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Stability Theory for Hybrid Dynamical Systems

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Abstract

This entry provides a short introduction to modeling of hybrid dynamical systems and then focuses on stability theory for these systems. It provides definitions of asymptotic stability, basin of attraction, and uniform asymptotic stability for a compact set. It points out mild assumptions under which different characterizations of asymptotic stability are equivalent, as well as when an asymptotically stable compact set exists. It also summarizes necessary and sufficient conditions for asymptotic stability in terms of Lyapunov functions.

Keywords Hybrid system • Asymptotic stability • Basin of attraction • Lyapunov function

Introduction

A hybrid dynamical system combines continuous change and instantaneous change. Instantaneous change is the only type of change available for variables like counters, switches, and logic variables. Instantaneous change may also be a good approximation of what occurs to velocities in mechanical systems at the time of an impact with a wall, floor, or some other rigid body. At other times, velocities evolve continuously. Continuous change is also natural for position variables, continuous timers, and voltages and currents. For mathematical convenience, it is typical in the analysis of hybrid dynamical systems to embed all of these variables into a Euclidean space, with the understanding that many points in the state space will never be reached. For example, a logic variable that naturally takes values in the set $\{\text{off}, \text{on}\}$ is typically embedded in the real number line where its two distinct values are associated with two distinct numbers, the only numbers that this variable will visit during its evolution.

A finite-dimensional dynamical system that exhibits continuous change exclusively is typically modeled by an ordinary differential equation, or sometimes a more flexible differential inclusion. A system that exhibits purely instantaneous change is typically modeled by a difference equation or inclusion. Consequently, a hybrid dynamical system combines a differential equation or inclusion with a difference equation or inclusion. A big part of the modeling effort for hybrid systems is directed at determining which type of evolution should be allowed at each point in the state space. To this end, subsets of the state space are specified where each type of behavior is allowed, like in the description of the heating system given above.

Though the behavior of a hybrid dynamical system can be quite complex and nonconventional, it is still reasonable to ask the same stability questions for them that might be asked about classical differential or difference equations. Moreover, the same stability analysis tools that are used for classical systems are also quite useful for hybrid dynamical systems. The emphasis of this entry is on basic stability theory for hybrid dynamical systems, focusing on definitions and tools that also apply to classical systems.

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Mathematical Modeling

System Data

A hybrid dynamical system with state x belonging to a Euclidean space $\mathbb{R}^n$ combines a differential equation or inclusion, written formally as $\dot{x} = f(x)$ or $\dot{x} \in F(x)$, with a difference equation or inclusion $x^+ = g(x)$ or $x^+ \in G(x)$, where $\dot{x}$ indicates the time derivative and x^+ indicates the value after an instantaneous change. The mapping f or F is called the *flow map*, while the mapping g or G is called the *jump map*. A complete model also specifies where in the state space continuous evolution is allowed and where instantaneous change is allowed. The set where continuous evolution is allowed is called the *flow set* and is denoted C, whereas the set where instantaneous change is allowed is called the *jump set* and is denoted D. The overall model, using inclusions for generality, is written formally as

$$x \in C \qquad \dot{x} \in F(x) \tag{1a}$$

$$x \in D \qquad x^+ \in G(x). \tag{1b}$$

Solutions

It is natural for solutions of (1) to be functions of two different types of time: a variable t that keeps track of the amount of ordinary time that has elapsed and a variable j that counts the number of jumps. There is a special structure to the types of domains that are allowed. A *compact hybrid time domain* is a set $E \subset \mathbb{R}_{\geq 0} \times \mathbb{Z}_{\geq 0}$, that is, a subset of the product of the nonnegative real numbers and the nonnegative integers, of the form

