Recursive Construction of Neural Networks with Long Periodic Behavior¹

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Extended Abstract

A neural network of size n , is a discrete dynamical system acting on $\Omega_n = \{-1, 1\}^n$, whose transition function, F_A , is given in term of an $n \times n$ real matrix $A = (a_{ij})$ as follows:

$$F_A(x) = \overline{sgn}(Ax); \quad (Ax)_i = \sum_{j=1}^n a_{ij} x_j \quad i = 1, \dots, n$$

$$\overline{sgn} : \mathbb{R}^n \rightarrow \{-1, 1\}^n, \quad \overline{sgn}(y)_i = sgn(y_i) \quad i = 1, \dots, n \quad sgn(u) = \begin{cases} 1 & u \geq 0 \\ -1 & u < 0 \end{cases}$$

We are interested in the reverberation neural networks, i.e. neural networks where each state of the system, after a finite number of steps become to himself (hypercube permutations). We study the question: *how many reverberation neural networks have really different dynamical behavior ?*. A related question is asked in [2] where it is defined an equivalence relation whose equivalence class are characterized and the number of elements in any class is proved to be $2^n n!$ where n is the size of the neural network, but nothing is done concerning the number of different class.

Our approach consists in to look the neural networks, in particular reverberation neural networks, as dynamical systems. In order to give a partial answer to our question, we define an equivalence relation which is as the equivalence relation of dynamical systems and we proved, by building recursively 2^n non equivalent neural networks, that there exists at least 2^n different class when one considers this relation in the set of the reverberation neural networks of size n .

We give a way to build from a threshold function $f : \Omega_n \rightarrow \Omega_1$ another threshold function $g : \Omega_{n+1} \rightarrow \Omega_1$ such that g acts over a vector $(x, u) \in \Omega_{n+1}$, excepte for four vector y_1, y_2, y_3, y_4 , as if only the n first coordinates, x , were considered, and then giving the value $f(x)$, moreover, the values $g(y_i), i = 1, 2, 3, 4$ are fixed by the construction. So, one can describe the dynamical evolution of g easily in term of those of f . Moreover, we show how to obtain threshold functions having an *a priori* desired behavior.

From these two constructions we give a recursive way for the construction of matrices, i.e., given a matrix A of size n satisfying some hypthesis we build two matrices B and C of size $n + 1$, satisfying also those hypthesis.

Later, we define an equivalence relation on P_n , the set of bijective function on Ω_n and we build a function η associating to each element in P_n a vector of size 2^n . We prove that two function F and G are equivalent iff $\eta(F) = \eta(G)$. Hence, B and C given by the construction define non equivalent neural networks because $\eta(F_B)$ and $\eta(F_C)$ are different. Later, we prove that given two non equivalent neural networks A and A' their extentions are also non equivalent. This fact says that growing the size of the neural networks by one unit, one can double the non equivalent neural network number. That explains because we find 2^n non equivalent neural networks.

Like a corollary, we build a neural network A of size n which has only a cycle of period 2^n .

Reference

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[2] M.Sato, C. Tanaka, (1982)

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