathematics education is for students to develop attitudes, beliefs and emotions that make them more likely to successfully use the mathematics they know, and to learn more mathematics, for personal and social benefit. (OECD, 2013, p. 42)

Highlighted in extracts such as the above is an acceptance of the close interplay between the learning of mathematics and a host of affective factors. This widespread acceptance can be seen as a driving force for continuing work concerned with subtle distinctions between different affective components as well as investigations germinated by different theoretical orientations.

2.2 About Beliefs, Affect, and Mathematics: What We Think We Know

Research on mathematics-related beliefs and affect can be clustered in many different ways. For example, groupings could be in terms of measurement of affective

factors; descriptive studies in which affective factors are of secondary interest and are overwhelmed by factors considered to be of greater importance; comparisons of affective and cognitive variables; and theoretical driven papers—which are often presented without any experimental data. Another option is to segment the field, and again not uniquely or unambiguously, under headings focusing on mathematics; on students (both internal and external factors); and on teachers (internal and external factors). Common to different approaches is the widespread acceptance that beliefs and other aspects of the affective domain cannot be measured directly, nor can they be readily observed. Instead they need to be inferred from behaviour, speech, or responses to specifically designed stimuli or instruments. How these measurements are devised, focused, administered, and interpreted inevitably affects the nature and scope of the belief or other affective construct “captured”.

From the above it is apparent that compiling a comprehensive overview of relevant research is no small task, nor one readily summarized within one chapter. As argued by Schoenfeld (2015, p. 395): “Research in the ‘affective domain’ is densely populated with overlapping constructs, partially commensurate methods, and somewhat contradictory findings”. Definitional imprecision is a further confounding factor.

In this chapter I am relying on descriptive snapshots of research on affect, drawn from three recently published and influential contributions to the field, to provide a succinct yet varied overview of relevant work. These are: *The Encyclopedia of Mathematics Education* (Lerman, 2014), *The ICME-13 Topical Survey on Attitudes, Beliefs, Motivation and Identity in Mathematics Education* (Goldin et al., 2016), and *Research in Mathematics Education in Australasia 2012–2015* (Makar et al., 2016). Pertinent summaries are presented in turn below.

2.2.1 *The Encyclopedia of Mathematics Education*

Three of the entries in this encyclopedia are of particular interest. These are the entries by Zan and Di Martino (2014), Liljedahl and Oesterle (2014) and Hannula (2014).

Focussing on attitudes in mathematics education, Zan and Di Martino (2014) reported that:

- Initially, research on attitudes tended to focus on the development of measuring instruments rather than on the theoretical definition of the construct.
- The correlation between attitudes and achievement in mathematics is typically not high.
- The importance of incorporating qualitative measures has been increasingly recognized.
- Attitude is now often considered “a flexible and multidimensional tool, aimed at describing the interactions between affective and cognitive aspects of mathematical activity”.

The authors indicated that they themselves favour an “interpretative approach based on the collection of autobiographical narratives of (a large number of) students”. Using this approach three dimensions were found to feature consistently: emotions, vision of mathematics, and perceived competence. More targeted intervention programs, Zan and Di Martino argued, can be devised using these distinctions.

Liljedahl and Oesterle’s (2014) covered teacher beliefs, attitudes, and self-efficacy in mathematics education. Their findings included:

- Various definitions are provided for both beliefs and attitudes.
- Research on teacher self-efficacy is “challenged by difficulties in clearly defining and measuring self-efficacy and its impact”.
- Both consistencies and inconsistencies have been reported between teachers’ espoused beliefs, enacted beliefs, actual beliefs, and the attributed beliefs that the researchers assign to them.
- Negative attitudes towards mathematics, which can be very difficult to change in adults, can interfere with teacher learning.
- Included among desirable teacher attitudes are curiosity, high motivation, and appreciation of “good” problems and elegant solutions.
- But: much is still unknown about the relationship between teachers’ attitudes and practice.