$$E = \bigcup_{i=0}^{J} ([t_i, t_{i+1}] \times \{i\})$$

for some $J \in \mathbb{Z}_{\geq 0}$ and some sequence of nondecreasing times $0 = t_0 \leq t_1 \leq \cdots \leq t_{J+1}$. It is possible for several of these times to be the same, which would correspond to more than one jump at the given time. A *hybrid time domain* is a set $E \subset \mathbb{R}_{\geq 0} \times \mathbb{Z}_{\geq 0}$ such that for each $(T, J) \in E$, the set $E \cap ([0, T] \times \{0, \ldots, J\})$ is a compact hybrid time domain. In contrast to a compact hybrid time domain, a hybrid time domain may have an infinite number of intervals, or it may have a finite number of intervals with the last one being unbounded or of the form $[t_J, t_{J+1})$; that is, it may be open on the right. A *hybrid arc* is a function x, defined on a hybrid time domain, such that $t \mapsto x(t, j)$ is locally absolutely continuous for each j; in particular, $t \mapsto x(t, j)$ is differentiable for almost every t where it is defined, and this mapping is the integral of its derivative. The notation "dom x" denotes the domain of x. Finally, a hybrid arc is a *solution* of (1) if the following two properties are satisfied:

1. For $\varepsilon > 0$, $(s, j), (s + \varepsilon, j) \in \operatorname{dom} x$ implies that $x(t, j) \in C$ and $\dot{x}(t, j) \in F(x(t, j))$ for almost all $t \in [s, s + \varepsilon]$.
2. $(t, j), (t, j + 1) \in \operatorname{dom} x$ implies that $x(t, j) \in D$ and $x(t, j + 1) \in G(x(t, j))$.

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For a hybrid system with no flow dynamics, each solution has a time domain of the form $\{0\} \times \{0, \ldots, J\}$ for some $J \in \mathbb{Z}_{\geq 0}$ or $\{0\} \times \mathbb{Z}_{\geq 0}$. For a hybrid system with no jump dynamics, each solution has a time domain of the form $[0, \infty) \times \{0\}$, $[0, T] \times \{0\}$, or $[0, T) \times \{0\}$ for some $T \geq 0$. No assumptions are made in this entry to guarantee existence of nontrivial solutions since stability theory does not hinge on existence of solutions; rather, it simply makes statements about the behavior of solutions when they exist. To ensure robustness of various stability properties, the following basic regularity assumptions are usually imposed.

Assumption 1 The data (C, F, D, G) satisfy the following conditions:

1. The sets C and D are closed.
2. The set-valued mapping F is outer semicontinuous, locally bounded, and $F(x)$ is nonempty and convex for each $x \in C$.
3. The set-valued mapping G is outer semicontinuous, locally bounded, and $G(x)$ is nonempty for each $x \in D$.

To elaborate further, a set-valued mapping, like F, is said to be *outer semicontinuous* if for each convergent sequence $\{(x_i, y_i)\}_{i=0}^{\infty}$ that satisfies $y_i \in F(x_i)$ for all $i \in \mathbb{Z}_{\geq 0}$, its limit, denoted (x, y), satisfies $y \in F(x)$. It is said to be *locally bounded* if for each bounded set $K_1 \subset \mathbb{R}^n$ there exists a bounded set $K_2 \subset \mathbb{R}^n$ such that, for every $x \in K_1$, every $y \in F(x)$ belongs to K_2; the latter condition is sometimes written $F(K_1) \subset K_2$. If C is closed, f is a function $f : C \to \mathbb{R}^n$ that is continuous, and F is a set-valued mapping that has the single value $f(x)$ for each $x \in C$ and is empty for $x \notin C$, then F is outer semicontinuous, locally bounded, and $F(x)$ is nonempty and convex for each $x \in C$.

Stability Theory

Definitions and Relationships

Given a dynamical system, predicting or controlling the system's long-term behavior is of primary importance. A system's long-term behavior may be more complicated than just converging to an equilibrium point. This fact motivates studying stability of and convergence to a set of points. For simplicity, this entry focuses on stability of sets that are *compact*, that is, they are closed and bounded. A variety of stability concepts are defined below. Each of these concepts applies to continuous-time or discrete-time systems as readily as to hybrid systems.

