

# Chapter 7

## Developing Mathematical Reasoning Using a STEM Platform



Andrzej Sokolowski

**Abstract** An interdisciplinary laboratory activity involved modelling and interpreting the motion of a rolling ball through the lens of algebraic representation. It was conducted with a group ( $N = 24$ ) of high school mathematics students. The participants used scientific methods to formulate an algebraic representation of a position for a rolling object on a horizontal surface. While traditional mathematical modelling activities are usually driven by provided data, the technique applied in this study is driven by the phenomenon itself, which serves as a means to verify if the derived algebraic function adheres to the observed behaviour. The results of the study showed that including scientific methods in mathematics interdisciplinary activities may serve as a means to activate, and stimulate, students' reasoning skills, and thus help them integrate the concepts of science and mathematics into a single coherent inquiry. While the study revealed benefits of using hypotheses in interdisciplinary activities, it also opened possibilities of utilizing interdisciplinary laboratories to improve students' mathematical thinking. Suggestions for instructional strategies, as well as suggestions for mathematics curriculum policy makers, are discussed.

**Keywords** Mathematical reasoning · Scientific inquiry · Modelling · STEM · Hypothesis

### 7.1 Introduction

Honey, Pearson, and Schweingruber (2014, p. 31) contend that, “learning science entails learning to express the behaviour of natural systems as mathematical models, making this form of integration not merely supportive of but indispensable to learning science.” However, the knowledge about a use of mathematics in science education is still fragmentary, and consensus on how to increase the rôle of mathematics in science, especially in physics, is not reached (Uhden, Karam, Pietrocola, & Pospiech, 2012). This study is an attempt to generate a learning experience that would exemplify the

---

A. Sokolowski (✉)  
Lone Star College, Houston, USA  
e-mail: [andrzej.sokolowski@lonestar.edu](mailto:andrzej.sokolowski@lonestar.edu)

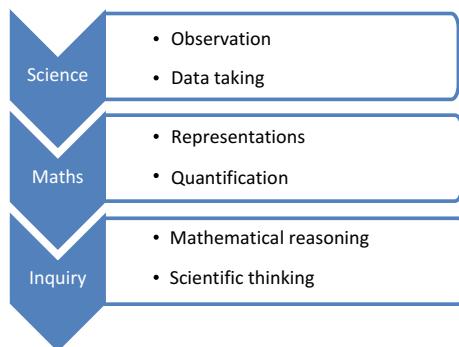
rôle of mathematics in science, and simultaneously provide a means of developing students' mathematical reasoning skills. More specifically, the objective of this study is to justify students' mathematical reasoning skills based on how they justified their selection of an algebraic function to describe the position of a rolling basketball.

Mathematical reasoning is characterised by activities such as looking for, and exploring, patterns to understand mathematical structures, and using available resources to solve problems (Schoenfeld, 1992). Mathematical reasoning, merged with scientific conduct possesses the capacity of advancing students' inquiry skills beyond memorisation of facts and procedures, and lead the learners to creating new knowledge (Sokolowski, 2018a). Hypothesis constitutes one of the initial stages of the scientific conduct. Although stating a hypothesis proposes an explanation, the way the investigator formulates and supports the hypothesis, can vary depending on his, or her, mathematical and scientific background. It is considered that the content of a hypothesis can serve as an instrument for justifying the level of the learners' mathematical and scientific reasoning skills and generate suggestions for its advancement.

Observation, experiment, discovery, and conjecture, are as much a part of the practice of teaching and learning mathematics as of any natural science (National Council of Teachers of Mathematics [NCTM], 2000). Several studies (e.g., Berlin & Lee, 2005) have shown that integrated curricula provide opportunities for more relevant learning experiences than traditional teaching methods. Interdisciplinary mathematical activities, that offer contexts during which students apply theorems in practice through experimentation, observation, and conjecture, present an excellent platform for following this recommendation. Through interdisciplinary activities, for example, modelling the periodic motion of an object attached to the spring, or using the motion of two carts, to investigate properties of a system of equations, students engage in observation, discovery, and algebraic model formulation by identifying patterns and using established criteria. Consequently, such experiences can lead to increasing motivation to learn both mathematics and science, and generate positive learner attitudes. Despite a wide diversity of learning opportunities provided by interdisciplinary education, the potential for developing students' mathematical reasoning skills, using interdisciplinary activities, is under-represented. This study attempted to determine whether using scientific methods, and more specifically, scientific hypotheses, in an interdisciplinary mathematics activity, can challenge students' mathematical reasoning.

Interdisciplinary mathematics education encompasses multidimensional types of integrated learning. One of the types of interdisciplinary activities is a block of Science, Technology, Engineering, and Mathematics (STEM). In this study, students will experience merging the attributes of a linear function with the properties of motion with a constant velocity. The activity will be conducted in a mathematics class. While STEM can merge several disciplines, research shows that mathematics is not fully exercised in that paradigm (English & King, 2015; Tytler, Prain, & Peterson, 2007), therefore attempts to increase its contribution are made. Since the primary type of inquiry in STEM activities gravitates toward scientific methods, the question that arose was whether inducing these methods in mathematical STEM activities,