In the section on affect, Hannula’s (2014) focus was primarily on emotional states and traits. Findings included:

- Mathematics anxiety is related to general anxiety, test anxiety, and low mathematics attainment.
- Students with a positive disposition towards mathematics tend to experience positive emotions more frequently and negative emotions less frequently than students with a negative disposition.
- Students’ affective and social interactions in mathematics classes are interdependent.
- Emotions may affect memory retrieval and performance.
- Emotions can often be identified from facial or behavioural expressions (and these may be more accurate indicators than self-report measures).
- Classrooms are “often emotionally flat, and boredom is one of the most frequently experienced emotions”.
- Features of the classroom climate and teachers behaviours which can enhance positive student emotions continue to attract research attention.

Collectively, the three entries cover a number of interacting, and at times overlapping, affective factors.

2.2.2 The ICME-13 Topical Survey on Attitudes, Beliefs, Motivation and Identity in Mathematics Education

In the introduction of the monograph compiled by Goldin et al. (2016), the view was re-iterated that research on mathematics-related affect is broad and varied. Core topics reviewed and discussed included “attitude towards mathematics, self-efficacy beliefs, teacher beliefs, mathematical identities, and mathematical motivation” (p. 23). Here I focus primarily on a number of the “open” questions identified and highlighted by the various contributors to this volume and their suggestions for further work to move the field forward. These included:

- Aim for further refinement of instruments used to tap attitudes to mathematics.
- Use the design and implementation of carefully planned cross- and inter-cultural studies to achieve consensus on putatively robust results.
- Plan and conduct longitudinal studies to monitor more stringently the efficacy of intervention studies.
- Aim for further explorations of the relationship between beliefs and practice, while taking into account possible confounding factors linked to different mathematics topics.
- Design contextual studies with a focus on factors influencing changes in beliefs, and particularly those set within teacher development programs and those incorporating e-learning environments.
- Build on small qualitative studies to extend defensible generalizations beyond the specific setting and sample initially studied.
- Build on in-the-moment perspectives to capitalize on researchers’ better control over personal and environmental variables.

2.2.3 Research in Mathematics Education in Australasia 2012–2015

In this publication it is the chapter on “Mathematics education and the affective domain” (Attard, Ingram, Forgasz, Leder, & Grootenboer, 2016) that is of special interest. Highly relevant, too, is the identification of largely neglected areas of research. Aspects identified as glaringly in need of further consideration included:

- Paying greater attention to sample and setting characteristics in order to facilitate and promote an evidence based synthesis of diverse findings.
- Interrogating and documenting if, and how, the widespread incorporation of technology into mathematics curricula has an influence on affective components. As Attard et al. (2016) noted: “Consideration of the impact of technology on this digital generation of teachers and students and the ways it influences their perceptions of and interactions with mathematics education is imperative” (p. 91).

- Exploring how the ready availability of new, user-friendly software can support the analysis of data gathered in affective studies.
- Capitalizing on advances in cognitive neuroscience to explore the impact of affective factor on mathematics learning.

Research on mathematics and the affective domain, it is apparent from the examination of the above publications, certainly warrants further carefully conceived and planned research.

Though not specifically highlighted in the excerpts taken from these publications, issues of gender often permeate—to a greater or lesser extent—research on mathematics and the affective domain. McLeod’s (1992, p. 580) observation that research on beliefs “comes mainly out of the work on gender differences in mathematics education” has lost none of its currency. Dowker et al. (2016, p. 7) likewise noted: “One of the factors that has received most study with regard to mathematics anxiety is that of gender”. In the overview provided in this chapter, the impact on the field of research on gender issues is incorporated where pertinent, and particularly in the discussion of approaches to the measurement of affect appropriated in mathematics education research.

2.3 About Beliefs, Affect, and Mathematics: New Directions?

It is not easy to gauge uniquely just how much progress has been made in recent decades in the field of mathematics education and affective research. It is, however, both revealing and informative to revisit some earlier work presented at the annual conference of PME, with its substantial number of international attendees and presenters.

2.3.1 *From the PME Archives*

The archival snapshots selected are taken from the records of three annual conferences held 10 years apart: in 1985, 1995, and 2005. Each presentation was designated in the PME Proceeding for that year as belonging under the umbrella of affective topics. The relevant headings comprised *teacher behaviour and attitude*; *beliefs*; and *affect, emotion, beliefs and attitudes*.