A compact set $\mathcal{A} \subset \mathbb{R}^n$ is said to be *Lyapunov stable* for (1) if for each $\varepsilon > 0$ there exists $\delta > 0$ such that for every solution of (1), $x(0, 0) \in \mathcal{A} + \delta\mathbb{B}$ implies $x(t, j) \in \mathcal{A} + \varepsilon\mathbb{B}$ for all $(t, j) \in \text{dom}\, x$, where $\mathcal{A} + \delta\mathbb{B}$ indicates the set of points whose distance to the set $\mathcal{A}$ is less than or equal to δ. In order for a compact set to be Lyapunov stable for (1), it must be *forward invariant* for (1), that is, each solution of (1) with $x(0, 0) \in \mathcal{A}$ satisfies $x(t, j) \in \mathcal{A}$ for all $(t, j) \in \text{dom}\, x$. However, forward invariance does not necessarily imply Lyapunov stability.

For a compact set $\mathcal{A} \subset \mathbb{R}^n$, its *basin of attraction* for (1), denoted $\mathcal{B}_{\mathcal{A}}$, is the set of points from which each solution to (1) is bounded and each solution to (1) having an unbounded time domain converges to $\mathcal{A}$, the latter being written mathematically as $\lim_{t+j \to \infty} |x(t, j)|_{\mathcal{A}} = 0$ where $|x(t, j)|_{\mathcal{A}}$ denotes the distance of $x(t, j)$ to the set $\mathcal{A}$. Each point that does not belong to $C \cup D$ belongs to $\mathcal{B}_{\mathcal{A}}$ since there are no solutions from such points. A compact set $\mathcal{A}$ is said to be *attractive*

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for (1) if its basin of attraction contains a neighborhood of itself, that is, there exists $\varepsilon > 0$ such that $\mathcal{A} + \varepsilon\mathbb{B} \subset \mathcal{B}_{\mathcal{A}}$. A compact set $\mathcal{A}$ is said to be *globally attractive* if $\mathcal{B}_{\mathcal{A}} = \mathbb{R}^n$.

A compact set is said to be *asymptotically stable* for (1) if it is Lyapunov stable and attractive for (1). A compact set is said to be *globally asymptotically stable* for (1) if it asymptotically stable for (1) and $\mathcal{B}_{\mathcal{A}} = \mathbb{R}^n$. It is useful to know that the basin of attraction for an asymptotically stable set is always open.

Theorem 1 *Under Assumption 1, if a compact set is asymptotically stable for (1), then its basin of attraction is an open set.*

A compact set $\mathcal{A} \subset \mathbb{R}^n$ is said to be *uniformly attractive* for (1) if it is attractive for (1) and for each compact set $K \subset \mathcal{B}_{\mathcal{A}}$ and each $\delta > 0$ there exists $T > 0$ such that for every solution x of (1), $x(0,0) \in K$ and $t + j \geq T$ imply $x(t,j) \in \mathcal{A} + \delta\mathbb{B}$. A compact set is said to be *uniformly globally attractive* for (1) if it is globally attractive and uniformly attractive for (1). Uniform attractivity goes beyond attractivity by asking that the amount of time it takes each solution to get close to $\mathcal{A}$ is uniformly bounded over initial conditions in compact subsets of the basin of attraction.

A compact set $\mathcal{A} \subset \mathbb{R}^n$ is said to *Lagrange stable* relative to an open set $O \supset \mathcal{A}$ for (1) if for each compact set $K_1 \subset O$ there exists a compact set $K_2 \subset O$ such that for every solution of (1), $x(0,0) \in K_1$ implies $x(t,j) \in K_2$ for all $(t,j) \in \operatorname{dom} x$. In Lagrange stability for the case $O = \mathbb{R}^n$, a bound on the initial conditions is given and a bound on the ensuing solutions must be found; this is in contrast to Lyapunov stability where a bound on the solutions is given and a bound on the initial conditions must be found.