**Fig. 7.1** Proposed pathway to develop students' inquiry skills

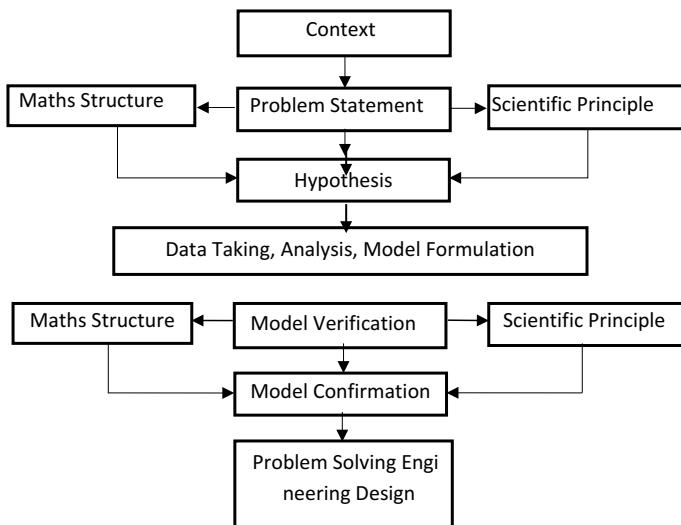


could serve as a catalyst for fostering students' mathematical reasoning. Although, at first, the idea seemed to propagate further, the scientific methods in mathematics, rather than mathematical reasoning in STEM, experimenting with such a pragmatic framework appeared to be a promising endeavour benefiting, not only students' mathematical thinking, but also helping them develop scientific inquiry skills, and a general disposition to undertake more complex STEM projects. The idea of merging the tools of mathematics with science, during this activity, is schematically illustrated in Fig. 7.1.

While the details of the pathway are further developed in Fig. 7.2, the main difference between what traditional mathematical modelling is, and what the proposed method offers, is the inductive character of the inquiry, and a constant intertwining between the contents under investigation. Application of mathematics is not driven by the provided data, but by observation and by a mapping of the behaviour of the physical quantities with the best fit to an algebraic function. This study will focus on investigating how students merge mathematics with science while formulating hypotheses for their investigations.

## 7.2 Theoretical Framework of the Activity Design

The activity was formulated using an integrated modelling scheme (see Fig. 7.2), that the author had previously developed, by meta-analysis of findings of the learning effects of interdisciplinary modelling activities (Sokolowski, 2015). It was argued that mathematics and science concepts need to be explicitly flagged, and linked, during the activities to enable a deeper analysis of their relationships and to strengthen their mutual interpretation. While the final product of the investigation is an algebraic function, the knowledge that students are to gain from the investigation is of a dual nature; the enacted algebraic representation is to serve as a means to learn more about the scientific nature of the experiment.



**Fig. 7.2** Integrated STEM modelling cycle (Sokolowski, 2015)

Mathematical activities can be structured in various ways, typically supported by mathematical modelling cycles. Analysis of several such cycles, (see, e.g. Greefrath & Vorhölter, 2016; Maaß, 2006) revealed that these cycles often lead students to finding a unique solution to a given problem, rather than providing a means for developing their scientific and mathematical thinking. Since the idea of using scientific methods has begun to gain more attention in STEM education, thus also in STEM activities conducted in mathematics classes, several theoretical frameworks emerged. For example, Kelley and Knowles (2016) designed an integrated STEM framework that included scientific inquiry as one of its main pillars, Kennedy and Odell (2014) suggested that high-quality STEM education programmes should promote scientific inquiry, that includes both rigorous mathematics and science instruction. The literature offers more examples of schematic diagrams, yet detailed practical examples are rarely found.

While providing students with opportunities to use science contexts as a platform to apply the tools of mathematics is not a new idea, and literature provides resources on organizing such activities (e.g., Berlin & Lee, op cit), using a hypothesis, along with the goal of inducing it in interdisciplinary mathematical activities to develop mathematical reasoning, is rare (Sokolowski, op cit). Several researchers (e.g., Crouch & Haines, 2004; Diefes-Dux, Zawojewski, Hjalmarson, & Cardella, 2012) pointed out that students' skills in formulating hypotheses, organising the proof, and validating elicited mathematical structures, are weak, but they did not suggest, explicitly, how to improve these skills. Formulating a hypothesis as a prediction of how a system's outputs depend on the inputs, is linked to identifying independent and dependent variables. Research on modelling in mathematics (Carrejo & Marshall, 2007) has reported students' difficulties with identifying variables, and

classifying the variables, to formulate mathematical constructs. Inviting students to hypothesise about laboratory outcomes induces the idea of predicting the behaviour of the experiment, and thus, it makes the students view the physical quantities as components of certain algebraic structures (functions) whose behaviour is consistent with that of the experiment in a mathematical sense. Figure 7.2 illustrates the general framework that makes the stages of connecting mathematical and scientific aspects of a real laboratory more explicit.

It is suggested that the problem statement be provided by the instructor, and students' tasks are to extract, and then merge, relevant concepts from both disciplines into one symbolic representation that is consistent with scientific behaviour of the variables of interest, and with further hypothetical analysis of the scientific nature of the experiment, as viewed through the properties of the algebraic form. The framework explicitly highlights these stages where scientific principles, and mathematical structures, are to be extracted from the laboratory context and integrated into a symbolic representation.

The instructional support offered to students' prompts, and self-directs, actions that supported formulation of the symbolic (algebraic) form of the basketball's position. While the scheme shows the flow of tasks in one direction, the students were encouraged to repeat it if the verification process disqualifies the elicited algebraic model. Prompts for revisions were provided in the instructional support.