The study by Fresko and Ben-Chaim (1985) was titled “Confidence, competence and in-service education for math teachers”. Using a pre-test and post-test quasi-experimental design, teachers attending in-service courses completed measures which tested their confidence with respect to subject matter skills and their confidence about their ability to teach topics included in the curriculum. More specifically, participants were required to solve a series of mathematics problems at a level

commensurate with that expected of their own students, were asked to indicate how confident they were that they themselves could solve the problems, and how confident they were about teaching the topics covered in the in-service course. The authors of the research report concluded that:

Many mathematics teachers in their sample needed to learn mathematics better. Thus content-oriented in-service programs are needed.

Teachers who are not confident about the subject matter they need to teach also tend to lack self-confidence. So, first teachers' level of subject matter knowledge needs to be addressed, followed by efforts to reinforce their self-confidence with respect to their subject matter competency and also with respect to their teaching capability.

The focus of the study reported by Yusof and Tall (1995) was captured economically in the title of their presentation: "Professors' perceptions of students' mathematical thinking: Do they get what they prefer or what they expect?" In this study, members of a university mathematics department were asked to complete a survey about attitudes to mathematics and attitudes to problem solving.

In the first instance the professors were asked to indicate the responses they expected from a typical student in their lecture group. Next, they were asked to respond to each item on the survey by indicating how they preferred that students would respond. The thrust of the survey items can be gauged from the findings presented by the authors and quoted in some detail below.

As well, interviews were used to probe in more detail the group's attitudes to problem solving. Perhaps not unexpectedly, the qualitative responses yielded information not necessarily congruent with that gathered via the surveys. Lecturers, it was concluded, preferred students to have a range of positive attitudes to mathematics but they expected reality to be different. Yusof and Tall (1995) wrote:

Although lecturers prefer students to have a range of positive attitudes to mathematics, they expect the reality to be different. They prefer students to see mathematics as solving problems, making sense, with students working hard, able to relate ideas without needing to learn through memory, having confidence, deriving pleasure, with low anxiety and fear, ready to try a new approach and unwilling to give up easily on difficult problems. On the other hand, they expect them to see mathematics as abstract, failing to understand it quickly, not making sense, working hard to learn facts and procedures through memory, unable to relate ideas, with less confidence, obtaining less pleasure, working only to get through the course, with anxiety, fear, seeking only correct answers, and ready to give up when things get difficult. (Yusof and Tall, 1995, p. 177)

"Teachers' beliefs of the nature of mathematics: effects on promotion of mathematical literacy" presented by Webb and Webb (2005) is the third article retrieved from the PME archives. In this study a variety of measures was used to encourage teachers not only to reflect on their own beliefs but also to make them explicit. To do this, participants (in-service teachers) were asked to complete a Likert-scale questionnaire based on videotapes of lessons they viewed. A sub sample of the larger group was interviewed individually to explore in greater depth their perceptions of the nature of mathematics and their own teaching practices. These views were then tested against the practices observed when the teachers were videotaped in their own classrooms. Webb and Webb (2005) concluded:

Data generated by videos support the findings of similar studies, i.e. that teachers' beliefs of the nature of mathematics are often not reflected in their practice. This has far-reaching implications for the implementation of compulsory mathematical literacy to (various) grades, as the mode of delivery is envisaged to be through contextual problem solving. (pp. 1–293)

Though conducted years ago, each of the studies reviewed, it can be seen, relied on an approach and an area of research that still resonate with current work and presentations at contemporary conferences and meetings. Well-tried avenues, at times supplemented with new approaches, continue to be pursued to progress our understanding of the interaction between affect and mathematics learning and instruction.

2.4 About Beliefs, Affect, and Mathematics: Testing the Boundaries

A diverse range of instruments has been used to measure affect and its role in the learning and teaching of mathematics. Self-report measures—with or without open-ended responses—are frequently used. Interviews, and particularly 'structured' interviews with their predetermined list of specific questions to be asked, are considered by some as little more than orally administered questionnaires. Unstructured interviews, on the other hand, can uncover views not anticipated in advance. A combination of these approaches—the semi-structured interview—is popular in mathematics education research. Interviews and surveys have been used as stand-alone measures and, at times, as a part of more diverse testing regime.