A compact set is said to be *uniformly asymptotically stable* for (1) if it is Lyapunov stable, attractive, Lagrange stable relative to its basin of attraction, and uniformly attractive for (1). A compact set is said to be *uniformly globally asymptotically stable* for (1) if it is uniformly asymptotically stable for (1) and $\mathcal{B}_{\mathcal{A}} = \mathbb{R}^n$. There is no difference between asymptotic stability and uniform asymptotic stability under Assumption 1.

Theorem 2 *Under Assumption 1, a compact set is uniformly asymptotically stable for (1) if and only if it is locally asymptotically stable for (1).*

As noted earlier, forward invariance does not imply Lyapunov stability. However, when coupled with uniform attractivity, Lyapunov stability ensues.

Theorem 3 *Under Assumption 1, a compact set is uniformly asymptotically stable for (1) if and only if it is forward invariant and uniformly attractive for (1).*

Asymptotic stability can be converted to global asymptotic stability by shrinking the flow and jump sets to be compact subsets of the basin of attraction. However, global asymptotic stability of a compact set $\mathcal{A}$ for $x \in C, \dot{x} = f(x)$ for each compact set C does not necessarily imply global asymptotic stability of $\mathcal{A}$ for $\dot{x} = f(x)$.

In some situations it is easier to assert the existence of a compact asymptotically stable set than it is to find one explicitly. In this direction, given a set $X \subset \mathbb{R}^n$, consider the set of points z with the property that there exist a sequence of solutions $\{x_i\}_{i=0}^{\infty}$ to (1) with initial conditions in X and a sequence of times $\{(t_i, j_i)\}_{i=0}^{\infty}$ with $(t_i, j_i) \in \operatorname{dom} x_i$ for each $i \in \mathbb{Z}_{\geq 0}$ such that $z = \lim_{i\to\infty} x_i(t_i, j_i)$. This set of points is called the ω-limit set of X for (1) and is denoted $\Omega(X)$.

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Theorem 4 *Let Assumption 1 hold. For the system (1), if X is compact and $\Omega(X)$ is nonempty and contained in the interior of X (i.e., there exists $\varepsilon > 0$ such that $\Omega(X) + \varepsilon\mathbb{B} \subset X$), then the set $\Omega(X)$ is compact and uniformly asymptotically stable with basin of attraction containing X and equal to the basin of attraction for X.*

Robustness

A given model (C, F, D, G) may have some mismatch with a physical process that it aims to describe. One way to capture some of this mismatch is to consider the behavior of solutions to a system with inflated data $(C_\delta, F_\delta, D_\delta, G_\delta)$, $\delta \geq 0$, defined as follows:

$$C_\delta := \{x \in \mathbb{R}^n : (x + \delta\mathbb{B}) \cap C \neq \varnothing\} \tag{2a}$$

$$F_\delta(x) := \overline{\text{co}} F((x + \delta\mathbb{B}) \cap C) + \delta\mathbb{B} \tag{2b}$$

$$D_\delta := \{x \in \mathbb{R}^n : (x + \delta\mathbb{B}) \cap D \neq \varnothing\} \tag{2c}$$

$$G_\delta := G((x + \delta\mathbb{B}) \cap D) + \delta\mathbb{B}. \tag{2d}$$

The notation $x + \delta\mathbb{B}$ indicates a closed ball of radius δ centered at the point x. Evaluating a set-valued mapping at a set of points means to collect all vectors that belong to the set-valued mapping at any point in the set that serves as the argument of the set-valued mapping. The notation "$\overline{\text{co}} F((x + \delta\mathbb{B}) \cap C)$" indicates the closed, convex hull of the set $\{f \in \mathbb{R}^n : f \in F(z), z \in (x + \delta\mathbb{B}) \cap C\}$. Note that $(C_0, F_0, D_0, G_0) = (C, F, D, G)$. More generally, the components of (C, F, D, G) are contained in $(C_\delta, F_\delta, D_\delta, G_\delta)$. The inflation data in (2) satisfy the regularity properties of Assumption 1 when (C, F, D, G) do.