The main science concept employed in this experiment was the idea of motion with a constant speed. While in physics, the concept of velocity is used to describe rate of change of an object's position, in this laboratory, the concept of speed was applied because the students were not supposed to describe the basketball's direction of motion but its magnitude only. The idea of modelling an object's speed to challenge students' mathematical reasoning, was selected for the following reasons:

- (a) Students who took part in the study already possessed a conceptual background for this concept from their physics classes.
- (b) Speed (the scalar version of velocity) is also often applied in mathematics textbooks (see Larson, 2005).
- (c) The idea of motion with a constant speed carries rather low cognitive load, thus no extra introduction to the mathematical modelling of this type of motion was necessary.

Since in science or physics courses the students used the terms speed and velocity to describe object's rate of change of distance or position, the instructor explained the differences to assure a cohesive terminology between both disciplines.

By merging the knowledge of different disciplines, STEM exploratory activity usually offers students an opportunity of deriving new knowledge (Sokolowski, 2018b). What was the anticipated new knowledge generated during this activity? In physics classes, students focus on investigating an object's constant rate of change of position over time, as evidence of uniform motion (e.g. Tipler & Mosca, 2007). However, in this interdisciplinary mathematics activity, in addition to hypothesising the form of the function based on the behaviour of the variables, the students constructed an algebraic function supported by observation, data taking, and graph



**Fig. 7.3** Epistemology of the lab activity

sketching. Students merged both streams of knowledge into one, coherent, representation, as depicted in Fig. 7.3.

Students were to formulate hypotheses for the stated problem, and also to reflect on the hypotheses by refuting or accepting them, considering the adherence of the data to the algebraic forms. Thus, while in physics, students investigate the properties of motion, and represent the properties with formulae, this activity represented the motion with algebraic functions, that provided more opportunities for extending the analysis of the motion beyond the physical domain of the experiment. As an example of such an extension, the students were required to use the functions to find either the time, or the position, of the rolling basketball. They were also to apply function transformations to construct new position functions based on pre-arranged new conditions. The purpose of this inclusion was to increase the interdisciplinary character of the investigation. More specifically:

- (a) Have the students build a tangible image of the experimental design as a basis for constructing new representations that would eventually help them with problem-solving.
- (b) Provide applications of function transformations in non-traditional context.

A short discussion is needed with the students about assuring that the motion of the basketball will be close to uniform. The ball will slow down due to air resistance and other factors, like the floor not being of a uniform texture. Because the ball will roll, rolling friction is needed and it helps to roll. To assure a uniform speed, the students might be asked to push the ball so that it moves 20 m but take the time measurements over first 10 m of its motion.

### 7.2.1 *Conduct of the Laboratory*

Before the laboratory activity, the students reviewed the basic properties of linear functions, along with sketching, and finding the constant rate, when the values of quantities were given. Due to time constraints, the properties of motion with a constant velocity, as seen from the physics point of view, were not explicitly reviewed, nor were they taught.

The teacher opened the lesson by informing the students that they would observe the motion of a rolling basketball, and then hypothesise the form of the function equation, record data (i.e., measure the time the rolling basketball passed certain

points), and formulate the function equation. The teacher demonstrated the motion in class, then, he handed out instructional supports to each student. After observing the motion, but still in the classroom, the students were invited to formulate their hypotheses. This arrangement was to assure that the students' hypotheses were individually formulated, thus, the data validity was secured. After the students finished formulating their hypotheses, the teacher divided the class into four groups of six students per group. Each group received four stopwatches, a tape measure, and a basketball. The teacher explained that each group must create a set of at least four co-ordinates that would depict the motion of the basketball with time and position as the variables. The distance of the motion of the ball was 10 m. Thus, some groups worked in the hallway. After plotting the points on a position-time axes, the students were asked to select a function, based on the data distribution, and then compute the coefficient of determination and correlation coefficient for the selected functions to justify, statistically, the choice of function. The teacher emphasised that although the data gathered by the groups would be the same, the best-fit curves, as sketched by hand, and their equations, might vary. The teacher briefly reviewed the technique of sketching the line of best fit. Once the students started organizing the process of data collection, the teacher took on the role of facilitator.

### 7.2.2 *Methods*

This study can be classified as one-group quasi-experimental (Shadish, Cook, & Campbell, 2002). Randomisation of participants was not possible due to the low school population in which the study was conducted. A quasi-experimental study shares many similarities with experimental design.

The study attempted to address the following research question: Can a verbal formulation of a hypothesis, and its verification, serve as a means of developing students' mathematical reasoning?

The participants in the study consisted of a group of 24 mathematics students (11 males and 13 females, age range 16–17 years) from a suburban high school. These students did not have a formal prior experience with modelling activities in their mathematics classes, nor with writing hypotheses. Five of these students (21%) were concurrently taking a physics course, and they had previously studied different types of motion in their physics class. The evaluation instrument consisted of an analysis of students' verbal formulation of hypotheses. The quality of the verbal responses served as an indicator of their quality of mathematical reasoning. The students then observed the experiment, collected data, and formulated algebraic representations that described the process of the experiment. They sketched the best-fit line by hand, and formulated the equation of the function. They also used technology to find a respective regression line. They computed the coefficient of determination for the best-fit line, reflected on the precision and accuracy of the data, and their hypotheses. Furthermore, they attempted to identify the sources of incorrect prior thinking in cases where their justifications were not supported by the data and the derived models.