Other methods adopted rely on the interpretation of, or inferences drawn from, observed behaviours. Information gleaned from physiological or neural activity is used in some work. As described below, if deemed opportune, researchers have not hesitated to introduce variations to previously published instruments, adapt existing instruments to new settings or to benefit from advances in technology. They have, however, not necessarily grappled with the attendant methodological or newly introduced ethical issues. Examples of commonly used instruments and strategies are presented below.

2.5 Measuring Affect

2.5.1 *Self-report Measures—Likert Scales*

Likert-item surveys, possibly with the option of open ended responses, are widely used to measure single or multiple aspects of affect. As Ruthven (2015, p. 393) has noted: "this is an unusually convenient technique which serves researchers well in generating results". The extent to which the tools actually reveal what they purport to measure remains a matter for debate, however.

Items commonly used in the internationally administered TIMSS and PISA testing programs have fuelled many Likert surveys, as have the Fennema-Sherman Mathematics Attitude scales.

The latter, which can be used individually or in their entirety, have been embraced particularly enthusiastically by the research community. Recent statistics (data accessed April 8, 2015) provided by the Journal for Research in Mathematics Education revealed that Fennema and Sherman (1976a) was identified as its second most accessed article, both in the short and long term (the previous three months and the previous three years respectively). Over the longer period monitored it was reportedly accessed 1870 times. In the more detailed Fennema and Sherman (1976b) the authors wrote:

Each scale assesses an attitude that has been hypothesized to be related to the study and/or learning of mathematics by males and females. The scales include (a) confidence in learning mathematics; (b) father, mother, and teacher scales measuring perceptions of attitudes toward one as a learner of mathematics; (c) effectance motivation in mathematics; (d) attitude toward success in mathematics; (e) mathematics as a male domain; (f) usefulness of mathematics, and (g) mathematics anxiety scale. (p. i)

The scales, collectively, reinforce the belief that mathematics learning is influenced by a variety of attitudes, strategically captured in the different sets of items. The scales have been adopted, and adapted, widely and persistently.

2.5.2 The Fennema-Sherman Scales and Gender Differences in Mathematics Learning

The mathematics as a male domain [MD] scale has been particularly influential in investigations of gender differences in mathematics learning. However, a key assumption that underpinned the development of the MD scale, namely that “the less a person stereotyped mathematics, the higher the score ... as it was assumed that the less a female stereotyped mathematics as a male domain, the more apt she would be to study and learn mathematics” (Fennema & Sherman, 1976a, p. 7) was challenged by Forgasz, Leder, and Gardner (1999). They argued that, with changing social circumstances, although many still considered mathematics to be a male domain some now thought of mathematics as a female domain. To overcome this uncertainty they produced two new instruments (Leder & Forgasz, 2002). The *Mathematics as a gendered domain* survey consists of traditional Likert type items clustered into three subscales: mathematics as a male domain, mathematics as a female domain, and mathematics as a neutral domain. When administered, the 48 items are presented in a random order. The *Who and mathematics* instrument contains items similar to those on the *Mathematics as a gendered domain* instrument but respondents are asked to indicate for each item if it applies more to boys than to girls (definitely/probably); there is “no difference between boys and girls”; or it applies more to girls than to boys (probably/definitely). The instruments are reportedly effective in tapping gender

differences in beliefs and attitudes about mathematics and its learning. Importantly, this example illustrates how theoretical assumptions can influence the content and response format of an instrument and, if flawed, distort or constrain the determination, formation and assessment of the affective domain.

Issues of definitional imprecision, as already discussed in some detail, plague the affective domain.

That this problem is not unique to the affective domain but is also raised with respect to describing gender identity also warrants a comment.

Those well versed in research on gender and mathematics education are well aware that both the terms sex differences and gender differences are readily found in the literature. Originally, the term sex differences was used uniquely and consistently. In more recent times, sex has more commonly been used to denote biologically-based differences. The usage of gender evolved following debates on whether all differences could or should be attributed to biology alone. Gender, as a term was consequently often used to describe differences between males and females that are not attributable to biology. For a detailed overview of the fluid terminology and shifting views in recent decades about gender diversity and sexual orientation see, for example, the definitions promoted by the American Psychological Association [APA] (2015).