Proposition 1 *If the data (C, F, D, G) satisfy Assumption 1 then, for each $\delta > 0$, the inflated data $(C_\delta, F_\delta, D_\delta, G_\delta)$ satisfy Assumption 1.*

From the point of view of asymptotic stability, the behavior of solutions to $(C_\delta, F_\delta, D_\delta, G_\delta)$ for $\delta > 0$ small is not too different from those of (C, F, D, G).

Theorem 5 *Under Assumption 1, if $\mathcal{A}$ is asymptotically stable with basin of attraction $\mathcal{B}_\mathcal{A}$ for the hybrid system with data (C, F, D, G), then for each $\varepsilon > 0$ and each compact set K satisfying $K \subset \mathcal{B}_\mathcal{A}$, there exist $\delta > 0$ and a compact set $\mathcal{A}_\delta \subset \mathcal{A} + \varepsilon\mathbb{B}$ that is asymptotically stable with $K \subset \mathcal{B}_{\mathcal{A}_\delta}$ for $(C_\delta, F_\delta, D_\delta, G_\delta)$.*

The robustness result of Theorem 5 has several consequences beyond the observations in the preceding examples. One of the consequences is the following reduction principle.

Theorem 6 *Under Assumption 1, if $\mathcal{A}_1$ is asymptotically stable with basin of attraction $\mathcal{B}_{\mathcal{A}_1}$ for the hybrid system with data (C, F, D, G) and the compact set $\mathcal{A}_2 \subset \mathcal{A}_1$ is globally asymptotically stable for the hybrid system with data $(C \cap \mathcal{A}_1, F, C \cap \mathcal{A}_2, G)$, then the compact set $\mathcal{A}_2$ is asymptotically stable with basin of attraction $\mathcal{B}_{\mathcal{A}_1}$ for the hybrid system with data (C, F, D, G).*

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Lyapunov Functions

Arguably the most common method for establishing asymptotic stability is known as *Lyapunov's method* and uses a Lyapunov function. A function $V : \mathbb{R}^n \to \mathbb{R}_{\geq 0}$ is a *Lyapunov function candidate* for (1) if it is continuously differentiable on an open neighborhood of the flow set C, it is defined for all $x \in C \cup D \cup G(D)$ (dom V denotes the set of points where it is defined), and it is continuous on its domain. Some of these conditions can be relaxed but are imposed in this entry to keep the discussion simple. Given a compact set $\mathcal{A}$ and an open set O satisfying $\mathcal{A} \subset O \subset \mathbb{R}^n$, a Lyapunov function candidate for (1) is called a *Lyapunov function for* $(\mathcal{A}, O)$ if:

(L1) For $x \in (C \cup D \cup G(D)) \cap O$, $V(x) = 0$ if and only if $x \in \mathcal{A}$.
(L2) For each $x \in C \cap O$ and $f \in F(x)$, $\langle \nabla V(x), f \rangle \leq 0$.
(L3) For each $x \in D \cap O$ and $g \in G(x)$, $V(g) - V(x) \leq 0$.

A Lyapunov function for $(\mathcal{A}, O)$ is called a *proper Lyapunov function for* $(\mathcal{A}, O)$ if, in addition,

(L4) $\lim_{i \to \infty} V(x_i) = \infty$ when the sequence $\{x_i\}_{i=0}^{\infty}$, satisfying $x_i \in (C \cup D \cup G(D)) \cap O$ for all $i \in \mathbb{Z}_{\geq 0}$, is unbounded or approaches the boundary of O.

The next result does not use Assumption 1, though the rest of the results in this entry do.

Theorem 7 *Let $\mathcal{A} \subset O \subset \mathbb{R}^n$ with $\mathcal{A}$ compact and O open. If there exists a Lyapunov function for $(\mathcal{A}, O)$, then $\mathcal{A}$ is Lyapunov stable for (1). If there exists a proper Lyapunov function for $(\mathcal{A}, O)$ then $\mathcal{A}$ is also Lagrange stable with respect to O for (1).*

We can also conclude asymptotic stability from a Lyapunov function when it is known that there are no complete solutions along which the Lyapunov function is equal to a positive constant.