## 7.3 Data Analysis

Students were expected to provide a hypothesis for the following problem: What type of function can be used to model the position of a basketball rolling along a floor if air resistance is ignored? Support your answer using your mathematics and science background.

All students selected a linear function to model the position of the rolling basketball. The depth of conceptual support, and the extent to which the students used mathematical and scientific terminology and reasoning, showed a range of diversity of reasoning across the students. Since the purpose of the study was to find out whether formulating hypotheses could be used to justify students' mathematical reasoning, the quality of the responses was not quantified but was clustered in two groups to reflect their common features. The students were asked to support their hypotheses using explicitly scientific and mathematical terminology, but their responses blended the terminology, of both of these disciplines, in verbal structures. The question that arose during the analyses of the responses was how to differentiate between students' scientific and mathematical reasoning? Is a different terminology used (scientific versus mathematical) sufficient? If so, would using, for example, constant rate, constitute mathematical or scientific reasoning? The literature about justifying the difference between mathematical and scientific reasoning is limited. As a reference, I have used schemes developed by national review physics committees. Viewing these recommendations, it was decided that making these distinctions was not necessary, because, for example, the term constant rate is used in both mathematics and science. Thus, to complete the preliminary analysis, separate categories for these responses, were not used, but rather, the quality and depth of the individual response were considered as a factor for reasoning evaluation. This analysis resulted in formulating two response groups. Group 1 constituted responses with more accurate properties of linear functions, such as, linear, or rate, or slope, and sufficient scientific support, like constant speed, and constant rate of change of position (these are summarised verbatim in Table 7.1) and Group 2 (see Table 7.2) included samples of more general responses, sometimes stating an overall dependence of the variables of interest.

### 7.3.1 Descriptive Analysis

Among the 24 responses, six students (25%) showed that they explicitly considered the properties of linear functions along with the principle of motion with a constant speed to support the function selection. These students (e.g., Student 2, 3, and 4 listed in Table 7.1) merged the attributes of linear functions effectively with patterns of observable motion, thereby making the justifications precise. They extracted the independent and dependent variables from the experiment, and embedded the variables in the algebraic rule formulation. The responses listed in Table 7.2

**Table 7.1** Responses that included properties of linear functions

Student	Response
1	If the ball is rolled on the floor, there would be a linear equation that has a value of b of zero with a positive slope
2	The function shown by basketball rolling on the floor should be linear because for every unit of time, the distance the basketball rolls increases by the same magnitude. Also, there is no horizontal acceleration, so the ball should be moving at a constant speed if the surface is frictionless
3	The function to represent the data would be linear because the position and time will increase at a constant rate
4	The ideal function to model the basketball position would be a linear function. If the force of friction is neglected, the ball would roll at a constant rate, making a linear function the best way to connect the points
5	A linear function because the rate of change will be constant
6	The graph will be linear due to the rate

**Table 7.2** Responses that did not include properties of linear functions

Student	Response
1	As the ball rolled along the floor, the ball would have a strong positive correlation because as each second passed the metres increased, until someone caught the ball
2	Just because the ball will eventually slow down, then that does not mean that a regression line will be a square root function or quadratic function
3	The line on the graph would be most likely linear because the amount of time it takes the ball to roll further would always go up
4	The function should be linear because the distance the basketball moves increases by the same magnitude
5	If the basketball runs across the floor, ignoring air resistance without stopping, the data shall compute only a linear function
6	The ball should plateau at some point because gravity forces the ball to stop rolling
7	If basketball is rolled at a constant speed, ignoring air resistance, then the data will produce a linear graph
8	It will be linear because as the time increases so would the distance unless there is a significant amount of friction
9	As the ball rolls, there are forces that act upon it causing it to gradually reduce its speed to a complete stop. Therefore, the equation would be a linear
10	If the ball increases in the distance, then time increases

**Table 7.3** Percentages of students using function attributes

Algebraic terms	Number of students and percent of the sample
Linear	18 (75%)
Constant	6 (25%)
Rate	5 (21%)
Slope	2 (8%)
Function	11 (44%)

show that the main attribute of linear functions—that is, a constant inclination (gradient) representing a constant speed or constant rate of change of distance over time—was not used very often by the students. This weakness might imply insufficient conceptualisation of the slope idea when it was first introduced to the students. While some students extracted the concept of a linear relation, between the ball's position and time, see for example Student 3 (Table 7.2), they did not convert these statements into a more precise mathematical representation. The remaining part of the group ( $N = 18$ , 75%) supported their claims even more loosely.

Five students from this group (28%), suggested using an increasing function due to an increase in the basketball's position which was correct. Their supporting statements though, lacked the specific terms that would warrant the application of the linear function. Thus, their claims were not evaluated, as satisfactory. Motion attributes, such as a constant rate, represented by a constant slope, were also not visible among this group of responses. The support statements were vague, showing these students' weaknesses in understanding the conceptual merit, and attributes, of linear functions. Table 7.3 summarizes the percentage of responses using the specific properties of linear functions for the entire group of students ( $N = 24$ ).

The students were also encouraged to compute the coefficient of determination, for the data distribution, to support their function selection, and to justify their hypotheses. The coefficient computed by using technology was high and in the range of 80–92% for all groups.