Recently the sufficiency of the male-female binary distinction has been further challenged and more nuanced refinements have been recommended:

In the April issue of AERA Highlights, AERA announced that members would soon have the option to select from an expanded list of gender identity categories when renewing their membership or joining the association... (A) two-step approach to collecting data on gender: the first being the collecting of data on the biological sex assigned at birth, and the second asking members how they describe their gender.... The wording of the two questions would be as follows:

Biological sex designated at birth (check all that apply):

Female

Male

Intersex

Biological sex not listed above (please specify): _____

Prefer not to answer

Gender (check all that apply):

Agender

Cisgender Man

Cisgender Woman

Gender Expansive

Gender Fluid

Gender Non-Conforming

Genderqueer

Man

Non-Binary

Transgender

Trans Man

Trans Woman

Trans/Trans*/Trans+

Two-spirit

Woman

Gender not listed above (please specify): _____

Prefer not to answer. (Levine, 2016)

How, or whether, these new categorizations will impinge on research on gender/sex differences in mathematics learning remains to be seen.

Also worth noting is the diverse range of personal and situational variables supposed to be linked to gender differences in mathematics learning. Factors invoked include stereotype threat, motivational differences, attribution theory, self-efficacy, autonomous learning, research on gender differences in a range of settings and endeavours is pursued seemingly unabated. Whether such a plethora of apparently different yet overlapping perspectives stimulates or obstructs a systematic study of the field is a question worth pursuing.

2.5.3 Projective Techniques

A variety of projective techniques has been used to tap affective factors. In this approach, respondents are shown an unstructured or ambiguous stimulus related to the attitude object of interest and are asked to react to this or explain what they see or think about it. Examples include a sentence completion task (“a good mathematics lesson is...”), a word association test, a picture preference test, a request to write a story (Anne/John came top of her/his mathematics class. Who will John/Anne tell about this?), or draw a picture in response to a particular cue. The technique is readily adapted for use with older or younger students and can yield powerful insights into their perceptions of mathematics and mathematics learning. The difficulty of achieving consistent scoring, and hence satisfactory reliability and validity, remains problematic but has not severely dented mathematics educators’ reliance on projective measures. It continues to be favoured by, for example, those concerned that explicit Likert items may draw responses thought to be socially or culturally acceptable rather than answers actually indicative of the attitudes held by the respondent.

2.5.4 Observations

Observations of behaviours in both structured and natural settings have also been used to infer attitudes about various objects or activities. When carried out by expert and objective observers, supported by a carefully designed observation schedule and

systematic recording, determination of affect is fairly consistent. However, as it is usually difficult to predict when pertinent behaviours will be observed this method is generally regarded as too inefficient and expensive to be used as a stand-alone measure. In educational research real time observations are often supported by audio- or videotaped recordings of the sessions observed. These tapes can then be analyzed and interpreted at length at a convenient time.

Attitudes to mathematical activities and learning have certainly been inferred from observations in a natural setting. For example, classroom teachers invariably take note of, and respond to, students' behaviours in class. While such observations cannot be termed a formal measure of affect they are, nevertheless, consciously or unconsciously, used to make inferences about students' beliefs about themselves and attitudes to their work.

2.6 Enhanced Measures

As already noted above, surveys and self-report measures are widely used to measure single or multiple aspects of affect. Technology facilitated adaptations of these approaches are increasingly being adopted. Some examples of technology enhanced research and already trialled for use in, and beyond the classroom, are presented next. That in such work ethical issues may arise beyond those inherent in traditional methods employed for data gathering is worthy of careful consideration but outside the scope of this chapter.

2.6.1 *Enhanced Self-report Measures*

In many Western homes and classrooms, tablet computers are now readily available. Much has been written elsewhere about the educational advantages and disadvantages of using these devices as teaching tools or aids. Of interest here, however, is their potential for use in research about affect and mathematics learning.

In a small study, Larkin and Jorgensen (2015) asked students in elementary school to use their tablet computers to video record mathematical activities and their reactions to them.