Theorem 8 *Let $\mathcal{A} \subset O \subset \mathbb{R}^n$ with $\mathcal{A}$ compact and O open. Under Assumption 1, if there exists a Lyapunov function for $(\mathcal{A}, O)$ and there is no solution x of (1) starting in $O \backslash \mathcal{A}$ that has an unbounded time domain and satisfies $V(x(t, j)) = V(x(0, 0))$ for all $(t, j) \in$ dom x, then $\mathcal{A}$ is uniformly asymptotically stable for (1). If the Lyapunov function is a proper Lyapunov function for $(\mathcal{A}, O)$, then the basin of attraction for $\mathcal{A}$ contains O.*

The simplest way to rule out solutions that keep a Lyapunov function equal to a positive constant is by finding a *(proper) strict Lyapunov function for* $(\mathcal{A}, O)$, which is a (proper) Lyapunov function for $(\mathcal{A}, O)$ that also satisfies:

(L2′) For each $x \in (C \cap O) \backslash \mathcal{A}$ and $f \in F(x)$, $\langle \nabla V(x), f \rangle < 0$.
(L3′) For each $x \in (D \cap O) \backslash \mathcal{A}$ and $g \in G(x)$, $V(g) - V(x) < 0$.

Theorem 9 *Let $\mathcal{A} \subset O \subset \mathbb{R}^n$ with $\mathcal{A}$ compact and O open. Under Assumption 1, if there exists a strict Lyapunov function for $(\mathcal{A}, O)$, then $\mathcal{A}$ is uniformly asymptotically stable for (1). If there exists a proper strict Lyapunov function for $(\mathcal{A}, O)$, then $\mathcal{A}$ is uniformly asymptotically stable for (1) with basin of attraction containing O.*

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While a strict Lyapunov function can be difficult to find, and this fact has motivated other more sophisticated stability analysis tools that have appeared in the literature, it is reassuring to know that whenever $\mathcal{A}$ is compact and asymptotically stable, there exists a proper strict Lyapunov function for $(\mathcal{A}, \mathcal{B}_{\mathcal{A}})$.

Theorem 10 *Under Assumption 1, if the compact set $\mathcal{A}$ is asymptotically stable for (1), then there exists a proper strict Lyapunov function for $(\mathcal{A}, \mathcal{B}_{\mathcal{A}})$. More specifically, for each $\lambda > 0$ there exists a smooth function V with $\operatorname{dom} V = \mathcal{B}_{\mathcal{A}}$ that $V(x) = 0$ if and only if $x \in \mathcal{A}$, $\lim_{i\to\infty} V(x_i) = \infty$ when the sequence $\{x_i\}_{i=0}^{\infty}$, satisfying $x_i \in \mathcal{B}_{\mathcal{A}}$ for all $i \in \mathbb{Z}_{\geq 0}$, is unbounded or tends to the boundary of $\mathcal{B}_{\mathcal{A}}$, and such that:*

1. *For all $x \in C \cap \mathcal{B}_{\mathcal{A}}$ and $f \in F(x)$, $\langle \nabla V(x), f \rangle \leq -\lambda V(x)$.*
2. *For all $x \in D \cap \mathcal{B}_{\mathcal{A}}$ and $g \in G(x)$, $V(g) \leq \exp(-\lambda) V(x)$.*

Summary and Future Directions

Under Assumption 1, stability theory for hybrid dynamical systems is very similar to stability theory for differential equations or difference equations with continuous right-hand sides. In particular, Lyapunov functions are a very common analysis tool for hybrid dynamical systems, though a Lyapunov function can be difficult to find in the same way that they are challenging to find for classical systems. With stability theory for hybrid dynamical systems firmly in place, future research is expected to exploit this theory more fully for the development of control algorithms with new capabilities.

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