Minimum Distance Pattern Classifiers Based On A New Distance Metric

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Abstract

A method for the design of minimum distance bipolar pattern classifiers based on a new distance metric between the bipolar patterns is described. The new distance metric is defined and its properties are demonstrated. Neural Networks with the Nearest-neighbor recall mechanism [2], trained according to this method, distinguish the images of a prototype pattern $\mathbf{X}^k$ from the images of the opponent prototype functions $\mathbf{X}^q$, taking into account not only the erroneous pattern elements but also their distinguishing ability. A Neural pattern classifier can be defined by R linear discriminant functions [1] $\{F_1(\mathbf{X}), \dots, F_R(\mathbf{X})\}$ with $F_k(\mathbf{X}) = \mathbf{X} * \mathbf{W}^k = x_1 * W_1^k + x_2 * W_2^k + \dots + x_n * W_n^k$ where, x_i are bipolar variables $x_i = +1, -1$ and the weights W_i^k are real numbers and $-1 \leq W_i^k \leq +1$. In the proposed neural net architecture, the discriminant function $F_k(\mathbf{X})$ is then, followed by an output function $T_k(F)$ defined as
$$T_k(F) = \begin{cases} 1 & \text{if } F_k(\mathbf{X}) > 1 - mND(\mathbf{X}^k)/2 \\ -1 & \text{otherwise} \end{cases}, \quad k = 1, \dots, R \text{ and } mND() \text{ is the new distance}$$

metric defined next. Let $\mathbf{X}^k = (x_1^k, x_2^k, \dots, x_n^k)$ be a prototype pattern with $k = 1, 2, \dots, R$ and $\mathbf{X} = (x_1, \dots, x_n)$ be an unknown pattern. Their Hamming distance $D(\mathbf{X}^k, \mathbf{X})$ can then be obtained from the Hamming Discriminant Function $H F_k(X) = x_1 * x_1^k + x_2 * x_2^k + \dots + x_n * x_n^k$ with $W_i^k = x_i^k$, $i = 1, \dots, n$, as $D(\mathbf{X}^k, \mathbf{X}) = (H F_k(\mathbf{X}^k) - H F_k(\mathbf{X}))/2$. Then, in the case of a finite set of R prototype patterns $\mathbf{X}^k$, $k = 1, \dots, R$, a new Discriminant Function $F_k(\mathbf{X}) = x_1 * W_1^k + \dots + x_n * W_n^k$ can be defined for every pattern $\mathbf{X}^k$, $k = 1, \dots, R$ with respect to its opponent patterns $\mathbf{X}^q$, $q = 1, \dots, R$ and $q \neq k$ as follows:

The weights W_i^k , $i = 1, \dots, n$ of the Discriminant Function $F_k(\mathbf{X})$ are calculated according to the formula

$$W_i^k = (x_i^k) * \left\{ \sum_{q \neq k} (1 - x_i^k * x_i^q) / (2 * R * D(\mathbf{X}^k, \mathbf{X}^q)) \right\}.$$

From the above formula we can see that the minimum value of a weight W_i^k will be equal to $W_i^k = 0$, in the case where $x_i^k = x_i^q$ for every $q = 1, \dots, R$ with $q \neq k$ and the maximum value of the weight W_i^k when $x_i^k = -x_i^q$ for every q , will be equal to $W_i^k = (2 * x_i^k) * \left\{ \sum_{q \neq k} 1 / (2 * R * D(\mathbf{X}^k, \mathbf{X}^q)) \right\}$. The properties of the new Discriminant Function can be demonstrated using the above formula of the weights W_i^k written as $W_i^k = (x_i^k) * \left\{ \sum_{q \neq k} 1 / (2 * R * D(\mathbf{X}^k, \mathbf{X}^q)) \right\} - \left\{ \sum_{q \neq k} (x_i^q) / (2 * R * D(\mathbf{X}^k, \mathbf{X}^q)) \right\}$.

According to the above, the output of the Discriminant Function $F_k(\mathbf{X}^k)$ for the pattern $\mathbf{X}^k$ will be equal to the maximum value $F_k(\mathbf{X}^k) = 1$. Also, the output of the Discriminant Function $F_k(\mathbf{X}^{k'})$ for the complement $\mathbf{X}^{k'}$ of the prototype pattern $\mathbf{X}^k$ will be equal to the minimum value $F_k(\mathbf{X}^{k'}) = -1$. Finally, the minimum New Distance $mND(\mathbf{X}^k)$ between pattern $\mathbf{X}^k$ and its opponent patterns $\mathbf{X}^q$, $q = 1, \dots, R$, $q \neq k$ will be equal to $mND(\mathbf{X}^k) = 1 - \max\{F_k(\mathbf{X}^q)\}$, $q \neq k$.

References

- [1] N. Nilsson. *Learning Machines*. McGraw-Hill, 1965.
- [2] P. K. Simpson. *Artificial Neural Systems*. Pergamon Press, 1990.