Two tents were erected, one in each year 3 and 6 shared space, to create a 'mathematical thinking space'. Students were able to enter the tent to record their video as they chose, within the boundaries of the usual classroom teaching requirements. This was usually during the times when students would be seated doing individual work or when they had completed set tasks; however, there was no requirement that all students must record a video. Students were able to use the video camera that comes standard with any iPad, to record their musings, and if they needed, they could also use the camera to take photographs of mathematics work relevant to their conversations. (Larkin & Jorgensen, 2015, p. 6)

Extra prompts, given as needed to probe students' attitudes and reactions to the task at hand, included questions such as "What would I tell my mum and dad or my principal about what I did in maths today?" or 'If maths were a food what type of food would it be?'. The experiences of students, and their reactions to them, were collected over a sustained period. The authors argued that the scope of the data obtained was far richer than would be gathered using more traditional methods.

Using the internet to administer surveys has also become increasingly feasible and popular. Even with limited resources, larger and geographically diverse samples can be reached promptly and efficiently. For example, building on information gathered through a face-to-face survey, Forgasz, Leder, and Tan (2014) used Facebook to reach an international sample to explore prevailing societal beliefs and attitudes in different countries about the gendering of mathematics and related issues. Mathematics, they reported from the data gathered in nine countries, was invariably believed to be important and worthy of continued study even when it was no longer a compulsory subject. Although the level of gender stereotyping of mathematics by parents varied somewhat by country, the authors concluded that "the consistency in the direction of the findings in support of the traditional male stereotype provides strong evidence that gendered perceptions of mathematics and related careers persist in many parts of the world" (p. 386). This finding, they added, serves yet again, as an important reminder that external factors should not be discounted by those aiming to counter gender stereotyped beliefs about students' proficiency in, and attitudes about, mathematics learning.

As foreshadowed earlier in this section, reliance on Facebook for sample recruitment can raise ethical issues as well as questions about the reliability and validity of the findings obtained. Advantages and disadvantages of this mode of recruitment are considered constructively by Kosinski, Matz, Gosling, Popov, and Stillwell (2016). How the ethics committee at a major international university wrestled with potentially additional ethical considerations associated with sample recruitment via social networking sites such as Facebook is described in some detail in Forgasz, Tan, Leder, and McLeod (2017).

2.6.2 *"Virtual" Observations*

The Experience Sampling Method [ESM] developed by Csikszentmihalyi (1997) enables "a virtual film strip of daily activities and experiences" (p. 15) to be provided. It has been used to tap affective factors over an extended period and, if used creatively, can serve as a substitute for real time observations. Furthermore, the obtained data is in a format that can be analyzed and interpreted at length at a convenient time.

The ESM requires participants to respond to signals and, at the time of contact, record their activities and reactions on specifically designed Experience Sampling Forms (ESFs). In studies involving university students (Leder & Forgasz, 2004) and secondary mathematics teachers (Forgasz & Leder, 2006), mobile phone and computer technology were exploited to send simultaneous SMS (text) messages to

participants in diverse geographic locations to indicate that it was time to complete ESFs. Signals, sent multiple times a day over one week, yielded a rich body of data about the individuals' activities, both mathematics related and more broadly. Data about the daily activities of males and females, and their behavioural and affective responses to them, were gathered in a natural setting, via self-report data, and in a less obtrusive and less resource intensive manner than possible through sustained observations or shadowing." The examples presented here are indicative of the ways in which traditional survey type self-report measures and instruments have been modified and extended in ways not conceivable without now readily available technology.

2.7 Physiological and Neurological Measures

Elsewhere I have written (Leder, 1985) that attempts to incorporate physiological indicators of affect with respect to mathematics learning and achievement are not a recent phenomenon—see, for example Dreger and Aiken (1957) who incorporated a measure of electrical skin resistance in their long-ago investigation of mathematics anxiety in college students or the study by Dew, Galassi, and Galassi (1984) who monitored college students' heart rate, skin conductance level, skin fluctuations, as they worked on mathematics problems. At that time I also noted that the use of such measures, particularly in a regular classroom setting, was likely to remain limited, as indeed it has.