The number of students, who conceptualised their selected function using sound mathematical terminology, ranged from 8% ( $N = 2$ ) who used the term slope, to 75% ( $N = 18$ ) who used the term linear. Perhaps the percentage would be higher if these students were instructed on what function attributes to look for, or how to use these attributes, to verbalize such justifications.

### 7.3.2 Inferential Analysis

One of the themes that emerged from the study was the effect of the interdisciplinary background—in this laboratory about kinematics—on supporting students' mathematical reasoning. It was anticipated that to mark the activity as a productive learning

experience, no scientific introduction of the idea of uniform motion was necessary. The laboratory results were not consistent with this assumption. While all students formulated the position function for the rolling basketball correctly, the way that they verbalised the scientific part of their responses, revealed a lack of sufficient scientific terminology and understanding of different types of motion. For example, Student 6 (Table 7.2) said, “The ball should plateau at some point because gravity forces the ball to stop rolling,” which is not entirely true, because it is the strength of resistance (e.g. the air resistance) that stops the basketball. While the force of friction is dependent on the force of gravity through the normal force exerted on the moving ball, the direction of the force of gravity was perpendicular to the ball’s velocity and therefore it had no direct bearing on the ball’s speed.

Hypotheses reflect closely on the problem under investigation. Thus, while formulating hypothesis in an interdisciplinary setting, students are expected to recall prior knowledge and provide a viable answer coherent with mathematical constraints and the natural behaviour of the system under investigation. Asking students to state hypothesis should help them to focus on a narrower area of inquiry. Since interdisciplinary mathematics activities can be dynamic, students can also be required to classify the quantities as dependent or independent. The integrated modelling scheme (see Fig. 7.1) suggested that students apply mathematical reasoning to describe the behaviour of the phenomena and support the description by using scientific reasoning. The task of merging both disciplines while formulating a hypothesis was explicitly addressed in the instructional support provided to the students.

The author believes that providing mathematics students with diverse cases to extract correct forms of algebraic functions, or their attributes, or theorems, in interdisciplinary activities, can serve as a means of improving their mathematical reasoning skills. The challenge is the extent to which similar tasks can be designed to assist teachers to understand the development of mathematical reasoning using interdisciplinary learning, and how to extract the prompts of high-quality reasoning skills from students’ work. The question of interest can also be the balance between, purely abstract mathematical understandings, and, the ability to apply the understanding in contexts. For instance, how to quantify students’ understanding of the Mean Value Theorem using the idea of a pure algebraic function versus using, for example, a velocity function and subsequent acceleration function? To what extent can mathematical reasoning be taught and be developed without a context? Alternatively, how do we measure the learning effects when pure algebraic representations are given, as opposed to using context to uncover the representations, and then analyse them?

## 7.4 In Search of Improving the Learning Experience

Analysing students’ responses, we have learned that reviewing, or even introducing, scientific background would benefit students’ learning experiences, and perhaps more efficiently help them integrate the knowledge of both disciplines. Therefore, discussing the contexts in more detail, prior to the laboratory is recommended. This

idea, is further supported, by considering the manner in which students, who were concurrently taking a physics course, formulated their hypotheses (there were five such students in the group, 21%). These students' responses (see Table 7.1, Student 2, 3, 4, 5, and Table 7.2, Student 8) displayed a higher level of understanding of motion, and their justifications for using linear functions to model uniform motion, were more consistent with the conditions of the motion. One could conclude that all students' reasoning skills would be deeper if a relevant review about properties of motion were provided. While the assumption that all students would realize that a constant speed calls for a linear function to be applied, the laboratory showed that not all students had that understanding. The laboratory also contained a section where the students reflected on their hypotheses, and provided possible sources of error.

The scientific method, by which scientists endeavour to construct an accurate representation of the world, consists of the following four elements: observation, formulation of a hypothesis that proposes an explanation of the phenomenon, the performance of an experiment, and the testing of the hypothesis. During the investigation, the rôle of a hypothesis is to confirm, or correct, an investigator's understanding of what is the content of the experiment. A hypothesis can also be perceived as a provisional idea that requires evaluation. For a valid assessment, it needs to be defined in operational terms referring to the specifics of the investigation, and it usually requires a formal scientific experiment designed and organised by the investigator. A confirmed hypothesis has the potential to become a law, and it is often expressed in a mathematical form (Simon, 2012). Moreover, a hypothesis can be perceived as the investigator's proposed theory, explaining why something happens, based on the researcher's prior knowledge (Felder & Brent, 2004). Thus, to increase the prestige of students' work, they can be asked to formulate a theory based on their discoveries. This task would serve as a factor strengthening their confidence, and encourage their own investigations. Formulating and proving, or disproving, a hypothesis might take various forms; for example, it can involve a statistical procedure, as developed by Fisher (1955), in which a null hypothesis is formulated and tested statistically to be rejected, or accepted, depending on whether results fall within an established confidence interval. Since, using statistical methods requires advanced analytical apparatus, this might not be accessible to all high school mathematics students, thus, these methods were not employed, but instead, students formulated their hypotheses verbally, using their mathematical and scientific background. A hypothesis can also take the form of existential statements claiming that some instances of the phenomenon under investigation have some characteristics and causal explanation (Popper, 2005). As an existential statement, a hypothesis can be formulated verbally and proved, or disproved, following the scientific approach, and this form was employed in this laboratory.

