A Generalized Recurrent Neural Network for Matrix Inversion

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Matrix inversion deals with computing the inverse matrix A^{-1} of a given $n \times n$ nonsingular matrix A , which has been widely used in a variety of applications such as control, robotics, and signal processing. This paper presents a generalized recurrent neural network for matrix inversion. The generalized recurrent neural network can use any monotone nondecreasing activation functions and its steady states are guaranteed to present the inverse matrix A^{-1} .

The dynamics of the recurrent neural network for matrix inversion can be described by the following matrix-valued differential and algebraic equations,

$$\begin{aligned}\frac{dV(t)}{dt} &= -\mu A^T U(t), \\ U(t) &= G[AV(t) - I],\end{aligned}$$

where A^T denotes the transpose of A , $V(t)$ and $U(t)$ are $n \times n$ activation state matrices of the neurons in the output and input layers respectively, $\mu > 0$ is a scalar gain parameter, $G[X] = [g_{ij}(x_{ij})]$ is a matrix of activation functions, and I is the identity matrix.

The architecture of the recurrent neural network consists of two layers and each layer consists of an $n \times n$ array of neurons. Each output neuron represents an element in the inverse matrix A^{-1} . The connection weight matrix from the input layer to the output layer is defined as $-\mu A^T$ and the connection weight matrix from the output layer to the input layer is defined as A . The biasing threshold (constant input) matrix in the input layer is defined as I and there are no biases for the neurons in the output layer. There is a functional transformation for each neuron in the input layer and an integral transformation for each neuron in the output layer.

The dynamical equations show that $v_{ij}(t)$ is connected with $u_{1j}(t), u_{2j}(t), \dots, u_{nj}(t)$ only and $u_{ij}(t)$ is connected with $v_{1j}(t), v_{2j}(t), \dots, v_{nj}(t)$ only. This pattern of connectivity shows the proposed recurrent neural network can actually be decomposed into n independent subnetworks. Each subnetwork represents one column vector of A^{-1} . The dynamical equations also indicate that the connection weight matrices are identical for each subnetwork.

Main Result: If $g_{ij}(x_{ij})$ is monotone nondecreasing with respect to x_{ij} (i.e., $dg_{ij}/dx_{ij} \geq 0$) and $g_{ij}(x_{ij}) = 0$ is equivalent to $x_{ij} = 0$ (i.e., $g_{ij}(x_{ij}) = 0$ iff $x_{ij} = 0$) for $i, j = 1, 2, \dots, n$, then the recurrent neural network for matrix inversion is asymptotically stable in the large and its equilibrium states represent the inverse matrix of a given nonsingular matrix (i.e., $\forall V(0) \in R^{n \times n}, \lim_{t \rightarrow \infty} V(t) = A^{-1}$).

The analytical result has been substantiated by simulation results.