It is also appropriate to consider here some early research purported to be on mathematics learning and brain activity. Once again it is useful to turn to work presented at annual PME conferences. Three presentations (Fidelman, 1983; Ransley, 1983; Yeshurun, 1983) with promising titles, placed under the overall heading of "Learning theories" and the further subheading "Neuropsychological theories", are found in the 1983 proceedings. Brief excerpts from each of these presentations are useful to capture the scope of the research actually conducted.

2.7.1 *From the PME Archives—Again*

In a paper titled "'Real life' numeracy, arithmetical competence and prediction from a neuropsychological theory", Ransley checked the effectiveness of some simple tests, administered by research assistants, to predict individual differences in performance among young school students experiencing learning difficulties and concluded:

Luria's theory of brain functioning **has led** to the development of a simple psychometric model with obvious relevance to the classroom. Measurement of competence in the model can be carried out using tasks which do not require any special formal knowledge and do not demand any particular higher order intellectual functioning. (pp. 112–113, emphasis added)

Under the title “Learning of non-standard arithmetic and the hemispheres of the brain”, Fidelman presented his investigation of the interaction between the two hemispheres of the brain, inferred from students’ attempts at a variety of mathematical proving tasks. Hemispheric activity was measured using:

- Counting of tachistoscopically represented dots as a test for the right hemisphere, and
- Counting of signs appearing rapidly one after another as a test for the left hemisphere.

The students’ responses to different problems were described, with answers to some tasks interpreted to be examples of “an inhibition of the right hemisphere by the left” (p. 119) and answers to others as “the inhibition of the left hemisphere by the right” (p. 119).

The paper “Practical applications of psychomathematics and neuropsychomathematics” by Yeshurun began with the definition of both psychomathematics and neuropsychomathematics, with the latter explained as “the study of the connection between brain functioning and behaviour” (p. 120). These new branches of research, Yeshurun stated, are “in the juncture of at least four sciences: psychology, mathematics, physiology, and education. For this reason progress is difficult and slow ... but there are some results. We want to deal here with their practical applications only” (p. 120). Reference was made to the importance of adapting teaching methods to promote the learning of new material, and to the role putatively played by the right or left hemisphere in processing material. Yet no direct evidence was presented to indicate how this might actually be achieved, or stimulated, in practice.

To summarize, while collectively these presentations represent, at a somewhat primitive level, tenuous efforts to capitalize indirectly on advances in cognitive neuroscience, the impact of affective factors on mathematics learning was not really considered implicitly or explicitly.

2.7.2 Some Contemporary Examples

While still attracting only limited research attention, the relevance of cognitive neuroscience to mathematics education, and within it the affective domain, is now increasingly recognized (see, for example, Goswami, 2004; Schlöglmann, 2008; Tzur & Leikin, 2015). That neuroscience can be used to improve education is enthusiastically advocated by Pincham et al. (2014). An issue faced by a teacher wishing to optimize her students’ learning of mathematics is used as an example of how this could be done. More generally, the authors recommended a four-stage process with the following elements: identify an educational need; develop a research proposal; test what is proposed in the classroom; communicate the results and, neuroscientist and teacher together, evaluate the outcome. Critically, they recommended, “(e)ducational neuroscientists must work with educators to draw on the educators’ wealth of practical knowledge regarding existing classroom practices and the feasibility of the proposed

project” (Pincham et al., 2014, p. 29) within the school setting. More widespread adoption of such a collaborative and interdisciplinary approach—not necessarily restricted to neuroscience—is a recommendation that can be fruitfully followed by those engaged in research on affect and mathematics education. A measure of how far research on affect per se has progressed in recent years is summed up evocatively by Cooper, Blackman and Kelly (2016):

Surely, when Thurstone pronounced that attitudes can be measured, he had no inkling that people’s evaluations would be assessed with changes in oxygenated blood flow or with event-related potentials measured at the scalp. Social neuroscience has opened new windows to the study of social processes in general, and attitudes in particular, that were not conceivable a few decades ago (p. 282)

The challenge to benefit, draw, and build on these newly opened windows is one which should not go unheeded by the mathematics education community. Collaborative, interdisciplinary work is likely to be required if mathematics education researchers, students, and teachers are to reap the full benefits of the new approaches pioneered in other fields.

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