The action of formulating a hypothesis is closely related to developing a prediction. While a hypothesis proposes an explanation for some puzzling observation, a prediction is defined as an expected quantitative outcome of a test of some elements of the hypothesis (Lawson, Oehrtman, & Jensen, 2008). Being of a quantitative manner, prediction can also be included in mathematical activities to support a hypothesis, and justify, for example, students' estimation skills. In this laboratory, the students

were asked to predict the magnitude of the speed in metres per second based on the observable motion. The range of their predictions varied more than expected, but this was accounted for by the fact that in everyday experience, these students use feet per second rather than metres per second to describe the rate of change of distance.

## 7.5 Discussion

The purpose of the laboratory was to have students experience the process of merging the content of mathematics and science in an interdisciplinary setting and generate new knowledge merging both disciplines. From a scientific point of view, students experienced that, motion with a constant speed, implies a constant slope. From a mathematics perspective, they learned that a constant rate of change, of quantities of interest, warrants application of linear functions.

This merging was to help students view the content of mathematics, not as a static subject bounded by procedures and facts, but as a dynamic process of creating new knowledge grounded in natural settings. Provision of instructional support guided the students through the merging stage, as well as supplying them with opportunities to reflect on their thought processes, while leaving room for their input. The idea of conducting the interdisciplinary activity was intended to assist students to link both disciplines, and the task of formulating hypotheses, was meant to provide prompts for evaluating the students' mathematical reasoning skills. Evidently, having students state a hypothesis, and reflect on it, served as a means of activating their thinking. While the students formulated their hypotheses before working on the activity, there is still an unanswered question about how the experience augmented their reasoning skills, in particular of those students whose responses were listed in Table 7.2. The question of how to best formulate the problem statement, or the prompts, during the activity, so that students' engagement and learning is maximized is not completely answered. Did observing the experiment, collecting the data, formulating the algebraic representation, and proving, or disproving, the hypothesis, deepen the integration of knowledge? Did it help to emphasise the importance of the mathematical aspect, or did it emphasise a discovery of motion with a constant speed? While either of the goals would be satisfactory, evaluating these effects would help with designing other interdisciplinary laboratories. The preliminary results of the study seem to be encouraging, but more such activities need to be designed and conducted to better reflect on the learners' cognitive processes.

There are several general recommendations that emerged from this study. First, there seems to be a need for developing assessment instruments that would evaluate students' skills of merging the concepts of mathematics with extracted quantities from interdisciplinary settings, while simultaneously promoting their creativity and reasoning. It is apparent that evaluating students' procedural skills does not provide the desired evidence. All of the students formulated the algebraic function correctly, however, based on their prior experiences, reasoning skills of only six of these students were rated as satisfactory. Being successful on solving problems that often

require finding a unique solution, does not necessarily prove adequate reasoning skills, but simply just proves that the student has adequate algebraic skills. Typical problem solving, which often requires memorisation of procedures, does not seem to support reasoning in a sense that students would use in their science classes. Designing mathematical assessment items that require the subtle area of merging mathematics with interdisciplinary, physical quantities, is a task worthy of further development.

The laboratory activity also revealed that, to encourage mathematical reasoning, there should be more emphasis on using function properties, or attributes, to select the most appropriate representation for given data. Statistical tests are necessary, but they do not provide a bridge between observable, or measurable, variables, and their graphical depiction, and the task of using statistical tests, often performed by technology, do not nurture the bridging. There should be review sections in mathematics textbooks designated to contrast various algebraic concepts, and challenge their applicability, and their limitations in different real situations. Sketching functions and finding their equations without context perfects procedural skills, but context-driven scenarios move the learning to a more sophisticated level. In such settings, the procedural skills will serve as a prelude to a deeper conceptual understanding of mathematical concepts, rather than an absolute means of generating the final product.

The element of hypothesizing, in this activity, had a dual purpose: it was intended to lead the students' thinking processes and serve as an instrument to justify their mathematical reasoning, and thus provide prompts for laboratory improvements. The effects of developing a hypothesis, along with conducting a scientific process, do not exhaust the learning objectives that an interdisciplinary laboratory can target. Simply stating that a hypothesis is correct, or incorrect, limits students' input and their share in the conduct of the laboratory. An interdisciplinary laboratory provides ample opportunity for students to reflect more deeply on their prior learning experiences, and correct, or modify, these experiences, if needed. How to accomplish this goal? After concluding the laboratory, students should be provided with problem-solving questions that would link the acquired experience with textbook problems, or other assessments, not necessarily referring to mathematics textbooks.

Finally, the data analysis revealed that more could be done in highlighting the interdisciplinary principle under investigation prior to the laboratory. Thus, supplying students with more tools, that they could have at their disposal, for the laboratory, would improve the laboratory outcomes, and would very likely be appreciated by the learners. For example, the categorisation of variables, as dependent or independent, is often related to plotting functions in the Cartesian plane, and such categorisation is explicitly given to students in their mathematics classes. Such a level of categorisation is not sufficient for successful integration of mathematics with other disciplines. It is seen that students' deeper understanding of the principle under investigation is necessary, to make the distinctions in real contexts. An interdisciplinary relationship provides the added benefit of improving students' general learning experiences, beyond the sole domain of mathematics. This feature however, cannot be easily implemented in mathematics daily routines because the curricula, scope, and sequences, are strictly designed. Perhaps, interdisciplinary activities should be conducted dur-

ing an independently formulated subject, called for example mathematical reasoning, where more time could be devoted to developing the contextual background and intertwining the concepts. Such courses, organised in parallel with mathematics, appear promising, and worthy of further consideration.

While hypotheses in mathematical interdisciplinary activities will most likely be verbalised with the aim of testing mathematical structures, and reflecting on how well the structures describe a system's behaviour, there is a need for a more detailed discussion on how to formulate a hypothesis in interdisciplinary mathematics activities. The final products of activities in mathematics classes can take various forms, ranging from formulating abstract mathematical representations to building artefacts. In the proposed integrated modelling cycle (see Fig. 7.1), it is assumed that the final product will be a mathematical representation, often in the form of an algebraic function. In this study, the stance was that the way the learner supports the hypothesis, along with using function attributes, and merging them with the system behaviour, can be used to justify students' mathematical reasoning as it pertains to understanding applications of linear functions to motion problems. Hypothesising in mathematics requires a conceptual understanding of functions, or ratios and proportions, domain, range, formulation of maximum, limits, intercepts, and so forth. For instance, if students are to formulate a function representing Newton's second law of motion, then in mathematics classes, their hypotheses will attempt to answer questions about the type of algebraic function that can be used to describe the mathematical relation between an object's acceleration and the net force. Students might predict a linear relation, because if the net force increases, so will the object's acceleration. A plotted graph will resemble a linear function, and its slope will show as the proportionality constant or as the object's mass. How will this process differ from what students learn in their science classes? In science classes, students hypothesise the type of proportionality that can be formulated due to investigating change of object's acceleration when the net force acting on it changes (Tipler, op cit). The term function, as studied in mathematics classes, is rarely used in science, and the scientific hypotheses will merely require the students to find if the relation is directly or indirectly proportional. Formulations of these hypotheses are mutually inclusive, and one can be perceived as a complement to the other, yet their natures are subject-domain dependent, and they appear to take a narrow meaning when they are discipline bounded. A proposed method of integrating the contents is to have students merge the hypotheses into one statement, consistent with mathematical terminology and rules, and with scientific laws.

It seems that merging both disciplines in a unified learning experience, and projecting the phenomenon analysis, using the methods of all subjects involved simultaneously, is a way to knowledge exploration and acquisition.

### 7.5.1 *Suggestions for Further Research*

The students contended that their hypotheses were correct, focusing on predicting linear graphs for the motion. They showed a high level of inaccuracy in time recording (the values of the coefficient of determination computed using a graphing calculator ranged from 80 to 92%) that affected the speed computations. They assumed that stating the statistical value of the error was sufficient, and therefore they did not attempt to correct their thinking, although this was assumed to take place.

While this deficiency can be eliminated, by designing a laboratory when the time could be detected by using a photo-gate, the answer how to correct or improve students' reasoning remains open. Will the students enhance their reasoning by conducting more similar interdisciplinary laboratories, or will they improve the reasoning by having the instructor pinpointing elements of the laboratory analysis and computations that were determinant to enrich their algebraic thinking?

The purpose of the study was to investigate students reasoning based on how they formulated their hypotheses, not how they corrected their thinking, therefore it was considered that the study served its purpose. Yet, the effects of interdisciplinary activities can be extended to include the measure of change, perhaps by interviewing students before, and after, the laboratory, and extracting changes in reasoning. This is certainly an area to pursue further.

The tasks of hypothesizing a possible graph for identified variables in a given experiment, along with providing verbal support, are included in science textbooks (see, for example, Etkina, 2010, p. 54) and they are included on national physics examinations (see AP Physics). According to reviewers, physics students have difficulties with producing such representations. More specifically, they do not only face difficulties with identifying and classifying variables, in a mathematical sense, but also with selecting a correct function, given by a curvature, or plot. These concepts are not frequently exercised in mathematics classes, although they represent core mathematical ideas. Instead, students analyse pure functions without contexts that probably supports learning by rote, not genuine understanding. Such teaching does not link mathematics with real world situations, and does not help students with applying these ideas in other subjects. Consequently, such teaching does not help the learners to broaden their view about applications of mathematics, and does not encourage them to study mathematics for understanding. Learners need to be provided with contexts, and be invited to extract the applications of theorems studied, from contexts. Can science communities be invited to initiate a better alignment of the mathematics they use in their science classes with mathematics that students study in their mathematics classes? There is certainly room for a change.

The task of verbalizing attributes of algebraic functions using contexts is silent in mathematical curricula, and it is seen that more attention could be designated to develop this skill. Students need to realize that each type of function has its own rules for sketching, or finding intercepts. More importantly, they need to realize that function attributes (e.g. existence of a horizontal asymptote, or a piece-wise nature) classify the functions to describe different phenomena, and vice versa, that a specific

property of the phenomenon's behaviour, will demand a specific algebraic representation. In typical mathematics textbooks, problems on certain kinds of function are usually clustered at the ends of chapters according to the types of function, or ideas, that the functions describe. Perhaps providing a diversity of assessments, where students would extract and pin-point specific attributes; e.g., constant percent rate, a local maximum or constant period of motion, would offer students more room to reason mathematically when context is provided.

The nature of the mathematical tools and systems of representation, available to students, determine the depth and breadth of learning about core ideas in science because mathematical forms correspond to forms of understanding natural systems (Quinn, Schweingruber, & Keller, 2012). It is expected that confirmed algebraic representations, resulting from similar investigations, can be further used to delve deeper into behaviour of scientific systems, thereby triggering opportunities for producing creative thinkers ready to generate new knowledge. This study, and also others from prior research, shows that students need to have a sufficient scientific background to be able to take advantage of the full range of learning benefits, that the method of integrating, offers. While verifying with students if certain concepts were covered, in the respective science class is a suggestion, this might not apply to students who are not concurrently enrolled in that particular science course. What would be the best solution? It seems that creating a separate course that would allow teachers, to devote extra time for bringing up the scientific contexts, would benefit the learners the most.

There are feasible ways of improving the quality of teaching and learning mathematics and science. It is hoped that this study offers suggestions that are worthy of being given further consideration.

## References

- Berlin, D. F., & Lee, H. (2005). Integrating science and mathematics education: Historical analysis. *School Science and Mathematics, 105*(1), 15–24.
- Carrejo, D. J., & Marshall, J. (2007). What is mathematical modelling? Exploring prospective teachers' use of experiments to connect mathematics to the study of motion. *Mathematics Education Research Journal, 19*(1), 45–76.
- Crouch, R., & Haines, C. (2004). Mathematical modelling: Transitions between the real world and the mathematical model. *International Journal of Mathematical Education in Science and Technology, 35*(2), 197–206.
- Diefes-Dux, H. A., Zawojewski, J. S., Hjalmarson, M. A., & Cardella, M. E. (2012). A framework for analyzing feedback in a formative assessment system for mathematical modeling problems. *Journal of Engineering Education, 101*(2), 375.
- English, L. D., & King, D. T. (2015). STEM learning through engineering design: Fourth-grade students' investigations in aerospace. *International Journal of STEM Education, 2*(1), 1.
- Etkina, E. (2010). Pedagogical content knowledge and preparation of high school physics teachers. *Physics Review, 6*(2), 020110.
- Felder, R. M., & Brent, R. (2004). The intellectual development of science and engineering students. Part 2: Teaching to promote growth. *Journal of Engineering Education, 93*(4), 279.

- Fisher, R. (1955). Statistical methods and scientific induction. *Journal of the Royal Statistical Society. Series B (Methodological)* 17(1), 69–78.
- Greefrath, G., & Vorhölter, K. (2016). *Teaching and learning mathematical modelling: Approaches and developments from German speaking countries*. New York: Springer International.
- Honey, M., Pearson, G., & Schweingruber, H. (Eds.). (2014). *STEM integration in K-12 education: Status, prospects, and an agenda for research*. Washington, D.C.: National Academies Press.
- Kelley, T. R., & Knowles, J. G. (2016). A conceptual framework for integrated STEM education. *International Journal of STEM Education*, 3(1), 1–11.
- Kennedy, T., & Odell, M. (2014). Engaging students in STEM education. *Science Education International*, 25(3), 246–258.
- Larson, R. (2005). *PreAlgebra*. Evanston, IL: McDougal Littell.
- Lawson, A. E., Oehrtman, M., & Jensen, J. (2008). Connecting science and mathematics: The nature of scientific and statistical hypothesis testing. *International Journal of Science and Mathematics Education*, 6(2), 405–416.
- Maaß, K. (2006). What are modelling competencies? *ZDM Mathematics Education*, 38(2), 113–142.
- NCTM. (2000). *Principles and standards for school mathematics*. Reston, VA: National Council of Teachers of Mathematics.
- Popper, K. (2005). *The logic of scientific discovery*. New York: Routledge.
- Quinn, H., Schweingruber, H., & Keller, T. (Eds.). (2012). *A framework for K-12 science education: Practices, crosscutting concepts, and core ideas*. Washington, D.C.: National Academies Press.
- Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics. In D. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 334–370). New York: Macmillan.
- Shadish, W. R., Cook, T. D., & Campbell, D. T. (2002). *Experimental and quasi-experimental designs for generalised causal inference*. Boston: Houghton Mifflin.
- Simon, H. A. (2012). *Models of discovery: And other topics in the methods of science* (Vol. 54). Berlin: Springer Science & Business Media.
- Sokolowski, A. (2015). The effects of mathematical modelling on students' achievement, a meta-analysis of research. *IAFOR Journal of Education*, 3(1), 93–114.
- Sokolowski, A. (2018a). Modeling acceleration of a system of two objects using the concept of limits. *Physics Teacher*, 6(1), 40–41.
- Sokolowski, A. (2018b). *Scientific inquiry in mathematics—Theory and practice: A STEM perspective* (1st ed.). New York: Springer.
- Tipler, P. A., & Mosca G. (2007). *Physics for scientists and engineers, Vol. 1, 6th: Mechanics, oscillations and waves, thermodynamics*. New York: Freeman Company.
- Tytler, R., Prain, V., & Peterson, S. (2007). Representational issues in students learning about evaporation. *Research in Science Education*, 37(3), 313–331.
- Uhlen, O., Karam, R., Pietrocola, M., & Pospiech, G. (2012). Modelling mathematical reasoning in physics education. *Science & Education*, 21(4), 485–506.

**Open Access** This chapter is licensed under the terms of the Creative Commons Attribution 4.0 International License (<http://creativecommons.org/licenses/by/4.0/>), which permits use, sharing, adaptation, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons licence and indicate if changes were made.

The images or other third party material in this chapter are included in the chapter's Creative Commons licence, unless indicated otherwise in a credit line to the material. If material is not included in the chapter's Creative Commons licence and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder.

